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PRINCIPLES OF ENGINEERING THERMODYNAMICS

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PREFACE

This work is presented as a basic and fairly comprehensive analysis of these aspects of Thermodynamics which are of concern in the design and operation of the many varieties of power producing and transforming machines which serve our modern civilization. The basic nature of the analysis is believed to conform with the trend in engineering education toward an increasing emphasis upon the fundamental sciences which underlie the multitudinous technical specialties.

The general character of the treatment is indicated by the following summarization of the five Parts into which the work is divided. Part I considers the First Law of Thermodynamics, with particular emphasis on an understanding of the distinctive characteristics of the several stored and transient forms of energy and of the energy equation as it applies to the innumerable steady-flow machines which are encountered in engineering practice. Part II considers the Second Law and the Carnot Principle, with emphasis on the availability of energy and the associated physical significance of the entropy function as an index of the unavailability of energy. Part III describes the physical properties of the vapors and gases and their mixtures. Part IV utilizes the principles and methods of the preceding parts in analyses of motive-power machinery and certain power-using apparatus, with attention both to ideal performances and to the character and reasons for the departure of actual performances from the ideal. For the interest of those who may be concerned with the correlation of the physical properties of fluids or with the further applications of thermodynamics in physics and chemistry, Part V develops the General Thermodynamic Equations.

The authors wish to record their sincere appreciation of valued suggestions and criticisms contributed by Professor C. Harold Berry, Dr. Edgar Buckingham, Dr. G. E. Grantham, Professor D. Kavanaugh, Dr. F. G. Keyes and Dr. S. A. Moss, and the late Professor G. A. Goodenough.

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NOTE TO THE STUDENT

Although the authors have employed every effort within their ability to present this work not only accurately but understandingly, it still remains that the student must contribute liberally in clear and logical thought. The subject is one which may not be learned simply by rote but may be mastered only by processes of consistent reasoning. The student is admonished therefore to approach the subject as one from which he may draw benefits only in proportion to the intelligent thought which he may devote to it. This is particularly true with respect to Parts I and II. A real understanding of those Parts, together with the information which is presented in Part III, will enable him to proceed easily and confidently through the remainder.

To facilitate the recognition by the student of the more important items which are presented in any chapter, a summarizing article is provided at its end, in which are epitomized the essential features of the chapter. Directly following that article a group of review questions and topics is provided. The student may well regard these as a means for self-test of his understanding of the preceding material. Numerous illustrative and test examples appear throughout the work. For convenience there is provided at the end of each chapter a tabular compilation of the symbols which have been employed in the chapter.

NOTE TO THE INSTRUCTOR

The instructor will find that in various general and detailed features the material departs somewhat from that which has been regarded as conventional. It is believed that the departures that have been made will be found advantageous. Reference to the Preface and to Article 103 will serve to familiarize the reader with the general scheme of development of the subject.

As a consequence of the adopted scheme of development the covering of the first two or three Parts may perhaps require more time than is usually devoted to their material. It is found, however, that remarkable dividends are paid by this time expenditure both in time savings in the subsequent developments and in engendering of a real understanding of the applications of the fourth Part. It might be remarked that the work has been so arranged that by making suitable omissions the instructor may readily adapt it to courses of varying objectives and scope.

The values that are employed for the properties of steam are those of the recent Keenan Steam Tables, published by the American Society of Mechanical Engineers. The symbols that are employed are in the main those recently adopted by the American Standards Association. By reason both of its rapidly increasing use and its definite pedagogical advantage, the term *enthalpy* has been used to designate that property which commonly has been given the misleading names total heat, heat content, et cetera.

GENERAL SYMBOLS

(At the end of each chapter there will be found a list of all general and special symbols which appear in the chapter.)

A = area, square feet.

C = coefficients, as of discharge.

c = specific heat.

D = density, pounds mass per cubic foot.

E = internal energy, B.t.u. per pound mass.

F = force, pounds.

H = enthalpy (total heat), B.t.u. per pound mass.

J = Joule's equivalent, 778 ft.-lb. per B.t.u.

k = ratio of specific heats (c_p/c_v).

M = mass, pounds.

M' = mass-rate, pounds per second.

m = molecular weight.

P = absolute pressure, pounds per square foot.

p = absolute pressure, pounds per square inch.

Q = energy transferred as heat (by conduction and radiation), B.t.u. per pound mass.

R = gas constant.

S = entropy per pound mass.

T = absolute temperature.

t = ordinary temperature.

U = velocity, feet per second.

V = specific volume, cubic feet per pound mass.

W = shaft-work, foot-pounds per pound mass.

CONTENTS

PART I

ENERGY

CHAPTER I

	PAGE
STORED ENERGY.....	1
1. General Classifications. 2. Potential Energy. 3. Kinetic Energy. 4. Composition of Matter. 5. Internal Energy. 6. Chemical Energy. 7. Properties. 8. Pressure. 9. Specific Volume. 10. Temperature. 11. Summary. 12. Review Questions.	

CHAPTER II

ENERGY IN TRANSITION.....	19
13. Processes. 14. Work. 15. Processes involving Work. 16. Heat. 17. Convection. 18. Processes involving Heat. 19. Reversibility. 20. Mechanical Reversibility. 21. Thermal Reversibility. 22. The Conservation of Energy. 23. Units of Energy. 24. Specific Heat. 25. Summary. 26. Review Questions.	

CHAPTER III

ENERGY EQUATIONS.....	42
27. Systems. 28. Energy Classifications. 29. Energy Equations. 30. Steady-Flow. 31. Steady-flow Energy Equation. 32. Enthalpy. 33. Applications of Steady-flow Equation. 34. Non-flow Energy Equation. 35. Applications of Non-flow Equation. 36. $-\int P dV$; Mechanical Effects. 37. Summary. 38. Review Questions.	

PART II

THE AVAILABILITY OF ENERGY

FOREWORD.....	71
---------------	----

CHAPTER IV

REVERSIBLE CYCLES.....	73
40. Cycles. 41. Reversible Cycles. 42. Carnot Cycle. 43. Reversal of Carnot Cycle. 44. Summary. 45. Review Questions.	

	PAGE
CHAPTER V	
THE CARNOT PRINCIPLE.....	83
46. The Carnot Principle.	
CHAPTER VI	
THE FUNDAMENTAL TEMPERATURE SCALE.....	88
47. The Kelvin Scale. 48. Realization of Kelvin Scale in Practical Thermometry. 49. Absolute Temperature. 50. Efficiency of Reversible Temperature Engine Cycles. 51. Summary. 52. Review Questions.	
CHAPTER VII	
ENTROPY, THE INDEX OF UNAVAILABILITY.....	103
53. Q_R/T_2 , Change of Entropy. 54. Entropy as a Property. 55. Entropy as a Coördinate. 56. The Entropy of Energy. 57. Summary. 58. Review Questions.	
PART III	
PROPERTIES OF FLUIDS	
FOREWORD.....	124
CHAPTER VIII	
LIQUIDS, VAPORS AND LIQUID-VAPOR MIXTURES.....	126
60. A Liquid and its Vapor. 61. Saturated Liquids and Vapors. 62. Superheated Vapor. 63. Subcooled Liquid and Wet Vapor. 64. Graphical Representations. 65. Simple State Changes. 66. Constant Volume Process. 67. Constant Pressure Process. 68. Reversible Adiabatic Process. 69. Irreversible Adiabatic Process. 70. Throttling Process. 71. Throttling Calorimeter. 72. Summary. 73. Review Questions.	
CHAPTER IX	
THE PERFECT GAS.....	155
74. P - V - T Relation. 75. Internal Energy. 76. ΔE as a Function of ΔT . 77. Enthalpy Change. 78. Relation between c_p and c_v ; between ΔH and ΔE . 79. E and H as Functions of P , V and k . 80. Entropy of Gas. 81. The Gas Constant. 82. Specific Heats. 83. State Changes. 84. Constant Volume Process. 85. Constant Pressure Process. 86. Isothermal Process. 87. Reversible Adiabatic Process. 88. Irreversible Adiabatic Process. 89. Throttling Process. 90. Polytropic Process. 91. Graphical Representations. 92. Summary. 93. Review Questions.	
CHAPTER X	
GAS AND GAS-VAPOR MIXTURES.....	190
94. Gas Mixtures. 95. Gas Constant of Mixtures. 96. Specific Heats of Mixtures. 97. State Changes of Gas Mixtures. 98. Gas and Vapor Mixtures. 99. Humidity. 100. State Changes of Gas-vapor Mixtures. 101. Summary. 102. Review Questions.	

PART IV

APPLICATIONS

	PAGE
FOREWORD.....	220

CHAPTER XI

FLOW THROUGH THE NOZZLE, ORIFICE, ET CETERA.....	222
104. General Forms of Nozzle and Orifice. 105. Energy Equation. 106. Nozzle Form for Reversible Expansion of Gas. 107. Critical Pressure. 108. Flow of Gas through Orifice. 109. Influence of Approach Velocity. 110. Actual Flow of Gas. 111. Venturi Meter. 112. Impact Tube and Pitot Tube. 113. Summary, Gas Flow. 114. Nozzle Form for Ideal Expansion of Vapor. 115. Actual Expansion of Vapor. 116. Flow of Vapor through an Orifice. 117. Diffuser and Ejector. 118. Summary, Flow of Steam. 119. Review Questions.	

CHAPTER XII

STEAM POWER PLANT CYCLES.....	273
120. Heat Engine Cycles. 121. The Rankine Cycle. 122. Thermal Efficiency, Rankine Cycle and Rankine Engine. 123. Factors Influencing Rankine Cycle Efficiency. 124. Modern Cycles. 125. Regenerative Feed-Heating Cycle. 126. Reheating and Reheating-Regenerative Cycles. 127. Multiple Fluid Cycles. 128. Type Efficiency. 129. Engine Efficiency and Steam Rate. 130. Actual Thermal Efficiency and Heat Rate. 131. The Energy ("Heat") Balance. 132. Summary. 133. Review Questions.	

CHAPTER XIII

RECIPROCATING STEAM ENGINE.....	323
134. Ideal Reciprocating Engine. 135. The Engine Indicator Diagram. 136. Incomplete Expansion Engine. 137. Non-expansion Engine. 138. Actual Cylinder Action. 139. Engine Governing. 140. Compounding. 141. The Uniflow Engine. 142. Summary. 143. Review Questions.	

CHAPTER XIV

STEAM TURBINE.....	350
144. General Characteristics and Classifications. 145. General Energy Relations. 146. Impulse Blading; Energy Analysis. 147. Impulse Blading; Kinematic Analysis. 148. Multi-staging; Impulse Turbine. 149. Pressure Staging. 150. Velocity Staging. 151. Reaction Turbine; Energy Analysis. 152. Reaction Turbine; Physical Characteristics. 153. The Condition Curve. 154. Summary. 155. Review Questions.	

CHAPTER XV

	PAGE
COMBUSTION.....	390
156. Quantitative Chemistry of Combustion. 157. Interpretation of the Analysis of Combustion Products. 158. Heating Value of Fuels. 159. Energy Equation of Combustion; Temperature Developed. 160. Distribution of Combustion Losses. 161. Summary. 162. Review Questions.	

CHAPTER XVI

INTERNAL COMBUSTION ENGINE.....	423
163. General Characteristics and Classifications. 164. Engine Energy Equation. 165. Otto Cycle, Efficiency Analysis. 166. Diesel Cycle, Efficiency Analysis; Dual Combustion. 167. Power Capacity, Engine Cylinder. 168. <i>T-S</i> Analysis of Engine Cycles. 169. Summary. 170. Review Questions.	

CHAPTER XVII

COMPRESSORS AND BLOWERS.....	459
171. Compressor Classifications. 172. Compressor Energy Equations and General Energy Analysis. 173. Isothermal Compression. 174. Adiabatic Compression, Single and Multi-stage. 175. Graphic Representations. 176. Index of Compressor Performance. 177. Mechanical Performance, Reciprocating Compressor. 178. Volumetric Efficiency, Reciprocating Compressor. 179. Centrifugal Compressor Performance. 180. Summary. 181. Review Questions.	

CHAPTER XVIII

REFRIGERATION.....	487
182. General Considerations. 183. Ideal or Carnot Refrigeration Cycle. 184. Refrigerating Capacity. 185. Practical Refrigeration Cycles. 186. Vapor Compression System. 187. Air Compression System. 188. The Absorption System; Refrigeration through the Supplying of Energy as Heat. 189. Summary. 190. Review Questions.	

PART V

CHAPTER XIX

GENERAL THERMODYNAMIC EQUATIONS.....	508
191. Foreword. 192. Exact Differentials; Point Functions. 193. Inexact Differentials. 194. The General Energy Equation. 195. Additional Thermodynamic Properties. 196. The Maxwell Relations and Other Relations between State Functions. 197. Thermal Capacities and Thermal Capacity Relations. 198. Expressions for dE and dH . 199. The Joule-Thomson Experiment. 200. Perfect Gas Properties. 201. Summary.	

PRINCIPLES OF ENGINEERING THERMODYNAMICS

PART I — ENERGY

CHAPTER I

STORED ENERGY

Considered in its broadest sense thermodynamics is the natural science which deals with energy and the transformation of energy. When so considered it has an extremely wide application in the general sciences of physics and chemistry. However there are certain limited aspects of the general science which are of particular interest and utility to the engineer in the design and operation of power production apparatus and of refrigerating, heating, ventilating, and allied equipment. Thus there has developed a subdivision of the general science which is known as **Engineering Thermodynamics**. To that division of the science this work is devoted.

1. The Concept and General Classifications of Energy. — Since thermodynamics is so fundamentally a study of energy and energy transformations it is inherently desirable first to review and coördinate our basic conceptions of energy. Every one recognizes in a general way what energy is but, as is true with all fundamental physical concepts, a wholly satisfying definition is not easy to make. Energy is sometimes defined in a narrow sense as the “capacity for performing work.” A broader definition is the “capacity for producing an effect,” and it is necessary to observe that the effect produced need in no sense be limited to the performance of work. On the contrary the capacity for producing effects may manifest itself in numerous ways and it is by this very variety of its manifestations that we are able to discern the various characteristic forms of energy, basing our recognition on the manner of effects which produce or are produced by an event. Similarly we measure amounts of energy in terms of the magnitude of those various effects. Evidently therefore a first step in our study would properly

be a recognition of the various forms in which energy may exist and of the methods for evaluating its amount in any of those forms.

For purposes of systematic study it becomes desirable to classify the several forms of energy according to certain general characteristics of those forms. This may be done in various ways but for present purposes a natural and very useful initial grouping is one into (a) **stored energy** and (b) **energy in transition**. This chapter will be devoted exclusively to the consideration of those manifestations or forms of energy which may be classified as stored in a substance or aggregation of substances. The manners in which transition of energy may take place from one substance or aggregation to another is the subject matter of a subsequent chapter.

One may imagine the earlier man obtaining a first concept of energy when he observed a falling rock producing, in its fall, some perhaps disastrous effect. He subsequently distinguished at least two features of this phenomenon: first, that associated with the rock when in an original elevated position there was a potentiality for falling and of producing an effect simply by virtue of its elevation above the earth; second, that associated with the velocity of the rock, whether acquired by fall or otherwise, there was likewise a very distinct ability to produce an effect. Rather later in scientific history the two forms of energy thus exemplified by the elevated rock and by the moving rock came to be known as, respectively, **potential energy** and **kinetic energy**. Also the scientific man has come to realize that these two are the only fundamental forms in which energy may be stored, although the storage may be associated not only with tangible bodies such as the rock and the earth but also with very intangible particles such as molecules and other still smaller divisions of matter.

Although the storage of energy in connection with **tangible** or **molar bodies** is a field covered by the science of mechanics some attention will nevertheless be given to it here in order to insure the attainment of certain viewpoints which later will be found to be essential.

2. Potential Energy of Molar Bodies. — Let us return to the simple illustration of the elevated rock, in connection with which one conceives a storage of potential energy, and observe several quite important aspects of the phenomenon.

(a) A first feature is that it is only because of the presence of the earth's mass near the rock, of a mutual attraction between the masses of the rock and the earth and of a separation between these two masses that there exists any storage of potential energy. Thus we are led to observe that, although one might say loosely that the energy is possessed by the rock, in a strict sense the energy does not at all reside in the

elevated rock alone but instead in a **system** consisting of the earth and rock together. Also we observe that energy is stored in the system only by virtue of the separation or configuration of the parts of the system and by reason of forces existing between those parts, whence it should be easy to extend our notion of the storage of potential energy to *any* system made up of separated parts between which there exist mutual forces. Science has not as yet been able to reveal a complete picture of the mechanism whereby such forces may exist between separated masses but for practical engineering purposes it fortunately is unnecessary to know the mechanism so long as we are able to recognize the force, to measure it and to predict qualitatively and quantitatively what it will do.

(b) A second aspect of the illustration lies in the fact that both the magnitude of the effect which might be produced by the rock in falling from a given elevation and also the physical effort which would be required to elevate the rock are proportional to the force of attraction between it and the earth and to its elevation, and thus proportional to the product of the two. The force of attraction between any object and the earth we call the **weight** of the object and frequently measure (in English-speaking countries) in the unit we know as the **pound**. The linear separation we measure in **feet**. Thus the amount of potential energy stored in the system when an object of given mass is at a given elevation above some selected reference plane is measured in terms of its weight in pounds and its elevation in feet, or in the unit known as the **foot-pound**. Specifically (if the elevation is not sufficiently great as to affect the weight),

$$\begin{aligned} &\text{Potential energy (foot-pounds)} \\ &= \text{Force (pounds)} \times \text{Distance of separation (feet)}. \end{aligned} \quad (1)$$

(c) It needs further to be recognized that in thus evaluating the stored potential energy there must first have been selected some standard or convenient **reference plane** from which to measure the elevation. As this plane is subject to quite arbitrary selection it follows that any quantitative statement of the potential energy of a system is in fact only a relative measure the magnitude of which depends on the location of the arbitrarily chosen reference plane.

(d) A fourth feature of the phenomenon is that the potential energy is *stored* in the system, and that it remains stored in constant amount so long as the object remains at a given elevation. Thus by specification of the condition or **state** of the system in terms only of the force existing and the separation of the parts we inherently, albeit perhaps indirectly, fix and specify the amount of energy stored.

Summarizing, we may say that potential energy is characterized by the following: (a) it is energy stored in a system rather than a body; (b) its magnitude is a function of the aggregation of the parts of the system; (c) the energy stored in a given state of a system may be measured and expressed only with respect to some reference state of the system and (d) statements of the state and of the energy storage associated with the state are virtually synonymous.

Example 1. A mass weighing 10 lb. is at an elevation of 40 ft. above a given plane, and another weighing 15.5 lb. is at an elevation of 35.8 ft. above a second plane which is 10 ft. below the first. Compare the amounts of potential energy stored in each case, computing each with reference to both planes, and discuss.

3. Kinetic Energy of Molar Bodies. — Returning to the simple example of the moving rock, we recognize it to be typical of any circumstance in which an effect may be produced by virtue of the velocity of a moving mass. Again we may make several important observations concerning such a phenomenon, observations which rather parallel those made in connection with the storage of potential energy.

(a) It is only by reason of a difference in the velocity of a mass and of a second mass with which the first may collide that an effect may be produced on their contact. Thus the notion of the existence of the kinetic energy must be associated not simply with a single moving mass but with a system made up of parts which possess velocities of different magnitude. However in terrestrial systems we are accustomed to ascribing a zero velocity to any object at rest relative to the earth and thus we may loosely assign the kinetic energy to the moving mass, albeit realizing that it is in fact assignable to the system made up of the object and the earth.

(b) Physical experience shows that the magnitude of the effect which might be produced by a moving body coming to rest, and likewise the effort which would be required to accelerate it from rest to a given velocity, depends on both the mass of the object and the velocity. To evaluate specifically the corresponding magnitude of the kinetic energy of the moving body we must have recourse to several other physical relations. These are: (1) that kinetic energy may be imparted to an object by the action of a force on the object, with consequent acceleration; (2) that the amount of kinetic energy imparted is proportional to the product of the force which produces acceleration and the distance through which it acts. Also, by the definitions of velocity and acceleration respectively,

$$U = \frac{ds}{dt} \quad \text{and} \quad a = \frac{dU}{dt},$$

where U = velocity at a given instant (feet per second).
 a = rate of acceleration (feet per second per second).
 s = distance through which the body moves (feet).
 t = time (seconds).

Combining these relations to eliminate the term dt ,

$$ds = \frac{U dU}{a}.$$

But the kinetic energy (foot-pounds) which is imparted to a body in accelerating it *from rest* to a velocity U equals the product of the force which goes to produce the acceleration times the distance through which the force acts, whence

$$\begin{aligned} \text{Kinetic energy (ft.-lb.)} &= \int_0^U \text{Force } (F, \text{ lb.}) \times ds \\ &= \int_0^U F \frac{U dU}{a} \\ &= \frac{FU^2}{2a}, \end{aligned}$$

for the circumstance where the force and the acceleration are constant.

Finally a *force of one pound* is defined as that force which will give a mass of one pound an acceleration of 32.17 ft. per sec. per sec., whence the force in pounds required to give a mass of M lb. an acceleration of a ft. per sec.² equals

$$F(\text{lb.}) = \frac{\text{Mass (lb.)} \times a(\text{ft. per sec.}^2)^1}{32.17}.$$

¹ There arises at times some unnecessary confusion from the engineer's use of the *pound* as a name for both a unit of *force* and a unit of *mass*. This usage is perhaps unfortunate but not unnatural nor necessarily disastrous. It originated of course because the gravitational force between an object and the earth is so convenient, although inexact, a measure of the mass of material in the object. The inexactness arises from the fact that the gravitational force on the object varies slightly with its location with respect to the center of mass of the earth. However for most practical purposes the variation may be neglected and the mass of the material of an object, expressed in pounds, may be taken to equal numerically the weight of that object, in pounds, when on or near the earth's surface. The error would rarely exceed $\frac{1}{4}$ of 1 per cent.

Perhaps a greater confusion arises from the insistence on the part of some writers that the acceleration (a) of a mass (M) when acted upon by a force (F) *must equal numerically* the ratio between the force and the mass, that is, $a = F/M$. It is much more useful simply to recognize that the acceleration is *directly proportional* to the ratio, whereby the acceleration, the force and the mass may be evaluated in any desired units and their relation may then be expressed numerically by the use

Thus we may rewrite the above expression for kinetic energy as

$$\text{Kinetic energy (ft.-lb.)} = \frac{FU^2}{2a} = \left(\frac{Ma}{32.17} \right) \times \left(\frac{U^2}{2a} \right) = \frac{MU^2}{64.34}, \quad (2)$$

where M = mass of moving object (pounds).
 U = velocity of object (feet per second).

(c) The kinetic energy associated with a moving part of a system is necessarily evaluated in terms of the relative velocity of that part with respect to some other part of the system. Thus any quantitative statement of the energy is essentially a *relative* one.

(d) The kinetic energy is *stored* in the system and remains stored in constant amount so long as the moving object remains at a given relative velocity. Thus by specification of the state of the system in terms only of the mass and velocity of the moving parts the amount of stored energy is inherently fixed and specified.

Summarizing we may say that kinetic energy is stored not in a body but in a system and that its magnitude is expressible only as a function of the relative velocity of the parts of the system.

The potential energy and kinetic energy stored in a system consisting of tangible or molar bodies are known jointly as **mechanical energy**. English-speaking engineers are in the habit of evaluating this energy in the foot-pound unit. However, it will appear later that any of the current units of energy might be employed.

Although it is essential that one maintain clearly the distinction between these mechanical forms of energy and the other manifestations of energy which will be considered later, it is equally important that one should perceive the close parallelism between potential and kinetic energy as manifested in tangible masses and those when associated with intangible particles such as systems of molecules, atoms, et cetera. Numerous subsequent paragraphs will be devoted to the development of a conception of energy storage in such systems in a manner which is quite similar to its storage in molar systems.

of a proper constant of proportionality (c), that is, $a = (cF)/M$. Thus if by definition a pound force shall accelerate a pound mass at the rate of 32.17 ft. per sec. per sec., then, in those units, $a = 32.17 F/M$; and similarly for any other system of units.

The units employed in this work are those which are more commonly used by the English-speaking engineer, namely, the pound mass and the pound force, and not the "slug" mass nor the "poundal" force. Therefore the proportionality constant 32.17 will appear consistently. The authors avoid the use of a gravitational symbol, such as g_0 , for this dimensionless constant in order to obviate a confusion of dimensions which frequently arises from the use of a gravitational symbol.

Example 2. An object the mass of which is 2 lb. has a velocity relative to the earth of 40.1 ft. per sec. What is its kinetic energy in ft.-lb.? If its travel should be resisted by a constant force of 10 lb., through what distance could it move before coming to rest and what would be the rate of deceleration?

Example 3. An object the mass of which is 7 lb. is acted on by a constant force of 7 lb. What will be the acceleration in ft. per sec.² and the velocity and kinetic energy if the force continues to act through a distance of 10 ft.?

Example 4. A jet of water is being delivered in a vertical direction from a nozzle at the earth's surface with a velocity of 95 ft. per sec. Neglecting any resistance due to air friction, compute the relative potential, kinetic, and mechanical energy of each pound mass of the water in the jet at elevations of zero, 93 and 140 ft. above the earth and the velocity of the jet at those planes. (Note — In so far as possible base computations on the energy relations.)

4. The Composition of Matter. — In the study of engineering thermodynamics there is, strictly speaking, no necessity for investigating the composition of matter but there is a very considerable advantage in doing so at least briefly, both for the purpose of reference in subsequent studies of the properties of various engineering fluids and for the immediate purpose of indicating in what manner energy may correctly be said to *reside in* or be *possessed by* those fluids.

According to the ideas which have been accepted for some years any homogeneous substance is made up of a myriad of extremely small particles each of which is similar to every other particle which goes to make up the original homogeneous substance. When this subdivision is carried as far as is possible without altering the particle in any manner except as to size the resulting ultimate particle is called the **molecule**. There exist about as many sorts of molecules as there exist sorts of substances in nature.

Molecules are in turn considered to be formed by various arrangements of still more minute particles or **atoms** of the elements, of which latter there are some ninety-odd. Finally, the atoms of these elemental substances are considered to be arrangements of only two sorts of fundamental entities, known as **electrons** and **protons**.² Thus an outline of the make-up of all matter would be —

Tangible substances — composed of molecules.

Molecules — composed of atoms.

Atoms — composed of electrons and protons.

² The investigation of the manner in which the atoms of the various elements are built up from the proton and electron has been for a number of years and still continues to be a foremost problem of physical research. Although this research is not at all complete it already has had a distinct influence on the general science of thermodynamics and quite probably will have a great future influence on the engineering division of the science.

5. Internal Energy. — It has been shown in what manner potential and kinetic energy may be stored in systems of tangible bodies by reason of the forces and the relative velocities which exist between the component parts of the system. We are now in a position to recognize further the possibility of the storage of such energy wholly within tangible material which is itself a system of molecules, atoms, et cetera.

Intimately associated with the general conception of the molecular composition of matter there are several allied and well substantiated conceptions which are of very fundamental interest. These are considered in some detail in the following.

(a) The first of these concepts is that in all substances the molecules are in a continual state of activity. This activity includes: (1) haphazard translational movement of a molecule through whatever space may be available, any molecule coming into close proximity and in a sense colliding more or less frequently with neighboring molecules and rebounding with perfect elasticity; (2) spinning of the molecule about axes through its center of mass; and (3) an activity internal to the molecule itself which may be due to relative motion of its atoms or of the electrons within the constituent atoms — a motion which is found to become more evident at higher temperatures and with which the radiation given out by hot substances is associated.

That the molecules are at least in active translational movement is well evidenced by many simple phenomena. Thus even in the case of a solid such as ice the homely example of the drying of frozen clothing on a line serves to indicate that a portion of the molecules must have been sufficiently active that they have been able to escape from the surface. This molecular activity is even more apparent in the liquid, vapor, and gas phases of a substance. In a liquid it is well substantiated by the phenomenon which is known as the Brownian movement, in which phenomenon microscopic particles of a suitable non-soluble solid may be observed to be actively joggled about in a liquid even when apparently the liquid is wholly dormant, the movement of the particles being due to molecular bombardment. Finally in the vapor and gas phases of a substance the rapidity with which the substance will diffuse itself throughout a container may well be regarded as evidence of active molecular movement.

In the light of the kinetic energy storage which we have seen to be associated with the motions of any part of a tangible system of bodies we must now conclude that in quite a similar manner kinetic energy may be and invariably is stored *within* the systems of active molecules which form any individual substance. In this connection it may be remarked further that, upon reasoning which needs not concern us here,

it is concluded that all character of molecular activity in a substance depends primarily on the temperature of the substance. As a result of this conclusion it is further concluded that any energy which on its reception by a substance acts primarily to raise its temperature is largely stored as internal kinetic energy.

(b) The second of the fundamental concepts associated with the molecular structure of matter is that in all substances there exist forces between the molecules which make up the composite mass. These forces would seem to be of greater magnitude in solid substances, less in yielding substances and liquids and of least magnitude in the vapors and gases.

If in this connection we now recall that the storage of potential energy in systems of tangible bodies is predicated wholly on the existence of mutual forces between the component masses of such a system we are immediately led to recognize that by reason of the intermolecular forces a potential form of energy must similarly be stored *within* the systems of molecules which make up any individual mass of a substance. This conclusion is well substantiated by our experience concerning the direction of energy transfer which is necessary to accomplish a transition of a substance from, say, its solid to its liquid phase, or from its liquid to its vapor or gas phase. The invariable experience is that a solid may be caused to pass to the liquid phase, or to *melt*, only by the supplying of energy in some manner, and that likewise the liquid may be caused to vaporize only if energy is furnished to it. As the fusion or the vaporization of a homogeneous substance is found to occur without change of temperature, and as molecular kinetic energy change is known to depend on temperature change, it is concluded that the energy which is supplied to a substance in effecting fusion or vaporization is stored primarily as internal potential energy in the system of molecules of the substance. Such energy storage will also accompany virtually all other changes of condition of a substance but these two processes offer the most spectacular instances.

It may have been observed that in the instances noted the storage of molecular potential energy has been accompanied by a progressive reduction of the magnitude of the intermolecular attractions. As a result of this phenomenon we may extend our ideas to include a state of a substance in which sufficient potential energy has been stored in the system that intermolecular attractions were entirely eliminated. This condition is found to be closely approached in highly superheated vapors and in the so-called permanent gases such as hydrogen, oxygen, nitrogen, et cetera.

The sole purpose of the foregoing has been the development of a

notion of the existence and storage of energy in the interior structure of all substances. It is essential for the purposes of the engineer that he shall acquire this concept clearly, for the reason that the practical application of the science of thermodynamics will be found to depend fundamentally on the evaluation of the *change* of that stored energy in the various state changes of the fluids employed in power production processes. However it needs also be recognized that, although the *absolute* amount of internal energy possessed by a given substance can not be evaluated, the *relative* amount may be and will be measured quite exactly with reference to that at some standard condition of the substance. We shall discover later that this energy is a definite function of observable characteristics of the body, such as its temperature and pressure, just as mechanical potential and kinetic energies were seen to be functions of observable characteristics of a system of bodies.

Just as the sum of the potential and kinetic energy stored in a system composed of tangible bodies was given the name **mechanical energy**, so a name should be assigned to the energy which is stored in a system composed of molecules. In the early stages of the development of science, prior to a realization of their real character, the name heat was assigned. However the energy associated with another phenomenon, that of energy transfer by radiation and conduction, was also called heat by the classical writers. As a result of this use of the same name to represent several separate physical concepts much unnecessary and unfortunate confusion of thought arose. With the subsequent development of the science of thermodynamics and of adequate recognition of the character of the energy stored in a system of molecules several distinctive names for that energy came into usage. Among them are *molecular energy*, *internal energy* and *intrinsic energy*. However, in spite of the introduction of these characteristic names, the earlier and dual usage of the word heat has persisted, as has also the confusion of thought. The authors are of a firm conviction that if such confusion is to be avoided it is essential always to use some one of the above distinctive terms as a name for that form of energy stored in a system of molecules by reason of their motion and configuration, and to reserve the name heat for another manifestation of energy which will be considered later. Therefore the reader will observe that throughout this work the stored energy will always be designated by the term **internal energy**, and never by the term heat.

The symbol employed for this class of energy will be the letter E , and as so employed will refer to the energy *per pound mass* of a substance.

Example 5. Discuss the probable character of the energy stored in a compressed spring; in a shaft under torsion or strain.

6. Chemical and Sub-atomic Energy. — In addition to the internal energy stored in systems of molecules by reason of their relative aggregation and velocities there are vast stores of energy bound up within the molecules and within the atoms. That energy which is stored within a substance by virtue of the forces which bind the atoms together in the molecule is called **chemical energy** and is properly included in the category of internal energy, since it clearly resides within the substance. Such energy is of outstanding interest to the chemist and is of very vital interest to the engineer in connection with the process we know as **combustion**, *that process being the only one which man has as yet been able to devise whereby he may gain access to, liberate and utilize economically the energy stores which have been accumulated in nature during the past aeons.* In point of fact the science of engineering thermodynamics is based almost wholly on the recognition of certain significant characteristics of the combustion process, and the major objective in the development of the science is the increase of that proportion of the energy released by combustion which may be directed to the service of man.

That energy which resides within the atom by virtue of the motion and position of the electrons and protons and which does not participate in what are known as chemical reactions is termed **sub-atomic energy**. An outstanding example of such energy is that released by the disintegration of radium. Although undoubtedly enormous quantities of sub-atomic energy are stored within the atoms of matter and this energy is of very great interest in modern physics, man has as yet been unable to devise any means for the controlled release and industrial utilization of that energy. Hence it is not of immediate interest in the science of engineering thermodynamics and will not be considered further. If or when means are devised for its utilization then the science may well become obsolete, as well as all present-day methods of power generation.

We are now in a position to summarize and classify in tabular form the various modes in which energy may be stored.

TABLE I
STORED ENERGY

- I. Energy associated with systems of Tangible Bodies
 - (A) Mechanical Potential Energy
 - (B) Mechanical Kinetic Energy
- II. Energy associated with systems of Molecules
 - (C) Internal (Molecular) Energy
- III. Energy associated with systems of Atoms
 - (D) Internal (Chemical) Energy
- IV. Energy associated chiefly with systems of Electrons and Protons
 - (E) Sub-atomic Energy
 - (F) Energy stored in electromagnetic or electrostatic fields

It is again emphasized that all of the above represent *stored* forms of energy. Any form in which energy may be stored should be assignable to one of these four major divisions.

Example 6. Classify according to Table I the form in which energy is stored in the following: —

Storage battery	Water reservoir
Ionized gas	Jet issuing from a nozzle
Surface tension of a liquid	Hot gas under pressure.

7. State and Properties of a System. — Throughout this work continual reference will be made to the **state** of a tangible system or of a fluid. In order to understand clearly the implications of the term it will be necessary to recognize the descriptive characteristics which serve to adequately designate state and which are indirectly specified by such designation of the state. To illustrate in the simple case of the earth-rock system — the state of that system was suitably defined when we specified the elevation of the rock relative to the earth, the mass of the rock and the magnitude of the force acting between the rock and the earth and the velocity of the rock relative to the earth. Scrutinizing this specification of the state of the system we observe the highly significant fact that *the very characteristics which thus have served to designate the state serve simultaneously to fix invariably the relative amount of the (mechanical) energy stored in the system.* This same feature will be observed in all subsequent devices employed for the designation of the state of engineering fluids, namely, that the information furnished must be of such character as simultaneously to fix directly or indirectly the relative amount of energy stored in the fluid.

To define in a parallel manner the state³ of a system of molecules (such as make up the fluids employed in engineering) would require intimate knowledge of the arrangement, the activities and the magnitude of the forces acting between each of the ultimate particles of which a substance is composed. We possess no such knowledge. Fortunately, however, experience shows that the usual states of those fluids with which we shall be concerned may be quite adequately described by the aid of certain characteristics of state which are known as **properties** of a fluid. Several of those properties we shall see to be direct external

³ In elementary physics the designation of the *state* of a substance frequently is taken to involve information only as regards whether the substance is solid, liquid or gaseous. It is evident that in these studies the meaning of the term will be much more extensive, any change whatsoever in the condition of a substance (such as simply a change of temperature) being considered a change of state. The more spectacular changes between the solid, liquid or gaseous condition will be known as changes from one *phase* to another.

evidences of the internal condition of the fluid and also to be ones for the measurement of which we have available simple engineering instruments. These more directly evidential and mensurable properties are the **pressure**, the **density** (or its reciprocal, **specific volume**), and the **temperature**.

Before considering these properties in detail let us observe several highly important aspects of the specification of the state of a fluid by the designation of its properties, to wit:

(a) Regarding a property as a characteristic which is determined by and does in itself assist in determining the state, we later shall rightly regard as properties certain quantities other than those listed above. Thus the relative internal energy store, E , in a pound of fluid will itself be designated as a property. Also we shall frequently borrow mathematical parlance and refer to properties as **state functions**, in view of the complete interdependence of the magnitude of the property and the state of the substance.

(b) The properties of a substance in a given state describe only that state and in themselves give no history of the manner or the processes by which the state has been attained. Thus a fluid may be caused to pass from one state to another by an infinite variety of processes but the properties at the second state afford no record of the character or sequence of the processes employed in arriving at that state.

(c) If a given combination of properties is reproduced the state of the system is reproduced in all particulars,⁴ no matter what the intervening history of the system may have been. Therefore if a system is caused to pass from the one state to the second the difference in the amount of energy which is stored in the system initially and finally is the same irrespective of the character of the processes which may have been employed in bringing about the state change.

8. Pressure. — One quite commonly thinks of pressure as something essentially exterior to a substance, that is, as a push or force per unit area exerted *on* rather than *by* the material. In thermodynamics, however, it is necessary that we acquire two viewpoints. Thus we may regard a pressure such as that in the cylinder of any engine either as due to a force impressed *on* the fluid by the piston or as due to a force exerted *by* the fluid on the piston and walls; and we may distinguish between these two viewpoints by designating the pressure as, respec-

⁴ When we consider that a given state of a fluid is reproduced we imply, not that the condition of each and every particle is the same as previously, but that, because of the complete haphazardness of the arrangement of the molecules, the average and effective properties and state of any finite mass made up of a myriad of these molecules will be the same.

tively, external or internal. From the latter viewpoint the pressure of a gas or vapor on the walls of its container is due to the continual bombardment of the surfaces by the moving molecules. Thus the pressure will depend on the number, mass, and velocity of the molecules and it would seem thoroughly reasonable, therefore, that pressure is a property which should provide significant contributory evidence of the molecular state of a substance and of its corresponding store of internal (molecular) energy.

The unit commonly employed by English-speaking engineers for designating pressure is the **pound (force) per square inch**. However, since the more usual linear unit is the foot rather than the inch, to secure consistency in subsequent equations it will ordinarily be necessary to convert to the **pounds per square foot** unit. Also the necessary basis for real pressure measurement will be the pressure relative to an absolute zero of pressure, or **absolute pressure**. This is in distinction to the evaluation of a pressure relative to that of the atmosphere, or **gage pressure**. Consequently, unless otherwise specified, any numerical values of pressure appearing throughout this work shall be taken to be **absolute pressure**. When gage pressure is specified without data as to the atmospheric pressure, that may be taken as the standard pressure of 14.7 lb. per sq. in. or 2116 lb. per sq. ft. The symbol employed for the absolute pressure in pounds per square foot will be the letter *P*.

9. Density and Specific Volume. — The viewpoint of **density** with which we shall commonly be concerned is that of the usual definition, the mass of a unit volume of a substance; and the units we shall employ are the mass in pounds and the volume in cubic feet. For the density thus expressed in pounds per cubic foot we shall use the symbol *D*.

The reciprocal of the density is the **specific volume**, or the volume occupied by a unit mass of the substance. This property will probably be referred to rather more frequently than is the density. For the specific volume in cubic feet per pound we shall use the symbol *V*.

Considering the density (or the specific volume) from the viewpoint of the molecular composition of matter it would appear that the density would depend on the number of molecules in a unit volume of a substance and the mass of those molecules. Thus it would again seem quite reasonable that the density should afford further contributory evidence of the molecular state of the substance and thus of the internal (molecular) energy store in the substance.

10. Temperature. — For present purposes we shall need to consider temperature simply as a property which assists in the designation of the state of a substance, deferring until later the exposition of the full

significance of the property and its very fundamental relation with the whole science of thermodynamics.⁵

The notion of temperature makes its first appeal to the senses only as a measure of the relative hotness or coldness of a substance. A related phenomenon which we also observe is the fall in temperature of a hotter substance and the simultaneous rise in temperature of a cooler one when the two are placed in contact or mixed. If the cooling of the hotter substance by contact or by mixing with the cooler one is considered in the light of the molecular theory we may readily conceive the process to be a transfer of molecular kinetic energy from the one to the other through the agency of molecular collisions, just as energy is imparted from one billiard ball to another by their collision. We are thus led to see a close relation between the temperature of a material and the average kinetic energy of its molecules, which in turn depends on the mass and velocities of the molecules. Therefore we may conclude that temperature is still another property which will contribute evidence as to the internal state of the material and its store of internal (molecular) energy.

The reader is presumably familiar with the temperature scales employed by the physicist and engineer, the Centigrade scale and the Fahrenheit scale. To measure temperature we necessarily make use of various secondary effects of change of hotness — effects such as change of volume in a fluid (mercury thermometer), change of internal pressure in a gas or vapor (constant-volume gas thermometer and vapor thermometer), change of electrical resistance (platinum or nickel resistance thermometers), thermoelectric effect (thermocouple), et cetera.

At this point let us again emphasize that each of the three properties thus far considered (pressure, density, or specific volume, and temperature) have been shown to provide valid external evidence of the internal molecular state of a substance and of its store of internal energy. Thus we may properly anticipate a fact which is well substantiated by experience, that, for the usual states of those fluids with which we shall be concerned, simultaneous information as to the value of the three will be wholly sufficient to designate the state of the fluid. In point of fact we shall see that the state will be adequately described if no more than two of these properties are evaluated.

Having thus recognized several properties by which the mean molecular state of a fluid may be adequately designated, there still remains the question of ascertaining for a particular state change of a given fluid the change in the amount of energy stored within the fluid. The

⁵ Consideration of those aspects of the temperature property forms a considerable portion of Part II of this work.

energy so stored we have seen to be of both the potential and kinetic forms but we must also recognize the impossibility of measuring either one directly, due to our meager knowledge of the internal configuration of engineering fluids. Therefore recourse must be made to indirect methods in which the energy accumulated in a fluid during a given state change is evaluated by measurement of the amount of energy supplied to it from some external source. Since that involves the conception and measurement of energy in transition it will be deferred until the following chapter, where attention is given to such energy.

11. Summary. — Engineering thermodynamics is that division of the general science of thermodynamics which considers from an engineering viewpoint the manners and means of energy storage and transition as such occur in the power plant, the refrigerating plant and allied installations.

Regarding energy simply as a capacity for producing an effect it would appear that the forms of energy would be as diversified as are the effects which might be produced. However analysis shows that all energy may be adequately classified into the two groups: **stored energy** and **energy in transition**; and that similarly there are but two fundamental forms of stored energy. These are termed potential energy and kinetic energy. The present chapter has considered only the manner in which energy may be stored.

Potential energy is that form of energy which is stored in a system by virtue of stresses existing between component parts of the system.

Kinetic energy is that form of energy which is stored in a system by virtue of relative motions existing between component parts of the system.

These forms of energy are pertinent not only to tangible masses of matter but also to matter in any and all of its intangible subdivisions. The forms of stored energy with which we shall be concerned are: (a) the **mechanical potential energy** stored in systems of tangible masses, (b) the **mechanical kinetic energy** likewise stored in systems of tangible (moving) matter, and (c) the **internal energy** associated with and stored in the less tangible systems of molecules and atoms which make up matter.

It needs to be recognized that, in attempting to evaluate the amount of energy stored in any system, any quantitative statement of the energy is essentially a relative one. Thus in tangible systems the stored energy may be expressed only in terms of the relative location and velocities of the component parts of the system. A specification which adequately designates those characteristics is said to describe the **state** of the system.

Since our knowledge concerning the exact configuration of systems of molecules is very meager indeed we are forced to describe the state of such systems by the aid of certain **properties**, which properly may be regarded as external evidences of the mean internal state. Experience has shown that, for the usual states of those fluids with which we shall be concerned, the state may be described with complete adequacy by specification of the **pressure**, **specific volume**, and **temperature** of the fluid, or by only two of these if the pair is properly selected. Subsequent studies will indicate the means for evaluating the relative energy store in the fluid at any state.

12. Review Questions and Topics. —

1. (a) Delineate the field covered by the science of engineering thermodynamics.
(b) Define energy; discuss the general attributes of stored energy; and classify stored energy as regards its two general forms.
2. Develop the concept of energy storage as potential energy in systems of tangible or molar bodies; discuss the outstanding features of such storage; and indicate the manner of computing the relative amount of potential energy assignable to the system in a given state.
3. (a) Develop the concept of energy storage as kinetic energy in systems of molar bodies; discuss the outstanding features of such storage; and indicate the manner of computing the relative amount of kinetic energy assignable to the system in a given state.
(b) What name is assigned jointly to the energy stored in systems of molar bodies as potential and kinetic energy of their parts?
4. Describe the accepted ideas regarding the composition of matter.
5. (a) Develop the line of thought by which the concept of potential energy storage may be extended to systems of molecules, illustrating in as concrete a manner as possible.
(b) Develop similarly the line of thought by which the concept of kinetic energy is extended to systems of molecules.
(c) What name will be assigned jointly to the potential and kinetic energy stored in systems of molecules and what symbol is employed? (What name will not be so assigned?)
6. (a) Describe briefly the manner in which energy is conceived to reside in sub-molecular systems.
(b) What is the sole process which man has been able to devise for liberating for industrial utilization the chemical energy stored in natural resources?
7. (a) What is involved in an adequate designation of the state of a system of molar bodies? Does such a description of state afford any information concerning the relative amount of mechanical energy stored in the system?
(b) Can the state of a system of molecules be described in like manner? If not, why not? What means are available for adequately designating the state of the fluids with which we shall be concerned?
(c) Does a specification of the state of a fluid imply any definite value of the relative amount of internal energy assignable to the fluid in that state? Does it imply any information concerning the manner of arriving at the state?
8. (a) Develop the line of reasoning by which the pressure existing on and in a fluid may be regarded as an evidence of its molecular state.

(b) What are the engineering dimensions of pressure and the units commonly employed by English-speaking engineers?

(c) Distinguish between gage and absolute pressure. What is the standard atmospheric pressure in pounds per square inch and per square foot?

9. (a) Develop the line of reasoning by which the density of a fluid may be regarded as an evidence of its molecular state.

(b) What are the engineering dimensions of density and what is the relation between density and specific volume?

10. (a) Develop the line of reasoning by which the hotness or temperature of a fluid may be regarded as an evidence of its molecular state.

(b) Describe various of the secondary effects of change of hotness which the engineer employs as a measure of temperature.

(c) Describe the Centigrade and Fahrenheit scales of temperature. Compute the equivalent of 68° F. on the Centigrade scale.

Symbols and Abbreviations, Chapter I

a — acceleration, feet per second per second.

D — density, pounds (mass) per cubic foot.

E — internal energy per pound mass of a fluid.

F — force, pounds.

M — mass, pounds.

P — pressure, pounds (force) per square foot.

s — distance, feet.

U — velocity, feet per second.

V — specific volume, cubic feet per pound (mass).

CHAPTER II

ENERGY IN TRANSITION

The objective of the preceding chapter was an adequate recognition of the manners in which we conceive energy to be stored in various sorts of systems. That was essential but it will be equally essential to develop similar concepts of the transitory forms in which energy may manifest itself during transition from one system to another. To develop those concepts and also to inquire into certain significant features of the transition processes is the objective of the present chapter.

13. Processes. — Recalling the definition of energy as the capacity for producing an effect it seems desirable that a convenient name should now be assigned to any physical occurrence during which, by any sort of transformation or redistribution of energy, an effect is produced. Such an occurrence will be known as a **process**.

The processes which occur in nature appear to be of almost infinite variety. However those which will be of concern in engineering thermodynamics are classifiable into two general groups: (1) those in which the energy transition occurs as or through the agency of **work**, and (2) those in which the transition occurs as **heat**. It will be the purpose of the following articles to indicate the identifying characteristics of these two types of processes. Also, to aid in the recognition of those characteristics and as a future aid in the intelligent analysis of more complex processes, several simple processes which are representative of the two general types will be considered in some detail. Each of these will be analyzed first with regard to the following essential features:

- (a) The physical character of the process.
- (b) The manner of the energy transition or transformation.
- (c) The form, amount, and location of the various kinds of stored energy prior to and after the process.
- (d) The changes brought about in the physical state of the system as a result of the process. In the course of these analyses we shall also be led to a recognition of another and an extremely important feature of any energy transition processes, their **reversibility** or **irreversibility**.

14. Work. — On scrutiny of the myriad of physical processes with which each one is familiar numberless of them will immediately appear to be characterized by the common feature that an effect and thus an energy transition is accomplished through the action of a tangible force through a tangible distance. Concerning the exact manner of thing it

is which we chose to call a force, it is unquestionably true that, just as science has as yet been unable to reveal a complete picture of the mechanisms by which energy is stored (see Art. 2a), so it has been able to tell but little of the intimate mechanism by which a force may act and by which energy may thereby flow. However it is equally true that for many practical purposes it is necessary only that we should be able to comprehend the notion of a force and should be able to recognize and classify the phenomenon of force action and to measure the energy quantities associated with that action. That fortunately we are able to do, and it is upon the basis of that recognition that we designate the general phenomenon of the transition of energy by the action of a tangible force through a tangible distance as the performance of **mechanical work**. We may therefore define **work** as the energy which is transferred by the action of a force through a distance.

In the consideration of real engineering processes we shall be primarily interested in the energy delivered by a turbine through its shaft or an engine through its piston-rod and shaft or in the energy supplied to the shaft of a pump or compressor. For that practical reason it will be convenient to assign the self-explanatory name **shaft-work** to such energy. For that energy we shall employ the symbol W . This we are accustomed to express in the foot-pound unit of energy, evaluating the energy delivered by the product of the force, in pounds, and the distance it acts, in feet.

In this connection an important distinction should be noted between work and the stored forms of energy. So long as energy of any form remains stored in a system and the forces in the system remain in static equilibrium no energy transition as work may take place. It is only when a force in the system is permitted to dominate or a dominant one is brought into play from without the system that action takes place, energy is transferred or transformed, and work is performed. The distinction may be further delineated, although from a rather different viewpoint, by a simple example such as the shaft which drives a ship. If, with the shaft at rest, a dominant twisting force comes into operation, the first action is an elastic deformation of the shaft and a storage of internal (molecular) energy by reason of that deformation. This stored internal energy persists in the shaft in unchanged amount so long as the force remains constant, as will also the mechanical kinetic energy which is stored if the shaft is permitted to acquire velocity. However, in distinction to this continuously stored energy, tremendous amounts of work energy may continuously *flow* through the shaft to some external system if the activating force remains in steady operation and the shaft is permitted to continue its rotation.

15. Some Simple Processes Involving Work. — To emphasize certain features of work processes a number of simple examples will be examined in some detail. The types of processes included are:

- (a) Work against gravity.
- (b) Acceleration of a free body.
- (c) Elastic deformation.
- (d) Work wholly against friction.
- (e) Work causing fluid turbulence.
- (f) Inelastic deformation.

Each process will be examined briefly with reference to the several features listed in Art. 13.

(a) *Work Against Gravity.* — The obvious example of this type of process is the vertical lifting of any body. Before the process starts the energy which will be required is stored in an external system in some form, which need not be stated. At the conclusion of the process the energy is in the form of mechanical potential energy stored in the earth-body system. The energy transition is by means of work through the agency of the lifting force acting through the vertical distance the body was lifted. The change in state of the system resulting from the process is manifested in the separation of the body and the earth.

(b) *Acceleration of a Free Body.* — Examples are the freely falling body or any free body accelerated by a steadily applied force. Considering the freely falling body, the process consists of the conversion of the stored mechanical potential energy of the elevated body into stored mechanical kinetic energy through the medium of the work done by virtue of the force of gravity, acting through the distance through which the body falls.

(c) *Elastic Deformation.* — A simple example is the compression of an elastic spring. In this instance the force required to accomplish a given compression will undoubtedly vary with the amount of compression, in which case the work required must be obtained by integrating the quantity $F ds$, that is, $\text{Work} = \int F ds$, where F is the force acting during any differential displacement and ds is the differential distance through which it acts. The energy supplied as work is stored as internal (molecular) energy in the spring by virtue of the elastic displacements of its molecules. The energy transformation is from some source of energy, not stated, into molecular energy through the medium of work.

(d) *Work Wholly Against Friction.* — Consider a process in which a solid body is dragged along a rough horizontal surface at a constant

velocity. The force applied will be that required to overcome friction and the work done will be the product of this force and the distance moved through. At the end of the process there will have been no storage of mechanical potential or kinetic energy. Instead, through the agency of friction, the work energy has been changed into molecular energy and resides in either the moving body or the surface, or both, as evidence by increase of temperature or by molecular displacements.

(e) *Work Causing Fluid Turbulence.* — This condition would be exemplified in any machine in which a paddle-wheel or a propeller is caused to rotate in and agitate a fluid. For simplicity let us consider a paddle-wheel immersed in an enclosed vessel of water. The immediate effect of the movement of the paddles is the production of motions consisting largely of swirls, eddies, or vortices in the whole mass of water and in numerous local portions. Such motions go by the name of *turbulence* and while they last represent mechanical kinetic energy. However any localized bit of this chaotic and disordered motion does not long persist as such but is quickly converted into haphazard molecular activity, which is manifested by a temperature rise of the water and which we recognize as internal energy. The energy transformations are thus those of work into mechanical kinetic energy of the turbulent fluid and then into internal energy.

(f) *Inelastic Deformation.* — Many processes resulting in inelastic deformation suggest themselves. Consider the compression of a lump of lead in a power press. Work, derived from some external source of energy, is required to cause the deformation. At the conclusion of the process the energy expended as work resides in the lump of lead and is evidently in the molecular form, as evidenced by increase in temperature of the metal.

Example 1. An object is dragged from rest up a rough inclined plane. The mass of the object is 27 lb.; the linear distance it moved was 50 ft. and in that distance it was elevated 10 ft. and acquired a velocity of 600 ft. per minute; the mean force or pull exerted was 12 lb. How much mechanical potential energy was stored as a result of the process and how much kinetic energy? How much work was done during the process and what became of this energy?

(270; 42; 600; 288 ft.-lb. to friction.)

Example 2. A spring requires a final force of 1000 lb. to accomplish a compression of 6 in. from its free state. The force required is proportional to the amount of compression. Draw a force-distance diagram for the process, show by an area on the diagram the work required, and compute the amount of work. (250 ft.-lb.)

16. Heat. — Although the delivery and efficient utilization of energy as work is a major concern of the mechanical engineer a second type of process which undoubtedly is of equal importance is that involving the transition of energy as **heat**. Included in this general category are two

rather distinctive phenomena. Both have the common and essential characteristic that they depend for their operation on the existence of a *difference in temperature* between the source of the energy and the receiver (just as energy transfer as work predicated the existence of a predominant force), but they differ in the modes by which the energy transfer takes place. The two phenomena are known as (a) **conduction** and (b) **radiation**.

(a) *Conduction*. — All are familiar with the common phenomenon of the warming of water in a container when the outside of the container is surrounded by a hotter region. If it is recalled that temperature is recognized as a manifestation of molecular kinetic energy (see Art. 10) it is not unnatural to conceive that the rise in temperature of the water is due to a continuous passing along of such energy from the hotter to the cooler region, possibly by elastic impacts between adjacent molecules of the conducting substance. However, irrespective of the mechanism by which the energy transition may be conceived to operate, this phenomenon is recognizable as representative of a great variety of processes in which a transfer of energy takes place from a body or a part of a body at a higher temperature to another at a lower temperature through a tangible intermediary, and takes place by reason of the temperature difference. Such a mode of energy transition is called heat conduction and the energy so transferred is known as **conducted heat**.

(b) *Radiation*. — We are equally familiar with such phenomena as the warming of an object when simply exposed to the sun or before an open furnace door — phenomena in which energy transition unquestionably occurs by reason of a temperature difference, just as in the case of conduction, but occurs between separated objects and without a tangible intermediary. The mechanism through which the transition occurs may be quite abstruse¹ but again a considerable variety of

¹ Such energy transfer is explained by the scientist as taking place through the agency of a certain group of electromagnetic waves which travel through space with

Frequency, kilocycles per sec.	Wave-length, meters	Name
5.5 — 22,500	55,000 — 13.3	Hertzian (radio) Waves
22,500 — 1.5×10^9	13.3 — 2×10^{-4}	Shorter Hertzian Waves
1.5×10^9 — 3.8×10^{11}	2×10^{-4} — 8×10^{-7}	Infra-red Rays
3.8×10^{11} — 7.5×10^{11}	8×10^{-7} — 4×10^{-7}	Visible or Light Rays
7.5×10^{11} — 2.5×10^{13}	4×10^{-7} — 1.2×10^{-8}	Ultra-Violet Rays
6×10^{12} — 2.5×10^{16}	5×10^{-8} — 1.2×10^{-11}	X-Rays
1.5×10^{15} — 1×10^{17}	2×10^{-10} — 3×10^{-12}	Gamma Rays
7.5×10^{18}	4×10^{-14}	Cosmic or Milliken Rays

processes are characterized by such an energy transfer through space between separated objects by reason of a temperature difference between the objects, and on that basis we are able to classify such processes. This mode of energy transition is called heat radiation and the energy so transferred is known as **radiant heat**.

In many of the engineering processes with which we shall be concerned the direct interest will lie, not in the exact mode of the energy transition (as to whether it is by conduction or by radiation), but only in the general feature that the transition is effected by a temperature difference. Therefore the generic name **Heat** will be assigned to *any energy transferred by reason of a temperature difference between an emitting and a receiving region*.

In this connection we would again emphasize the necessity for maintaining clearly in mind the distinction between the stored forms of energy and energy in transition. Let the distinction between stored molecular energy and heat be illustrated by the simple example of a bar one end of which is in a hotter region and the other end in a cooler one. Unquestionably the amount of stored molecular energy per unit mass at the hotter end of the bar may materially exceed that at the cooler end but the amounts of energy so stored will persist unchanged so long as the temperatures and molecular construction of the bar remain constant. However, this stored energy is in pronounced distinction to the large amounts of energy which will be delivered continuously *as heat* via the bar from the hotter to the cooler region.

The authors would reiterate (see Art. 5, closing paragraph) their strong conviction that in the study of thermodynamics much unnecessary difficulty and confusion have been experienced through the loose and indiscriminate usage of the term *heat* to mean both energy in transition by conduction or radiation and the internal molecular energy residing in a substance. The reader should now be in a position to realize clearly the utter difference in the characteristics of the two entities and the corresponding necessity for separate designating terms. Only by maintaining a rigid, consistent and single-meaning use of terms,

the speed of light. The entire gamut of those waves is subdivided into classes according to their cyclic frequency or their wavelength, also according to their cause or effects, as in the table. All are originated by electronic action in the emitting source but their effect when striking a material substance depends on both the nature of the substance and the frequency of the wave. Those of a certain limited range which lies between the Hertzian waves and light waves are capable of inciting molecular activity when encountering material substances, so that the energy of the wave is partially or completely absorbed and converted into molecular energy within the substance. Such waves are frequently called heat waves and the energy which is so transmitted and delivered is designated as radiant heat.

even at the risk of appearing pedantic, may needless confusion be avoided.

17. Convection; Other Methods of Energy Transition. — In addition to the transition of energy as work and as conducted or radiant heat there needs to be recognized the unique agency by which energy which is stored in a substance and remains so stored may simultaneously be transported from one location to another by the physical conveyance of the substance from the first to the second location. Illustrative of such a process would be the flow of a fluid (as furnace gas flowing to and through a boiler or steam flowing through a pipe line), by virtue of which the potential, kinetic and internal energy stored in a given mass of the fluid is transported when the mass is itself transported. This phenomenon of the transportation of stored energy through the agency of flow is known as **convection**.²

The several foregoing methods of energy transition, that is, work, heat, and the transportation of stored energy by convection, are the only ones with which the science of engineering thermodynamics is concerned. However for completeness we may list all of the general forms in which transient energy is recognized. Table II provides such a listing.

TABLE II
ENERGY IN TRANSITION

- V. Energy transferred by Tangible Agencies
 - (G) Mechanical Work
 - (H) Stored Energy transported by Flow (Convection)
- VI. Energy transferred by Molecular Activity
 - (I) Heat delivered by Conduction
- VII. Energy transferred by the activity of Electrons
 - (J) Heat delivered by Radiation
 - (K) Energy delivered by all electromagnetic waves other than radiant heat waves
 - (L) Energy transmitted by Electric Current

It is observed that classes K and L are the only additional ones listed. The various phenomena associated with the electromagnetic wave have been indicated in the footnote of Art. 16. The electric current, which is sometimes described as a bodily movement of electrons through a conducting wire or other medium, is obviously a prime consideration of electrical engineering.

² Intimately associated with the transportation of energy by convection is a simultaneous transition of energy in the form of the work which is done in effecting the flow of the fluid against pressure. Such energy we shall now designate as *flow-work* but we shall defer further attention to this energy until processes involving flow are under detailed consideration. See Art. 31.

Example 3. Make a diagram indicating the arrangement or sequence of the principal items of machinery in a steam-electric power plant. For each machine designate the forms in which the stored energy enters and leaves, and denote the locations where energy is in transition, as in shaft-work, heat, et cetera.

18. Some Simple Processes Involving Heat. — To illustrate processes involving heat transition two quite simple ones will be examined.

(a) *Adding Heat to a Mass of Metal.* — The energy may be supplied by radiation (as through exposure to the sun) or by conduction (as through contact with a hotter substance). Upon reception of the energy the temperature rises and the energy becomes stored as internal energy. Also if the metal expands in any degree some portion of the energy supplied is concurrently delivered as external work by reason of the expansion against the surrounding atmospheric pressure. The energy transformation is described as one from heat to internal energy and work.

(b) *Adding Heat to a Gas at Constant Pressure.* — During the process the temperature of the gas will rise and by reason of the characteristics of a gas it must necessarily expand if the pressure is to be maintained constant. Thus some of the energy supplied remains stored in the gas as internal energy and some is concurrently delivered from the system as work, in an amount which will depend on the change of volume and the existing pressure.

19. Reversibility in Energy Transition Processes. — It will have been observed that all processes have three features in common, namely, that they start with a system in a given state and with a certain quantity of energy existing in a given form and location, that the sequence of events and energy transformations constituting the process occurs, and that at the end of the process the system is in a different state and the energy is distributed in changed forms and locations. We are now in a position to recognize another and an extremely important characteristic of any such process.

This characteristic is known as the reversibility or irreversibility of the process, by which is meant respectively the possibility or impossibility that

(a) *the process, after completion, might be caused to occur in exactly a reverse order, whereby the immediate system and any associated system should be returned from its last to its initial state; and*

(b) *that all of the energy which was transformed during the process should also be returned from its final to its original form, amount, and location.*

It will be our purpose in the several subsequent articles to consider in some detail the conditions under which reversibility might conceivably exist and the circumstances which prevent its existence. In that investigation it may appear that, following the dictates of experi-

ence, the attainment of complete reversibility as defined above is both quite inconceivable and perhaps undesirable in an active world. One may therefore question the utility of further attention to the subject, but the reader is assured that its consideration is of most fundamental and far-reaching importance both in the theoretical study of thermodynamics and in engineering practice and thus merits our critical attention.

The two basic criteria of reversibility stated above will serve to test any process whatsoever, whether mechanical, thermal, chemical or electrical. However our immediate attention is confined to mechanical and thermal processes. Therefore, to acquire an intimate realization of the full meaning of reversibility as it pertains to those processes and to recognize more exactly the particular factors which contribute to their reversibility or irreversibility, let us scrutinize again the simple processes already considered in Articles 15 and 18 but at this time with particular reference to the conditions affecting their reversibility.

20. Reversibility of Mechanical Processes.— In the following each process of Art. 15 will be considered with the object of determining if or how a complete reversal of events and of the energy transformation might ideally be accomplished and also of classifying such mechanical circumstances as definitely preclude reversibility. The first process is considered in such detail as appears necessary for the development of certain basic ideas. Only sufficient attention is given to the others as is required for a recognition of the application of those ideas in the various circumstances involved.

(a) *Work Against Gravity.*— To assist in analyzing the process of work against gravity let us consider a specific mechanism made up of an object, line, and pulleys, such as shown in Fig. 1. To elevate the body there would be required the action of a total force, F_1 , at the end of the line equal to the weight of the body *plus* such an excess force, ΔF , as would be required to overcome any frictional resistance in the rotating pulleys, the line, et cetera, that is, $F_1 = \text{weight} + \Delta F$. Thus, to have performed an elevation through distance s , energy must have been supplied as work from an associated external system in the amount,

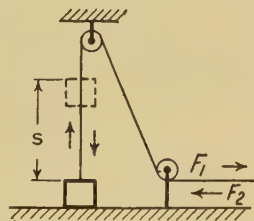


FIG. 1

$$\text{Work supplied} = F_1 s = (\text{weight} + \Delta F)s$$

Of this energy the amount (weight $\times s$) would be stored as potential energy in the object and the amount ($\Delta F \times s$) would be diverted by

friction in the moving parts and stored as molecular energy in those parts or dissipated to the atmosphere.

It is very easy indeed to conceive the reversed procedure, that of letting the body return from the elevated to the original level through a sequence of locations which exactly retrace the original path, thereby returning the system to its original state and thus conforming to the first criterion of reversibility. However, when we investigate the energy relations of the reversed process we observe that the net force, F_2 , which the body might exert at the end of the line while returning would equal the weight of the body *less* the force required to overcome friction, $F_2 = \text{weight} - \Delta F$. Thus the net energy returnable as work to any associated external system becomes

$$\text{Work returnable} = F_2 s = (\text{weight} - \Delta F)s.$$

Therefore the second criterion of reversibility has been violated, the work returnable being less than the work originally supplied by the amount $2 \Delta F s$. The irreversibility is clearly due to the friction, and it seems wholly reasonable to draw from this simple example the general conclusion that *so long as any mechanical friction exists in the moving parts of a mechanism any process performed with that mechanism is mechanically irreversible.*

To investigate the circumstances whereby reversibility might ideally be attained we may observe that as the requisite increment or decrement of force, ΔF , becomes smaller and approaches zero as a limit the difference between the energy supplied and that returnable, $2 \Delta F s$, likewise becomes smaller and approaches zero as its limit, whence the process would approach reversibility as the frictional resistances approach their limiting value zero. Entire absence of mechanical friction in the moving parts of a mechanism is therefore a derived but substantial criterion for the reversibility of any process performed with the mechanism. In that sense we recognize that the approach to reversibility connotes an approach to *ideal* conditions, and it is to this significant attribute of reversibility that we shall direct our subsequent attention.

(b) *Acceleration of a Free Body.* — In process (b), the falling body might be pictured as being redirected upward by suitable reversing guides, whereby, under ideal frictionless conditions, it might be caused to return to its original elevation, its kinetic energy be wholly restored to the original potential form and the process be effectively reversed. Again the existence of any friction will perforce make the process irreversible.

A parallel example would be the swinging bob of a pendulum, in which the energy of the system is alternately in the potential and the

kinetic form as the bob passes from its highest to its lowest position. In that mechanism when arranged with a bearing of minimum frictional resistance and oscillating in a vacuum we have one of the most nearly reversible of man-made devices. A wholly reversible mechanism of this character would be known as a perpetual motion machine of the *third class*.³

(c) *Elastic Deformation*. — In the case of the elastic deformation of a spring, perfect elasticity would permit perfect reversibility in so far as concerns the spring alone, the molecular energy stored in the spring being wholly returnable as mechanical energy. Other than perfect elasticity would clearly preclude reversibility.

(d) *Work Wholly Against Friction*. — In the case of work employed wholly to overcome frictional resistance in dragging an object, it is evident that by an additional supply of work from some external source the body might be returned to its original location and that it might be otherwise returned to its original state. However, experience dictates that in no conceivable manner could the molecular energy which was generated in the first process be returned as work and be caused to retract the body. Much less could it be caused to revert as work to the original source of energy supply. Thus the process is most emphatically irreversible.

(e) *Work Causing Fluid Turbulance*. — In the turbulent agitation of a fluid it again is inconceivable⁴ that the molecular energy finally acquired by the fluid as a result of the process could act to reestablish the turbulent motion of the fluid, that those turbulent motions even if established could act to cause rotation of the propeller against a resisting force, or that in any manner whatsoever could the molecular energy

³ Three classes of perpetual motion machines are conceivable. A machine of the first class is one which would be alleged actually to create energy, and thus violate what we know as the principle of the conservation of energy. A machine of the second class would permit an irreversible mechanical process to reverse; for example, it would presume that the molecular energy produced in dragging a block along a rough surface could act to return the block to its original position. We shall see later that this presumption violates what is known as the second law of thermodynamics. A machine of the third class would be any mechanical device in which no friction would exist. Such a device does not violate any principle or law but in the light of experience is highly improbable of attainment.

⁴ In developing our ideas of the conceivable and the inconceivable our experience leads us to describe as inconceivable that which we feel to be infinitely improbable. However it has been very aptly remarked that the relatively few centuries during which man's experience has been accumulated cover entirely too short a period, in comparison with the limitless aeons of time, to justify the drawing of the rigid conclusions which we so confidently promulgate. In point of fact, who of us is competent to deny that at some period in eternity absolute reversibility has been or may be the rule?

serve to return to the original source the work energy which had been supplied by that source. The complete irreversibility of the process is therefore evident.

(f) *Inelastic Deformation.* — It is likewise evident that inelastic deformation is a wholly irreversible process. Although the material might perhaps be reshaped and otherwise returned to its original state, that could be done only by the expenditure of additional energy rather than by an automatic process in which the original work energy supply would be returned.

The above processes may be grouped into two typical classes which may well be recognized now. This classification pertains to the general location of the frictional resistances. In processes (a), (b), and (d) the resistance exists more particularly between moving parts of a mechanism with which the process is performed; in (c), (e), and (f) the irreversibility is attributable more particularly to conditions within a material or fluid which enters into the process. In subsequent consideration of various processes which are somewhat more germane to engineering thermodynamics it will be necessary to emphasize this distinction. For that reason irreversibility due to frictional resistance between moving parts of a mechanism will be termed **external mechanical irreversibility**, and that due to frictional resistance, turbulence, et cetera, within or more closely associated with a material or fluid itself will be termed **internal mechanical irreversibility**.

Example 4. In both the lifting and the fall of a 200-lb. piledriver, hammer movement is restricted by a 20-lb. frictional force. If the lift and the fall were 20 ft. each, how much work was required to lift the hammer, how much energy was returnable at the instant of the blow, how much energy was dissipated as friction, and what was the efficiency of the process? Discuss the reversibility of the process.

21. Reversibility of Thermal Processes. — We are not yet in a position to appreciate the significance of reversibility as an *ideal* condition in processes involving the transition of energy as heat, that is, in thermal processes. However, we may at least observe the characteristic circumstances which would permit or preclude reversibility in such processes. For testing of thermal reversibility the same general criteria apply, that the energy transition shall have been performed under such circumstances that the process might be performed in the reverse direction in such a manner as to restore all parts of the system to their original state, and all quantities of energy to exactly their original form, amount, and location.

As a vehicle for illustrating the factors which control thermal reversibility we may select any convenient heat transfer process, such as that of adding heat to a gas at constant pressure (process *b* of Art. 18).

Let us assume the transfer to take place through the head of a cylinder (with piston) which contains the gas, the energy supply coming from some source at temperature T , all as portrayed in Fig. 2.

Recalling that our basic conception of energy transition by the agency of heat predicated a difference in temperature between the emitting and receiving substances, it follows that during the original heat flow process the gas would necessarily be at some temperature T_1 which is less than that of the source, that is, $T_1 = T - \Delta T$. Experience indicates that the requisite temperature difference, ΔT , would depend on the character of the material which formed the cylinder head, the rate of energy flow, et cetera.

Similarly, the reversed process of heat transition from the gas to the original source would require that the subsequent temperature of the gas, T_2 , should exceed that of the source, that is, $T_2 = T + \Delta T$ (or $T = T_2 - \Delta T$). It follows that, to complete the reversed process, either the temperature of the gas must exceed its original temperature or the temperature of the source must be less than its original, thereby precluding the reattainment of original states in all parts of the system and precluding reversibility. Thus it develops that the irreversibility of thermal (heat-flow) processes is attributable to the very temperature difference between source and receiver which we regard as the necessary characteristic of heat transition, and in that respect thermal reversibility would appear as an unattainable condition. Such it may well be, but we should observe simultaneously that it might be *approached* as a limiting condition if the necessary temperature difference should become infinitesimally small and approach zero as a limiting value. The acceptance of this viewpoint will be a requisite in the subsequent consideration of certain thermal processes which will be conceived to approach thermal reversibility as an ideal limiting condition.

The characteristic condition of the foregoing example was the transition of energy by conduction or radiation between a source and a region or fluid at different temperatures. A parallel but distinctive condition is that of the actual physical mixing of a hotter and a cooler fluid, with subsequent arrival of each at a median temperature for the mixture. The essential equivalence of such a process to the first will be evident if the two fluids are considered not to mix initially but first to be brought into contact with the opposite sides of the surfaces of any sort of heat interchanger, such as a condenser or a feed-water heater, with subsequent mixing after their temperatures have become the same. Even if

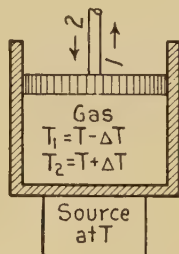


FIG. 2

the several fluids might be separated it is inconceivable that the energy which has been acquired by the cooler fluid could flow back to warm the other. The condition for thermal reversibility is thus violated and the fundamental irreversibility of the mixing process is apparent.

These two thermally irreversible processes are representative of the two general types which will be encountered in engineering thermodynamics. To assist in distinguishing them, irreversibility due to energy transition by conduction or radiation between regions or fluids at different temperatures will be termed **external thermal irreversibility**; and that due to energy transition between two fluids at different temperatures which is accomplished by their physical mixing will be termed **internal thermal irreversibility**.

22. The Principle of the Conservation of Energy. — It was remarked in Art. 1 that the capacity for producing effects may manifest itself in numerous ways and that it is by this very variety of its manifestations that we are able to discern the various characteristic forms of energy, also that we measure amounts of energy in terms of the magnitude of those various effects. In the several foregoing articles we have given attention to the characteristic forms in which energy may be stored in systems or may be transferred from one system to another. From previous physical experience we have also recognized the methods of measuring stored mechanical energy and energy in transition as work. We are now in a position to consider in some detail the general methods of measuring and evaluating amounts of energy of the less tangible forms. To do so, it will first be necessary to recognize the basic principle upon which such measurements are founded.

In a long experience in attempting to measure energy in its various forms the scientist has learned that whenever a certain measured amount of energy of one particular character is transformed during the course of an event, and perhaps apparently disappears, that energy is never actually lost or destroyed but invariably reappears in some other form and in the same quantitative amount as the original energy. This important fact of experience, namely, that *energy may exist in many varied and interchangeable forms but may not be quantitatively destroyed or created*, is known as the **Principle of the Conservation of Energy**. This principle is one of the few basic laws upon which the engineering sciences are built.⁵ It is as a result of this principle that

⁵ Although modern physics indicates that energy is stored in and *as* matter itself and in certain processes may be evolved from matter, this aspect of the Conservation Principle need not concern us in the applications of the principle to engineering processes.

man is able to establish units for the measurement of energy in its various forms and directly or indirectly to measure energy in any form.

Because of the very fundamental association of this principle with the science of thermodynamics it is frequently known as the **First Law of Thermodynamics**. Part I of this work is devoted principally to an exposition of the full import of this Principle or Law and to the development of practical methods for its application.

23. Units of Energy. — We have already observed that changes of stored potential and kinetic energy of tangible objects and amounts of work are readily and naturally measured in the foot-pound unit. Since changes in the internal energy of a substance are so distinctly changes in the total potential and kinetic energies of the systems of molecules, atoms, et cetera, which make up the substance, it would not appear unnatural also to measure in the same unit the changes in internal energy or relative amounts of that energy. On the other hand, molecules and atoms are scarcely tangible bodies and it is not evident how direct measurements of their energies can be made since we possess such meager knowledge concerning the details of the internal arrangements and activities in even the simplest sorts of substances. It is equally inevident how direct measurement of quantities of energy in transition by such intangible methods as conduction and radiation might be made in the foot-pound unit.

To meet this condition the general line of procedure which has been followed is that of measuring the amount of energy input as work which is required to produce a given state change in a fluid, whereby, on the basis of the Conservation of Energy Principle, one will simultaneously have evaluated the increase of internal energy which corresponds to a given state change, assuming of course that proper provisions have been made for preventing or accounting for any other transition of energy. One classical experiment which follows that line of procedure is practically that described in process (*e*) of Art. 15. More specifically the experiment consists of the violent agitation of a known mass of water in a container by means of a paddle wheel driven in such a manner that the work input may be accurately measured. In performing the experiment extraordinary care is taken to prevent as far as possible or to account for any escape of energy from the container as heat. The work energy input will appear almost wholly⁶ as an increase in the

⁶ Inasmuch as an increase in temperature of water is accompanied by a change in its specific volume — an increase except between 32° and 39° F. — a small proportion of the energy input (about 1/1000 of 1 per cent) goes out as work in “pushing back” the surrounding atmosphere.

molecular energy store of the water and will be evidenced by a rise in the temperature of the water.

Experiments of this character indicate that about 140,000 foot-pounds of work are required to raise the temperature of one pound of water at atmospheric pressure through the 180 degrees (Fahrenheit scale) between the freezing point and the boiling points of water, or an average of about 778 foot-pounds per degree temperature rise.⁷ Thus it is to be concluded that the average change in the internal energy of a pound of water per degree Fahrenheit change of temperature between 32° and 212° F. is closely 778 foot-pounds.

Since the amount of energy required to accomplish this unit change in temperature in a unit mass of water is considered to be a convenient unit of energy it has been so adopted and is called the **British thermal unit** (B.t.u.). Also engineers have acquired the custom of evaluating both internal energy and heat in this unit. Thus at least two units are employed currently by the engineer for the measurement of energy: the foot-pound and the B.t.u. However, it must be recognized that, just as one might use the dollar or the cent as a monetary unit and use them indiscriminately so long as the relationship between the two is known, so the engineer may use these two units of energy indiscriminately so long as he knows that

$$1 \text{ B.t.u.} = 778 \text{ foot-pounds (approximately).}^8 \quad (1)$$

The experiments of Dr. Joule conducted in the middle of the last century were the first to establish a numerical relation between the foot-pound and the B.t.u. as defined above, and for that reason the conversion factor 778 is commonly called **Joule's Equivalent** and is represented by the symbol *J*.

The foot-pound and B.t.u. are undoubtedly the energy units most used by the mechanical engineer but others are in common use by other engineers and scientists. The one used universally by the scientist

⁷ It should be remarked that the energy requirement per degree temperature is not strictly constant, and that this average figure is closely representative only of the rise between 63° and 64° F.

⁸ A difficulty of thus relating the B.t.u. and the foot-pound by means of physical tests of the properties of water is that the relation is subject to variation with each separate physical determination by individual experimenters. In order to avoid this difficulty and make the B.t.u. unit of energy independent of the properties of any fluid, the International Steam Table Conference meeting in London in July, 1929, recommended that the relations between all energy units be fixed by definitions rather than by experiment. Specifically, the Conference recommended that the calorie be defined as exactly 1/860 of a watt-hour. In effect, this procedure results in *defining* the B.t.u. as 778 ft.-lb. The authors strongly indorse this procedure.

in general is the **joule**, likewise named for Dr. Joule. In value,

$$1 \text{ joule} = 0.7376 \text{ ft-lb.} = 0.000948 \text{ B.t.u.} \quad (2)$$

Several other widely used energy units are those derived from the units of power, which in themselves are derived from energy units. The most used engineering units of power are the horsepower and the kilowatt, which are defined respectively as the transition of energy at the rate of 33,000 foot-pounds per minute and the transition at the rate of 1,000 joules per second or 44,256 foot-pounds per minute. The derived units of energy are the **horsepower-hour** and the **kilowatt-hour**, which are defined as the amount of energy involved in the transition of energy for a period of one hour at the rate of, respectively, one horsepower and one kilowatt. By making the proper substitutions it may be shown that

$$1 \text{ horsepower-hour (hp-hr.)} = 2545 \text{ B.t.u.} \quad (3)$$

$$1 \text{ kilowatt-hour (kw-hr.)} = 3413 \text{ B.t.u.} \quad (4)$$

The foregoing definitions may well be memorized, as well as the relations of equations (1), (3), and (4).

Example 5. In one type of experiment employed to ascertain the relation between the foot-pound and the B.t.u. a gasoline engine equipped with a very accurate dynamometer was arranged to drive a paddle that was immersed in water which was circulating through a thoroughly heat-insulated tank. If the measured power input by the engine was 9.8 horsepower and if it was found that when water was supplied to the tank at the rate of 104 lb. per minute its temperature rose 4° F. while passing through the tank, what value would these data indicate for Joule's equivalent?

Example 6. In determining the "calorific value" of a fuel a common procedure is to burn a small, known mass of the fuel in a closed container which is immersed in a known mass of water and to measure the resulting temperature rise produced in the water. If in such a test for an oil fuel the mass of the fuel was 0.0015 lb. and 14.3 lb. of water were warmed through 1.94° F., what was the calorific value of the fuel?
(18,500 B.t.u. per lb.)

Example 7. Derive the above conversion factors, 2545 and 3413.

Example 8. List, define and give the foot-pound and B.t.u. equivalents of various other units of energy which are currently employed in other sciences.

Example 9. An internal-combustion engine uses oil of 19,000 B.t.u. per lb. calorific value and consumes 100 lb. per hour when developing 200 hp. What is the efficiency of the engine?
(26.8 per cent.)

Example 10. If a power plant uses 20,000 B.t.u. of chemical energy in the fuel per kw-hr. output, what is its efficiency?
(17.1 per cent.)

24. Specific Heat. — When energy is delivered to or withdrawn from a substance *as heat* (by conduction or radiation) and during the process the temperature of the substance changes, the ratio of the quantity of heat interchange per unit mass of the substance to the

change in temperature is called the **specific heat**, c , of the substance for the particular process. Symbolically,

$$c = \frac{Q}{\Delta t}, \quad (5)$$

where c = specific heat of the substance for the process in question.

Q = energy interchange as heat during the process, per unit mass of substance, regarded as positive if incoming and negative if outgoing.

Δt = change of temperature of the substance, regarded as positive if increasing and negative if decreasing.⁹

In the case of solids and liquids there is only one characteristic process with which we shall be concerned, namely, the heat interchange while the substance is at constant pressure. Therefore when the specific heat of such a substance is quoted it may be taken implicitly to be the specific heat *at constant pressure*. With gases, however, there is in practice an infinite variety of processes or state changes of the gas during which energy may be transferred as heat, frequently under circumstances where energy transition as work occurs simultaneously and affects correspondingly the temperature change. Thus the specific heat of a given gas may have an infinite range of values. For definiteness, however, there is commonly quoted for any particular gas its specific heat for a *constant volume* state change and that for a *constant pressure* state change, the two being designated respectively as the *specific heat at constant volume* (c_v) and the *specific heat at constant pressure* (c_p).

Not only is the specific heat of a substance dependent upon the type of process but also, for a given process, it may vary with other conditions, such as the instantaneous temperature at which the process is operating. In that case, to determine the specific heat *at* a particular temperature we must pass to differential changes of temperature, dt , in

⁹ In this work Q will as usual be expressed in B.t.u. and Δt in degrees Fahrenheit. However it would be found that the numerical value of c is the same if any *consistent* system of units are employed for Q and Δt .

From the definition of the B.t.u. the value of c for water at 63.5° F., or the average between 32 and 212° F., is unity, so that for any substance its specific heat as above defined equals numerically the ratio between the amounts of heat required per degree temperature rise *in the substance* and *in water* at 63.5° F. Consequently $Q/\Delta t$ is sometimes called the "thermal capacity" of a substance and its specific heat is considered to be the foregoing ratio. The need for the two terms is not essential and it seems to be more direct and convenient to employ the definition of specific heat as it has been given above. We say, therefore, with many writers, that specific heat = $Q/\Delta t$.

place of Δt and write

$$c = \frac{d'Q}{dt}^{10} \quad (5a)$$

Transposing, $d'Q = c dt$, and for a finite rise from temperature t_1 to temperature t_2 ,

$$Q = \int_{t_1}^{t_2} c dt \quad (5b)$$

This expression shows the use to which information concerning the specific heat is frequently put, namely, that of computing the amount of energy transferred *as heat* during the performance of a specified process. If the specific heat varies with the temperature (as it usually does) a functional relation between c and t must be known before the expression may be integrated. A functional relation which is frequently found to be suitable is of the form,

$$c = a + bt + ft^2,$$

where a , b , and f are coefficients which are determined by experiment.

Example 11. Assume the specific heats of a certain gas to be constant and of the following magnitudes; $c_v = 0.27$, $c_p = 0.35$. How much heat is required to increase the temperature of 100 lb. of the gas through a range of 50° F., both at constant volume and at constant pressure? Can you account for the difference in the energy required for the two state changes? (1350; 1750.)

Example 12. For a given gas, $c_p = 0.2154 + 0.00002t$. Find (a) the instantaneous value of c_p at 0° F. and at 1000° F.; and (b) the heat transferred per pound of gas during a temperature rise at constant pressure from 0° to 1000° F. (0.2154; 0.2354; 225.4.)

Example 13. A gas is compressed in a cylinder in a process during which the temperature rises 200° F., the work required for the compression is 55,000 ft-lb. per pound of gas and 40 B.t.u. are abstracted per pound as heat by a water jacket surrounding the cylinder. What was the mean specific heat *for this process*? (Note that heat abstracted is negative in sense while temperature rise is positive in sense; also recall that specific heat is defined as $Q/\Delta t$ and *not* as $\Delta E/\Delta t$.) (- 0.20.)

25. Summary. — Having observed in Chapter I the various characteristic forms in which energy may be stored in various systems it is

¹⁰ The symbol $d'Q$ represents the quantity of energy transferred as heat per pound of the substance during the infinitesimal temperature change dt . This symbol is employed instead of the symbol dQ in order to emphasize that, while dt represents a small *change* in the property temperature, $d'Q$ does not represent a change in any property but simply a *quantity* of energy. In mathematical parlance, t , being a property, is a state or point function and thus a function of other properties of the state, whereby dt is an *exact* differential. In distinction, Q is not a property nor is it a function of any property of the state, whence $d'Q$ is an *inexact* differential. This distinction is a very necessary one for the mathematical treatment of thermodynamics which is presented in Part V.

essential that acquaintance be gained with the several transitory forms in which energy may manifest itself during processes of transition from one system to another. Those transitory manifestations which will be of concern in engineering thermodynamics are classifiable into two general forms: **work** and **heat**.

Work is defined as energy in process of transition through the agency of a tangible force acting through a tangible distance. As such it is to be clearly distinguished from those stored forms of energy which exist perhaps by reason of forces existing between various parts of a system but not acting to effect a transition or redistribution of energy.

Heat is defined as energy in process of transition by reason of the existence of a temperature difference between an energy-emitting region and an energy-receiving region. The general category of heat transmission includes both transmission by **conduction** through a tangible intermediary and transmission as **radiation** by electromagnetic waves through space. Again, a definite distinction needs to be maintained between the stored internal energy and these transitory manifestations of energy which are designated as heat.

There are other important methods of energy transition but the only additional one with which the student of engineering thermodynamics is directly concerned is the transfer of stored energy by means of a physical transportation of the substance or system in which the energy is stored. This process is known as **convection**.

Aside from the physical characteristics of various engineering processes there is a further characteristic which needs to be clearly recognized. That is the **reversibility** of the process, or the possibility that the process might be caused to occur in exactly a reverse order and manner, whereby all energy which was transferred or transformed during the original process might be restored to its original form, amount, and location. The existence of friction or turbulence in any mechanical process would preclude the possibility for *mechanical* reversibility. The existence of a finite temperature difference in any thermal process would preclude the possibility for *thermal* reversibility. The notion of reversibility will appear as an extremely valuable concept in subsequent analyses of actual engineering processes.

The **Principle of the Conservation of Energy** is a formulation of the important fact of scientific experience to the effect that energy may exist in many variable and interchangeable forms but may not be quantitatively created or destroyed. Because of its basic association with the science of thermodynamics it is frequently known as the **First Law of Thermodynamics**. It is only by the recognition and application of this Principle that the scientist or engineer is enabled to determine

the amounts of energy which are involved in the numerous engineering processes with which he is concerned and in the various state changes of the substances which may enter into those processes.

By reason of the variety of the manifestations of stored energy and of transient energy the engineer has (unfortunately) acquired the habit of evaluating various forms of energy in various units. However it is to be recognized that all of these units are expressible in any other unit so long as their mutual relationship is known. The units which are in current use among English-speaking engineers are the **foot-pound**, the **British thermal unit**, the **horsepower-hour** and the **kilowatt-hour**. The relationships between several of these units are:

$$1 \text{ B.t.u.} = 778 \text{ ft.-lb. (Joule's equivalent)}$$

$$1 \text{ hp-hr.} = 1,980,000 \text{ ft.-lb.} = 2545 \text{ B.t.u.}$$

$$1 \text{ kw-hr.} = 3413 \text{ B.t.u.}$$

The foot-pound is defined as the energy delivered by the action of a force of one pound through a distance of one foot, the B.t.u. as 1/180th of the energy required to raise one pound of water from the freezing point to the boiling point of water at standard atmospheric pressure (see foot-note 8), the horsepower-hour as the energy delivered in one hour when delivered at the rate of one horsepower (= 33,000 ft.-lb. per minute), and the kilowatt-hour as the energy delivered in one hour when delivered at the rate of one kilowatt (= 1,000 joules per second = 1.34 horsepower = 44,256 ft.-lb. per minute).

The **specific heat** of a substance is defined as the ratio between the *quantity of heat* in transition to or from the substance during a particular process (usually evaluated in B.t.u. per pound of the substance) and the simultaneous temperature change of the substance (usually evaluated in degrees Fahrenheit). A given substance might have as wide a variety of specific heats as there are processes into which it might enter, this by reason of the variety in the amounts of energy which might enter or leave as work. However in quoting the specific heat of solids or liquids the only implied process is that at constant pressure. Two specific heats are commonly quoted for a gas: the process when the gas remains at constant volume, and that when the gas remains at constant pressure. The specific heat of a substance will differ not only with the process but also with the temperature level at which the process is operating, generally increasing with increase of the temperature level.

26. Review Questions and Topics. —

1. Define a process and illustrate.
2. (a) Define the general term mechanical work and the particular term shaft-work.

(b) How is energy which is delivered as work evaluated and in what unit of energy is it most naturally expressed?

(c) Distinguish between work and stored mechanical energy. At the conclusion of a process involving energy transition as work does the energy *exist as work* in the system?

3. (a) When the progress of a process is impeded by frictional resistance of any character what is the immediate destination of the work energy which is required to overcome the friction?

(b) When a fluid is involved in a process and turbulence is set up as a result of the process, what is the immediate destination of the work energy which went to establish the turbulent motion?

4. (a) Define the general term heat and the particular terms conducted heat and radiant heat.

(b) Distinguish between heat and stored molecular energy. At the conclusion of a process involving energy transition as heat will energy *exist as heat* in the system?

5. (a) Describe the process of energy transportation by convection.

(b) List and classify all of the general forms or manifestations of energy in transition.

6. If energy were being supplied as heat to a gas contained in a vessel, what might be the several destinations of the incoming energy?

7. Define reversibility or irreversibility in energy transition processes, indicating the two criteria by which the reversibility of any process may be tested.

8. (a) Illustrate the application of the two test criteria for reversibility in the case of some process in which energy is in transition as work.

(b) Distinguish between external and internal mechanical reversibility.

(c) Can mechanical reversibility be attained? How may it be approached? In what respect may it be regarded as an ideal condition?

9. (a) Illustrate the application of the test criteria for reversibility in the case of some process in which energy is in transition as heat.

(b) Distinguish between external and internal thermal reversibility.

(c) Can thermal reversibility be attained? How may it be approached?

10. (a) Define the Principle of the Conservation of Energy.

(b) Define the First Law of Thermodynamics.

(c) What is the origin of this Principle or Law?

11. (a) List and define the various alternative units of energy which are employed by the engineer and quote the relative magnitude of each in terms of the others.

(b) Might quantities of energy in transition as heat be measured in foot-pounds, in joules? Might quantities of energy stored as mechanical potential energy be measured in B.t.u.?

(c) Describe the classical experiment for the determination of Joule's equivalent.

(d) Discuss two methods of defining the B.t.u. Derive Joule's equivalent from the International Steam Table Conference definition of the calorie.

12. (a) Define specific heat, and write the defining equations for both finite and infinitesimal temperature changes. Distinguish between the symbols d' and d as employed in the latter equation.

(b) Explain the dependence of the specific heat of a given substance on the character of process which may contribute to the temperature change.

(c) What character of process is usually implied in quotation of the specific heat

of a solid or liquid? For what character of processes are the specific heats of a gas commonly quoted?

(*d*) Will the temperature level at which a process occurs influence the specific heat of the substance involved in the process?

Symbols and Abbreviations, Chapter II

B.t.u. = British thermal unit of energy.

c = specific heat for any process. (c_v , constant volume; c_p , constant pressure.)

F = force, pounds.

Hp-hr. = horsepower-hour (= 2545 B.t.u.).

J = Joule's equivalent (= 778 foot-pounds per B.t.u.).

Kw-hr. = kilowatt-hour (= 3413 B.t.u.).

Q = heat, in B.t.u. per unit mass (pound) of a fluid. ($d'Q$, small quantity of heat.)

s = distance, feet.

T = temperature; t , temperature in degrees Fahrenheit (dt , infinitesimal change of t).

W = shaft-work, foot-pounds.

CHAPTER III

ENERGY EQUATIONS

In Chapters I and II there have been developed conceptions of the various ways in which energy may be stored and by which it may be transferred. In those developments, aside from a general recognition of the Principle of the Conservation of Energy and of its significance as a medium for correlating the several units of energy, the attention to the energy concepts has been *qualitative*, rather than *quantitative*. With those concepts now clearly in mind we are in a position to proceed with an investigation and formulation of the *quantitative* energy relations which must maintain in any of the processes which we shall encounter in subsequent studies. The most convenient method of such formulations is the energy equation. It will therefore be the present purpose to consider the principles and methods by which such equations are written.

27. Systems. — The term **system** has already been used quite frequently to designate a particular aggregation of tangible objects, of molecules, of atoms, et cetera, which were able to receive energy from external sources, to store that energy in various forms, or in turn to serve as a source for the distribution of energy to objects or associated systems outside of the particular system. For the purposes then in hand it was essential to emphasize the detailed character of the more or less simple aggregations which constituted the system, in order to visualize more easily the internal phenomena associated with the energy manifestations. In the subsequent studies of the more involved processes occurring in actual machines it frequently will be neither convenient nor necessary to inquire into the intimate character of the assemblage to or from which energy may pass and in which it shall be stored, it being required only that the passage or storage be adequately recognized. For that reason we may pass to a broader viewpoint of a *system* as simply *a region within which some aggregation of matter is contained, which acts as a reservoir of energy and to or from which energy may be delivered.*

Adopting that viewpoint we observe that an engine cylinder, a boiler, a compressor or even a complete power or refrigerating plant comes within the category of a system. The term will thus be employed hereafter as the generic designation of a region containing one or another of such particular pieces of apparatus.

28. Energy Classifications. — Before proceeding with the main purpose of the chapter it is advantageous to bring together in tabular arrangement the several forms of stored and transient energy which have been discussed individually in the preceding chapters. Such a compilation appears in Table III.

TABLE III
SUMMARY OF THE FORMS OF ENERGY

STORED ENERGY (Chapter I)	ENERGY IN TRANSITION (Chapter II)
(A) Mechanical Potential Energy (tangible in its associations) (B) Mechanical Kinetic Energy (tangible in its associations)	(G) Mechanical Work (tangible in its associations) (H) Stored Energy in Transportation by Convection (flow)
(C) Molecular Energy } = Internal (D) Chemical (atomic) } Energy Energy	
	Heat { (I) Energy transferred by con- duction (through molecu- lar agencies) (J) Energy transferred by radi- ation (through the agency of electromagnetic waves)
(E) Sub-atomic Energy (F) Energy stored in electromagnetic and electrostatic fields	(K) Energy of other electromagnetic waves (radio, light, etc.) (L) Energy of the electric current

In this table the demarcation between stored energy and energy in transition is still emphasized. Also there is suggested by the arrangement of the horizontal division lines a further grouping of the energy forms. Thus, mechanical potential and kinetic energy, mechanical work, and convection have been assembled into a common group by reason of their mutual association with tangible agencies and with mechanical phenomena. The two forms of internally stored energy, molecular energy and chemical energy, are likewise associated because only these two will be involved in the processes which we shall consider, the former in all processes and the latter in combustion processes. Since energy in transition by conduction and energy radiation, that is, as heat, comprise the two additional forms of energy in transition with

which we shall be concerned they naturally are grouped together. It is essential for the future studies that the student shall have an adequate concept of these several energy forms.

Those additional forms of energy in storage and in transition which are beyond the scope of engineering thermodynamics are assembled in the last group. Processes involving these are the materiel of the sciences of physics and electrical engineering.¹

29. Energy Equations. — For all developments of quantitative energy relations one must refer directly to the **Principle of the Conservation of Energy**. An alternative and equivalent statement of that principle is made if one says that, *accounting for all forms in which the energy may appear, the total amount of energy which enters any region or system must equal the amount which leaves the region plus any accumulation or minus any diminution of the energy stored within the region*. This statement of the principle in the form of an equality suggests immediately the accounting of the energy quantities involved in any process by means of the **energy equation**. Such an equation will in fact be found to be one of the most useful and illuminating devices by means of which we may investigate and analyze the performance of any piece of power or refrigerating plant equipment.

The above statement of the Principle when written as an equation appears as follows:

$$\begin{aligned} \text{Energy entering a system} &= \text{Energy leaving the system} \text{ plus any} \\ &\text{accumulation or minus any diminution in the amount of} \\ &\text{energy stored within the system.} \end{aligned} \quad (1)$$

For the reason that there will be developed several special forms of the equation which are more directly applicable to various typical sorts of processes the above equation will be designated as the **General Energy Equation**.

There are two characteristic conditions under which energy transformations take place in the processes of engineering thermodynamics. Undoubtedly the one with which we shall be more frequently concerned in practice is that in which a fluid medium is flowing steadily through some device, such as steam through a boiler or a turbine, while energy transitions and fluid state changes take place. Processes conducted under such conditions are called **steady-flow processes**. For those processes the system includes both the device through which the fluid

¹ In this connection we should observe that modern physics points to the conclusion that all matter as well as all energy is in the ultimate analysis, electrical, that is, it may be accounted for wholly in terms of electrons.

flows and that portion of the fluid within the device at any particular instant.

The second condition is that in which a fluid medium which is employed in a process is enclosed in a cylinder or its equivalent and while confined therein undergoes various state changes, accompanied by various energy transitions. Typical of such a condition would be the expansion of the steam in a reciprocating engine after cut-off, or any state change of quiescent atmospheric air. The characteristic feature of such a process is that the medium does not flow in nor out of the system during its state change. Processes conducted under such conditions we shall therefore call **non-flow processes**. For such processes the system may include only the fluid or the fluid and its container, as circumstances may dictate.

These two distinctive sorts of processes will be considered in the succeeding articles, as well as the special forms of the general energy equation which may be used most conveniently in analyzing those processes.

30. Steady-Flow. — Since the extensive adoption of rotating machinery such as the steam turbine, the centrifugal pump, et cetera, in the modern power plant a distinctly major portion of the devices used are of such character that the fluids employed enter, pass through, and leave the device in a more or less steadily flowing stream. Even in the case of the reciprocating machine, although the processes performed in the cylinders are clearly intermittent, a multiplicity of cylinders or the use of receivers and pipes of sufficient size will render the flow essentially uniform at the inlet and outlet.

Involved in the conception of ideally steady-flow are several conditions which may be stated briefly as follows:

(a) The motion of the fluid is continuously progressive in one direction, as distinguished from oscillating motion — a condition which will always be met in the processes studied.

(b) The fluid fills the entire channel through the device at any section — a condition which will likewise be met.

(c) At any particular cross-section in the channel the same state of the fluid (as revealed by its pressure, temperature, and specific volume) and the same stream velocity exist continuously, although it is to be understood that a continuing change in the state and velocity of the fluid *as it proceeds from section to section* is both permissible and usual. This condition of constancy of state at a given section may not be absolutely satisfied under actual operating conditions, particularly when the “load” on a machine is varying materially. It is sufficiently satisfied when operating conditions are reasonably steady.

(d) All particles of the fluid at any section are moving in non-intersecting stream lines and with equal velocities — in distinction to the actual conditions in which the path of a given particle is usually distinctly irregular and the velocities of particles at various points through a cross-section will differ appreciably. The error in computing kinetic energy which would result from this deviation from the ideal might perhaps amount to one per cent at a given section but it is practically compensated when only *changes* in velocity and kinetic energy from section to section are involved.

31. Energy Equation for Steady-Flow Processes. — Quite evidently there is a very great variety of engineering devices and machines which are alike only in the characteristic that one or more fluids flow steadily to, through, and from the device when it is in uniform operation. A complete list of such apparatus would be quite too extensive for setting down here, but a few may be listed as follows:

Blower	Furnace
Boiler	Nozzle
Compressor	Pump
Condenser	Turbine
Engine, steam	Valve
Engine, internal-combustion	Venturi meter

Evidently these devices perform widely divergent functions and take very diversified forms. The purpose will be to evolve a comprehensive energy equation which may be employed in the analysis of any one of them. Unquestionably such an equation must be one in which terms are provided for designating and evaluating all quantities of energy which may enter, leave, or accumulate within the system. To devise such an equation will require a clear recognition of all manners and forms in which the various energy quantities may appear.

It will be convenient first to investigate the manner in which energy will be delivered to and from a device simply by reason of the flow of the fluids to and from the system, or as it has been termed, the energy delivered by convection (Art. 17). In Fig. 3 let the rectangular enclosure represent the device, whatever it may be, to, through, and from which the fluids flow. For simplicity let us initially consider a device through which a single fluid passes, entering through the passage on the left and leaving to the right. Cross-sections in the pipes at entrance and exit are denoted as (1) and (2) respectively, and the areas at those sections (in square feet) as A_1 and A_2 . It will be recognized that by reason of the steady character of the flow the same relative mass of fluid will be continuously contained within the region and also that for

each unit mass entering at (1) a unit mass must also leave at (2). The mass-rate of flow of the fluid, in pounds per second, we shall designate by the symbol M' . The mean elevation of the fluid at the two sections, with reference to some arbitrarily selected datum plane, is Z_1 and Z_2 .

As regards the state of the fluid at the two sections, there is no evidence that the state will be the same at the second section as at the first, but rather a high probability that the states will differ by reason of some process undergone by the fluid while passing through the device.

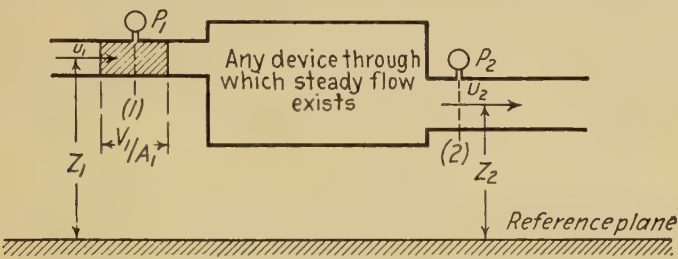


FIG. 3

Therefore the properties of the fluid at the two sections, such as the pressure (P , lb. per sq. ft.), specific volume (V , cu. ft. per pound mass), temperature (t), internal energy (E , B.t.u. per pound mass), et cetera, will be distinguished by the subscripts (1) and (2). The mean velocity (U , ft. per sec.) will be taken as the volume rate of flow ($M'V$, cu. ft. per sec.) divided by the area, that is, $U_1 = M'V_1/A_1$ and $U_2 = M'V_2/A_2$. In this connection note that

$$M' = \frac{U_1 A_1}{V_1} = \frac{U_2 A_2}{V_2}. \tag{2}$$

This simple but important relation will frequently be referred to as the **Continuity Equation**, or the **Continuity of Flow Equation**.

Investigating conditions at section (1), it appears that the stored energy which is being transported to the region by virtue of the flow of the fluid will be

(a) *Mechanical potential energy* by reason of the relative elevation of the fluid at the section, in the amount of Z_1 ft.-lb. per pound mass of fluid and $M'Z_1$ ft.-lb. per second (see Art. 2).²

(b) *Mechanical kinetic energy* by reason of the velocity of the fluid and

² To be exact, this energy quantity is $Z_1 g/32.17$ ft.-lb. per lb. and $M'Z_1 g/32.17$ ft.-lb. per sec. In this instance the error in presuming the ratio $g/32.17$ to have a numerical value of unity is wholly negligible.

in the amount of $\frac{U_1^2}{64.34}$ ft.-lb. per pound mass of fluid entering, or $\frac{M'U_1^2}{64.34}$ ft.-lb. per second (see Art. 3).

(c) *Internal molecular (and chemical) energy* in the fluid by reason of its internal state and in the amount of E_1 B.t.u. (JE_1 ft.-lb.) per pound mass of fluid entering or $M'E_1$ B.t.u. per second.

These quantities of stored energy have been imparted to the fluid during its prior history in manners of which we need have no information. It is sufficient, although essential, simply to recognize and evaluate their quantities.

Another manner in which energy is delivered to the system by reason of the flow needs now to be recognized and evaluated. To do so let us consider a transverse "slice" of the fluid which is enroute through the entrance passage, such as the slice enclosed between the dotted lines at section (1) in Fig. 3. It may be illuminating to regard this slice as the effective equivalent of a movable piston in the passage—a piston which is imposing a continuous and active force on the fluid ahead of it and thereby impelling the steady entry of that fluid into the system. The implication of a consequent continuous entry of energy into the system as flow is maintained should be evident.

The amount of this energy entry per pound mass of the fluid which enters the system will equal the product of the total force acting on the cross-sectional area of the passage times the distance through which that force must act to accomplish the entry of a pound of the fluid. The magnitude of the force is the product of the pressure per unit area at section (1), P_1 lb. per sq. ft., and the cross-sectional area of the section A_1 sq. ft., or P_1A_1 lb. (force). The distance moved for the entry of the pound of fluid will equal the linear length of the passage which would be required to contain the mass of one pound and this will equal the volume of one pound, V_1 cu. ft. per pound, divided by the cross-sectional area of the section, or V_1/A_1 ft. per pound. Thus the work required per pound, being the product of the force times the distance, becomes $(P_1A_1) \times (V_1/A_1)$ or P_1V_1 ft.-lb. per pound mass of fluid entering.

It should not be difficult now to recognize that any cross-section in the moving stream of fluid is the effective equivalent of the piston face and thus that this manner of energy transition is an invariable concomitant of the *flow* of a fluid in a passage. For that reason we shall hereafter designate energy so transported as **flow-work**. We have seen that its amount in foot-pounds per pound mass of fluid flowing may be uniquely evaluated simply by taking the product of two *properties* of the fluid at a given section, namely, the absolute pressure and the specific volume. Thus we may write that

(d) The energy entering the region by reason of the entry of the fluid against the existing pressure at the entrance section, that is, as flow-work, equals P_1V_1 ft-lb. per pound mass of fluid entering, or $M'P_1V_1$ ft-lb. per second.

Terms have thus been provided for indicating and evaluating all the conceivable manners in which energy may enter a steady-flow device by reason of the flow of a fluid into the system through the inlet passage (1). In an exactly parallel manner energy will depart from the device by reason of the flow of the fluid out of the system through the exit passage (2). The energy quantities associated with the efflux of the fluid may be written directly by analogy with the foregoing. Thus the corresponding items will be

(e) *Mechanical potential energy*, in the amount of Z_2 ft-lb. per pound mass of fluid.

(f) *Mechanical kinetic energy*, in the amount of $\frac{U_2^2}{64.34}$ ft-lb. per pound mass of fluid.

(g) *Internal (molecular and chemical) energy*, in the amount of E_2 B.t.u. per pound mass of fluid.

(h) *Flow-work* in the amount of P_2V_2 ft-lb. per pound mass of fluid.

The above terms, being ones which arise wholly from the flow of the fluid, are therefore pertinent to any steady-flow device and each may be expected to appear (unless its magnitude should be negligible) in any formulation of the steady-flow energy equation.

Referring again to Fig. 3 it appears that, having taken account of all manners of energy entry to the device via section (1) and of energy departure from the device via section (2), it remains to account for any manner of energy entry and (or) departure which may occur *between* those two sections. To discern the terms which may be required in order to account for such energy observe that the device through which the fluid flows might be one which is equipped with a shaft through which energy might pass as *shaft-work*, whence it is necessary that the comprehensive energy equation shall contain terms for designating and evaluating such work. Similarly the device might be one to or from which energy might pass by heat conduction or radiation, whence the equation must contain further terms by which such *heat energy* might be designated and evaluated. Referring more concretely to several distinctive classes of devices.

(i) If we consider the device which makes up the system to be for example a pump, it is evident that the motivation of the pump will require the furnishing of energy to the system as the *shaft-work* necessary to drive it, this energy being supplied from some external agency.

The amount of such energy we shall designate as W_{in} ft-lb. per pound of fluid or $M'W_{\text{in}}$ ft-lb. per second.

(j) If the device were a steam turbine, the sole function of which is the delivery of energy as *shaft-work*, the amount of energy so delivered from the system we would designate as W_{out} ft-lb. per pound of fluid or $M'W_{\text{out}}$ ft-lb., per second.

(k) In an analogous manner the device might be a steam boiler, through the tubes of which large quantities of energy will pass into the system as *heat*. The amount of such energy we shall designate as Q_{in} B.t.u. per pound of fluid or $M'Q_{\text{in}}$ B.t.u. per second.

(l) Finally, if the device were perhaps a radiator, energy would similarly leave the system as *heat*. Its amount would be designated as Q_{out} B.t.u. per pound of fluid or $M'Q_{\text{out}}$ B.t.u. per second.

Terms have thus been provided for indicating all of the conceivable manners or forms in which energy may enter or leave a system in those steady-flow processes encountered in engineering thermodynamics. Referring to the General Energy Equation (1) of Art. 29 it now appears that attention must still be given to the possibility of the accumulation or diminution of energy *stored within* the system.

If we recall that the specification of steady-flow (Art. 30, c) asserts the persistence of a certain amount of fluid in a device and a given state of the fluid at any specific point in its travel through the device, it will be evident that there can be no such accumulation or diminution of stored energy within the system. This conclusion may perhaps be made more clearly evident if one considers such a device as a steam boiler which has been brought up to a steady steaming condition. Although there is unquestionably an accumulation of energy while the boiler and its contents are being brought from cold to a steady steaming condition, if one considers a *steadily* operating boiler and its contents it will be evident that from moment to moment there is neither increase nor decrease in the amount of energy stored therein. The same line of thought will apply to any steady-flow device and therefore in the steady-flow energy equation no terms need appear to account for an accumulation or diminution of energy stored within the system.

We have thus provided terms for evaluating the energy quantities in any conceivable form in which they may enter or may leave a steady-flow device and we have disposed of the question of the accumulation or diminution of stored energy within the device. We may therefore proceed to set down in specific terms the General Energy Equation of Art. 29 as it applies to steady-flow processes. For the reason that the foot-pound is somewhat the more fundamental unit of energy we shall write the equation in foot-pound units, but both in foot-pound per pound

mass of fluid entering and in foot-pound per second. The former will be more useful for the case of a device through which only one fluid is flowing, the latter for one through which several fluids are flowing. The equations become

$$Z_1 + \frac{U_1^2}{64.34} + (JE_1 + P_1V_1) + W_{\text{in}} + JQ_{\text{in}} =$$

$$Z_2 + \frac{U_2^2}{64.34} + (JE_2 + P_2V_2) + W_{\text{out}} + JQ_{\text{out}}, \text{ ft-lb. per}$$

pound of fluid, or,

(3a)

$$M'Z_1 + \frac{M'U_1^2}{64.34} + M'(JE_1 + P_1V_1) + M'W_{\text{in}} + M'JQ_{\text{in}} =$$

$$M'Z_2 + \frac{M'U_2^2}{64.34} + M'(JE_2 + P_2V_2) + M'W_{\text{out}} + M'JQ_{\text{out}},$$

ft-lb. per second.

(3b)

The equation as thus written is thoroughly comprehensive but perhaps appears somewhat formidable. Fortunately it develops that in the application of the equation to actual engineering devices several eliminations and simplifications may be made. To enumerate:

(a) In the devices of engineering thermodynamics the difference in elevation of any two cross-sections of the fluid stream is usually not large and, with rare exception, the difference in the corresponding mechanical potential energy quantities is insignificant in comparison with the magnitudes of the other terms in the equation. For that reason the mechanical potential energy terms (Z_1 and Z_2) may immediately be eliminated.

(b) As a matter of convenience the two shaft-work terms (W_{in} and W_{out}), and the two heat terms (Q_{in} and Q_{out}) may well be condensed into single terms, W and Q , which shall represent the *net* energy transfer to or from the device as shaft-work or as heat, respectively. In so doing, however, there seems to arise a question as to the side of the equation on which these composite terms should be placed, although in point of fact their location is immaterial so long as due intelligence is exercised in their interpretation. The authors follow the practice of placing them on the left side of the equation, in which case either term as so located would necessarily be algebraically *positive* if the net energy transition were an *input* of energy to the system, and algebraically *negative* if the net transition were an energy *departure*.

Having thus discarded the mechanical potential terms in the above comprehensive energy equation and having combined the several work terms and the several heat terms, the equation may be written in the

somewhat more simplified form

$$\frac{U_1^2}{64.34} + (JE_1 + P_1V_1) + W + JQ = \frac{U_2^2}{64.34} + (JE_2 + P_2V_2) \quad (3c)$$

(c) It will perhaps have been observed that in any steady-flow process the terms E and PV will invariably appear, and each on both sides of the equation. This arises from the fact that in the flow of a fluid to or from a system energy is inevitably transported both as molecular energy and as flow-work. Likewise it will be observed that the magnitudes of both terms are fixed wholly by the state of the fluid, the one being the single property E and the other the combination of the two properties P and V . It would therefore seem thoroughly logical and a distinct convenience if the several terms might be combined into a single term, *which term will be as distinctly a property of the fluid as is any of its component parts*. That combination is discussed further in the following article.

32. Enthalpy. — We have recognized in the immediately preceding paragraph: (a) that without exception the E term and the PV term will always appear jointly in the energy equation as applied to any steady-flow process; (b) that each term is either a property or the product of two properties, whence the sum of the two terms is equally a property and thus invariably fixed by the state of a fluid; and (c) that their combination as a sum is therefore both convenient and logical. There would thus seem to be no question of the utility and propriety of introducing this additional property or state-function of an engineering fluid which shall be defined as the sum of the internal energy of the fluid and the product of its absolute pressure and its specific volume.

Having thus created the property out of other well-defined properties it is necessary to assign to it a name, a symbol, and a suitable unit of measurement. Various names have been used in technical literature but the one employed by numerous modern writers, and adopted by the authors is the term **Enthalpy** (en-thäl'py). Similarly various symbols have been assigned to the property. We shall select the letter H . Finally, it is customary among engineers to express the enthalpy in the B.t.u. unit of energy and we shall follow that practice. Therefore, recalling that internal energy E is also commonly expressed in the B.t.u. unit but that the product PV has the dimensions of energy³ in foot-

³ To avoid possible future confusion it should be noted here that, although PV has the *dimension* of energy per pound mass and measures energy *in the case of flow*, in certain other cases it need have no energy significance and may be considered simply as the product of the two properties. Torque or moment likewise has the dimension of energy but does not measure energy except in the case of rotation.

pounds per pound, we may write that

$$\text{Enthalpy, } H = E + PV/J, \text{ B.t.u. per pound mass of fluid.} \quad (4)$$

Of the other names which are employed for this property two which have had wide usage are **total heat** and **heat content**. Although this usage must be recognized the authors avoid it for reasons which parallel those for their absolute divorcement of the terms *internal energy* and *heat* (see Arts. 5 and 16). The term *total heat* leads to the erroneous impression that increase of enthalpy may be effected only by the supplying of energy as heat and that change of enthalpy always measures the amount of energy in transition as heat. The term *heat content* leads to the same incorrect impressions and further erroneously implies that the energy transferred during an increase of enthalpy of a fluid is necessarily *contained* in the fluid, whereas only the internal energy is *contained* or stored in the fluid, the PV not being *contained* in any sense. In recognition of such arguments several writers have employed such terms as "thermal potential" and "thermal head." Willard Gibbs, the eminent physicist who first recognized the significance and value of the composite property, called it the **X (chi)-function**.

Having thus eliminated the mechanical potential energy terms in the energy equation for steady-flow processes, having condensed the several shaft-work and heat terms into single terms and having substituted for the $(E + PV/J)$ terms the composite property enthalpy (H), we may now write the equation in final form as

$$\frac{U_1^2}{64.34} + JH_1 + W + JQ = \frac{U_2^2}{64.34} + JH_2, \text{ ft.-lb. per pound.} \quad (5)$$

This form of the equation will be found to provide all the terms necessary for the energy analysis of any steady-flow process encountered in engineering thermodynamics. In view of its repeated use in the subsequent work it is regarded as highly essential not only that it should be memorized but also (and more important) that the basis and significance of each term should be thoroughly understood.

33. Applications of the Steady-Flow Energy Equation.—In the following the steady-flow equation is applied to several typical engineering machines. These applications are not made with the intent of developing thorough-going analyses but only with the purpose of exemplifying the general applicability of the equation, of indicating the methods employed in adapting it to the particular circumstances under which particular devices operate, and of illustrating the further abbreviations in the equation which may be permissible under those particular circumstances. In each instance the procedure will be to scrutinize the

outstanding conditions of operation with the view of determining which terms of the equation are unquestionably pertinent and which might perhaps be omitted, thereby evolving an appropriate adaptation of the complete equation.

(a) *Steam Boiler*.—The objective of the boiler is the impartation of energy to the entering water, this energy having been supplied originally as chemical energy in fuel, released by combustion and eventually delivered to and through the boiler tubes by conduction, that is, as heat (Q_{in}). Although the entering feed-water and leaving steam unquestionably possess some velocity and therefore mechanical kinetic energy, these energies in practice are either relatively low or differ by a relatively small amount, whence it may be a permissible approximation to neglect them. There certainly is no shaft among the appurtenances of a boiler, through which energy might enter or leave as shaft-work, wherefore that term is wholly irrelevant. With such conditions the particular adaptation of the steady-flow energy equation to the boiler would become

$$JQ_{in} = JH_2 - JH_1, \text{ ft.-lb. per pound of water, or} \\ Q_{in} = H_2 - H_1, \text{ B.t.u. per pound of water.}^4$$

Example 1. Tables of the "Properties of Steam" quote the following values:

	Specific volume	Enthalpy
Dry steam at 100 lb. per sq. in. abs.	4.408	1186.6
Water at 100 lb. and 100° F.	0.016	68.2

Compute the amount of energy which would need to be supplied in a boiler for the production of 1 lb. of dry steam at 100 lb. abs. from feed-water entering at 100° F. Separate this energy into the amount of increase in internal energy per pound of the fluid and the excess of the flow-work required to deliver the steam over that required for the entry of the feed-water. (1118.4; 1037.2; 81.2.)

(b) *Steam Engine or Turbine*.—The sole objective of the engine is the delivery of energy as shaft-work. If the machine is well insulated only a minor and probably negligible amount of energy leaves it as heat, except as friction in the bearings et cetera may cause some energy distraction and heat emission. As in the boiler, it may be a permissible approximation to neglect the mechanical kinetic energy terms if the

⁴ It is recognized in this instance that the amount of heat supplied to the system equals the change of the enthalpy property for the fluid. The designation of enthalpy change as change in *total heat* or *heat content* (Art. 32) is undoubtedly an echo of this particular circumstance. However it should also be recognized in the succeeding applications of the equation that persistence in such designation becomes awkward and misleading.

velocities of the entering and leaving steam are not high or differ by a minor amount. With these considerations the energy equation of the engine would become:

$$W_{\text{out}} = J(H_1 - H_2) - JQ_{\text{radiation, etc.}}, \text{ ft-lb. per pound of steam.}$$

Example 2. The steam entering a given turbine is superheated steam at 200 lb. per sq. in. abs. pressure and 450° F. It leaves as dry steam at 20 lb. per sq. in. abs. For those states Steam Tables quote enthalpies of 1240.0 and 1156.0 B.t.u. per lb., respectively. Compute the work output in foot-pounds per pound of steam passing: (a) if any energy emission as heat may be neglected, and (b) if such emission via the casing and bearings is 10 per cent of the enthalpy decrease in the turbine steam. (65,400; 58,800.)

(c) *Nozzle, Orifice or Venturi Meter.* — The outstanding characteristic of any of these devices is the large accretion of velocity which takes place as an accompaniment of the flow of the fluid. The shaft-work term is again irrelevant. Any energy emission or accession as heat should be so very small as to be negligible, whence the equation would become

$$\frac{U_2^2}{64.34} - \frac{U_1^2}{64.34} = J(H_1 - H_2), \text{ ft-lb. per pound of fluid.}$$

Example 3. A steam nozzle is supplied with dry steam at a pressure of 100 lb. per sq. in. abs. and in expanding into the atmosphere the enthalpy of the steam is decreased to 1058 B.t.u. per lb. What is the velocity of discharge, presuming that the entering velocity of the steam was negligible? (Find the enthalpy of the entering steam in the data of example 1.) (2540.)

(d) *Throttle.* — A throttle may be described in a general way as any obstruction placed in a line through which a fluid is flowing. There is no appreciable heat interchange to or from the fluid during its flow and no shaft-work is done or is supplied. As regards the velocity of the fluid — although there may be a velocity increase in the constricted passage-way immediately at the throttle — that velocity is largely dissipated directly thereafter. Therefore if one considers sections in the line ahead of and after the throttle the corresponding velocities and kinetic energies may usually be taken either to be negligibly small or to differ by a negligible amount. The equation would thus reduce to the relation

$$H_1 = H_2, \text{ B.t.u. per pound of fluid.}$$

(e) *Water-Jacketed Air Compressor.* — The essential characteristics of this machine are, from our present viewpoint, that shaft-work is required to drive it and that two fluids enter and leave the system, the

air and the water. For the latter reason it will be advantageous to write the energy equation initially in terms of the energy per second (as in equation 3b of Art. 31.) In writing the equation the subscripts "a" and "w" will be employed to designate the air and the water respectively.

Again the velocities and kinetic energies of the air and water entering and leaving may be quite moderate and differ negligibly. Energy may be dissipated by radiation from the cylinder and bearings in appreciable amount. On this basis the energy equation might initially be written as

$$\begin{aligned} M_a'(JH_1)_a + M_w'(JH_1)_w + M_a'W_{in} \\ = M_a'(JH_2)_a + M_w'(JH_2)_w + M_a'JQ_{\text{radiation}}, \end{aligned}$$

in which equation the shaft-work input and the heat emission are expressed in terms of energy per pound of air delivered by the compressor. Since the work required to drive the compressor is probably the item of major interest it may be advantageous to solve the equation for the work per pound of air. Thus

$$W_{in} = J(H_2 - H_1)_a + \frac{M_w'}{M_a'}J(H_2 - H_1)_w + JQ_{\text{radiation}},$$

ft.-lb. per pound of air.

An alternative and convenient viewpoint in scrutinizing the compressor would be to consider only the air and to regard the energy which passes through the inner cylinder walls to the cooling water as energy in transition as heat. In that light the equation becomes

$$W_{in} = J(H_2 - H_1)_a + JQ_{\text{to jacket, radiation, etc}}$$

Example 4. In the test of a water-jacketed air compressor it was found that the shaft-work required to drive the compressor was 60,000 ft.-lb. per pound of air delivered, that the enthalpy of the air leaving was 30 B.t.u. per pound greater than that entering, and that the energy removed by the circulating water was 40.5 B.t.u. per pound of air. From these data compute the amount of energy which must have been dissipated as heat to the atmosphere from the bearings, cylinder walls, etc.

5150 ft.-lb. per pound of air.)

The foregoing examples are merely typical of the many and important applications of the steady-flow energy equation met in engineering thermodynamics. Undoubtedly it will have been observed that the enthalpy property is an intensely valuable and significant one, its change being involved in every process taking place coincidentally with flow. It may therefore be reasonably anticipated that the evaluation of the enthalpy change for various processes with various fluids will

form an important part of the subsequent study of the application of thermodynamics to engineering processes.

(f) *Application in Hydraulics.* — It may be of interest to note that the well-known **Bernoulli theorem** which is used so extensively in hydraulics is a special form of the steady-flow energy equation of thermodynamics as that might be applied to the flow of a liquid. If in equation 3a of Art. 31 we write for PV its equivalent P/D (where D is density, lb. per cu. ft.) and if we collect the terms E , W and Q on the right side of the equation it becomes

$$Z_1 + \frac{P_1}{D_1} + \frac{U_1^2}{64.34} = Z_2 + \frac{P_2}{D_2} + \frac{U_2^2}{64.34} + [J(E_2 - E_1) - W - JQ].$$

If we consider the particular process of the flow of a liquid of sensibly constant density through such a device as a pipe, orifice, nozzle, Venturi meter, et cetera, in which devices there is no energy transition as shaft-work and no sensible exchange of energy as heat, and if we further agree to neglect any increase in internal energy in the liquid arising from friction and turbulence, so that the entire bracketed term disappears, we have the expression:

$$Z_1 + \frac{P_1}{D} + \frac{U_1^2}{64.34} = Z_2 + \frac{P_2}{D} + \frac{U_2^2}{64.34}$$

which will be recognized as an expression of the *Bernoulli theorem*. It appears clearly that the so-called "heads" of that theorem are strictly measures of mechanical energy per pound mass of the liquid, and it should also appear that the so-called "head losses" discussed in hydraulics are amounts of mechanical energy which are transformed to internal energy by reason of friction and turbulence and depart from a system as internal energy or as heat.

34. Energy Equation for Non-Flow Processes. — Although the steady-flow processes predominate in importance in practical engineering, there are a number of processes which take place under circumstances such that the fluid involved is not flowing. Representative of such processes would be the expansion or compression of a fluid *while retained within* a cylinder, as for example the expansion of the steam in an engine cylinder after the cessation of admission and before the beginning of its ejection (between "cut-off" and "release"). Another example would be a state change taking place in a still atmosphere. Also we shall have occasion to analyze several idealized engine cycles which may conveniently be conceived to take place in their entirety while a fluid is retained within the engine cylinder. State changes or processes occurring under such circumstances have already been designated as **non-flow processes**.

Let us recall that the *general* energy equation (Art. 29) is:

$$\text{Energy in} = \text{Energy out} \left\{ \begin{array}{l} + \text{ Accumulation} \\ - \text{ Diminution} \end{array} \right\} \text{ of the energy stored within the system.}$$

As we endeavor to particularize this equation for, say, a non-flow state change taking place in a stationary cylinder, we observe that any mechanical potential and kinetic energy terms are irrelevant, as is also any flow-work term. However, we must recognize that energy might enter or leave the system by conduction through the cylinder walls, that is, as heat. Also if the cylinder were closed by a piston which may be caused to move by or against an external force acting through a piston rod, connecting rod and crankshaft an agency is present whereby energy may be delivered to or from the system as shaft-work. More broadly, any expansion or compression of the fluid in the cylinder against or by an external force would predicate the transition of energy as work.

As regards the accumulation or the diminution of energy within the system — if perchance there were an excess of energy delivery to the system over the energy departure that energy must assuredly be stored within the system. The only available site for such storage is the internal structure of the fluid itself, with perhaps the material of the container as well, that is, as an increase in the internal energy. Similarly any excess of energy outgo over input would necessarily be at the expense of the internal energy of the fluid (or of the fluid and the material of the container).

With these aspects of the process in mind and employing the same symbols as have already been used the comprehensive energy equation for non-flow processes would become

$$W_{\text{in}} + JQ_{\text{in}} = W_{\text{out}} + JQ_{\text{out}} + J(E_2 - E_1). \quad (6)$$

Combining the several work terms and the several heat terms in the same manner as they were combined in the steady-flow equation, the non-flow equation finally appears as

$$W + JQ = J(E_2 - E_1) \text{ ft-lb. per pound} \quad (7)$$

in which $W = W_{\text{in}} - W_{\text{out}}$, or the *net* energy transition as work; and $Q = Q_{\text{in}} - Q_{\text{out}}$. Again, this arrangement of the work and heat terms would require that energy *entering* by either agency should be taken as algebraically *positive*, and energy *leaving* as *negative*. However, any arrangement of the equation would be workable so long as it were given an intelligent analysis and interpretation.

The latter formulation of the equation is the one which will be employed hereafter. It should be memorized and thoroughly understood.

It will be observed that it is identical with the steady-flow equation in all respects except for the elision of all terms which originate or have significance by reason of flow (the terms Z_1 and Z_2 , $U_1^2/64.34$ and $U_2^2/64.34$, P_1V_1 and P_2V_2).

35. Applications of the Non-Flow Energy Equation. — In the following the non-flow energy equation is applied to several types of processes which have been selected, not as being representative of processes taking place in any distinctive pieces of apparatus, but rather as those which will portray certain distinctive sorts of state changes. These are: (a) an **adiabatic** state change, (b) a **constant-volume** state change, (c) a **constant-pressure** state change, and (d) an **isothermal** state change.

(a) *Adiabatic State Change.* — An *adiabatic* state change is defined as *one during which no energy is supplied to or departs from a system by radiation or conduction, that is, as heat.* Thus the state changes of the fluid during its expansion in the nozzle, orifice or throttle of Art. 33 would be designated as adiabatic processes, albeit those particular processes were ones accompanied by flow. A non-flow adiabatic process may be conceived to occur in a thoroughly insulated cylinder closed by a movable piston, the state change in the fluid being accomplished by compressing the fluid through the agency of energy supply as work or by permitting its expansion and thus the delivery of work. The successive compressions and expansions of the air in such a process as the propagation of sound waves are virtually adiabatic.

Although strictly adiabatic processes occur very rarely if ever, they are quite closely approached in certain actual processes and, more important, they are of outstanding importance as an idealized conception to which we shall need to refer most frequently in subsequent analyses.

Since in the adiabatic process there is no energy transition as heat the term Q disappears from the energy equation and it thus becomes

$$W = J(E_2 - E_1)$$

in which $(E_2 - E_1)$ measures the change of the stored internal energy in the fluid and container, B.t.u. per pound of fluid.

(b) *Constant-volume state change.* — A state change of a fluid which takes place in a closed container of constant volume, such as a closed tank or a cylinder with piston locked in position, would obviously be a *constant-volume* state change. The work term is clearly of zero magnitude, all energy transition to or from the system being as heat. Thus the energy equation reduces to

$$Q = E_2 - E_1, \text{ B.t.u. per pound of fluid.}$$

(c) *Constant-pressure state change.* — A constant pressure state change would be one undergone by a fluid contained in a cylinder and under such circumstances that the fluid pressure on the piston is permitted to remain constant while energy is added to (or removed from) the system as heat. An equivalent process would be one taking place in a portion of the atmosphere while under constant atmospheric pressure. In practically all engineering fluids the maintenance of the constant pressure would require that the fluid be permitted to expand as heat energy is supplied, with the resultant action of a force on the piston or on the surrounding medium and therefore the concurrent emission of energy from the system as work. All terms in the energy equation thus are retained, or

$$JQ + W = J(E_2 - E_1).$$

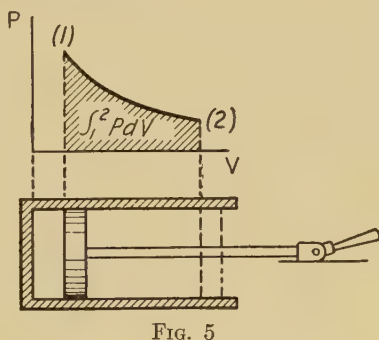
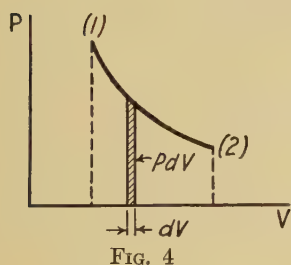
(d) *Isothermal state change.* — An isothermal state change is defined as one during which the temperature of the fluid is maintained constant. The state change might be accomplished by the supplying of energy as heat, in which circumstance practically all fluids would require that the fluid be permitted to expand (with concurrent energy emission as work) if temperature rise is to be avoided. All terms of the energy equation thus appear, or

$$JQ + W = J(E_2 - E_1).$$

36. — $\int_1^2 P dV$; Mechanical Effects. — It is to be emphasized that all of the various formulations of the energy equation which have been presented in the foregoing articles have been developed wholly without concern as to whether the processes considered were actual and therefore mechanically irreversible (by reason of the inevitable friction of moving parts of a machine or the equally inevitable friction and turbulence of a flowing fluid), or whether they were idealized in the feature of being mechanically reversible. The equations are therefore wholly applicable to both the real and the ideal processes, and as such may perhaps seem to be correspondingly the more useful. Be that as it may, we shall find that it is not only convenient but also absolutely essential that methods be devised whereby the mechanical terms in those equations may be more definitely evaluated for processes which are idealized to the extent of being mechanically reversible. One of the most useful methods which have been developed to that end is the evaluation of the $-\int_1^2 P dV$ for the process.

If one plots on pressure-volume coördinates the simultaneous values of the absolute pressure and the specific volume of a fluid as it passes

through the sequence of states encountered in any process (whether non-flow or steady-flow), a graphical representation of the process is obtained which we shall see to have certain unique characteristics. Such a representation is portrayed in Fig. 4, the state change of the fluid being one from an initial state indicated by point (1) to a final state at point (2). If on this figure any ordinate P is taken to measure the mean pressure of the fluid while undergoing any infinitesimal volume change dV , then the product $P dV$ is the differential cross-hatched area and the integral of this product between the initial and final states as limits $\left(\int_1^2 P dV\right)$ obviously measures the total area *under* the state change curve. It will be our purpose to investigate the useful significance which may be attached to this integral and area in mechanically reversible processes, both non-flow and steady-flow.



(a) *Non-Flow Processes*.—Let us first consider a *non-flow state* change taking place in a cylinder with piston, that is, in some mechanism such as that represented in Fig. 5. Above the cylinder there is redrawn a P - V diagram similar to that of Fig. 4 and portraying the sequence of states of the fluid as the piston travels from the position shown in full lines to that shown by broken lines. The abscissas are to be taken to represent the total volume of the fluid in the cylinder at any piston position. For simplicity let it be assumed that a 1-lb. mass of fluid is contained in that volume.

If the moving parts of the engine mechanism were unretarded by frictional resistance then the energy delivered as shaft-work during the indicated travel of the piston would be measured by and equal to the energy delivered to the inner piston face. That energy would equal the product of the force F exerted on the face by the fluid pressure and the distance s through which the piston moved. If the pressure and force should perchance have been constant the work would equal the

simple product of the two. If the pressure and force should vary the energy would needs be obtained by integration, that is, work to piston face = $\int_1^2 F ds$. Let us consider the latter relation.

The total force acting on the piston face would equal the product of the absolute pressure of the fluid (P , lb. per sq. ft.) times the piston area (A , sq. ft.) or $F = PA$. Therefore, work to piston face = $\int_1^2 PA ds$. But the product $A ds$ measures the differential change in volume dV of the space back of the piston and thus of the fluid within that space. Thus we would say in general that the work equals $\int P dV$. However an expansion of the fluid would predicate a final volume V_2 of the fluid greater than the initial volume V_1 and thus an algebraically positive value for the definite integral between the limits V_2 and V_1 , whereas an expansion also predicates a *delivery* of energy as work from the system and we have considered it advantageous to regard all energy delivery as negative in sense. Therefore it is necessary that the equation should be made consistent in the above respect by changing the sign of the integral and writing the equation as

$$\text{Work} = - \int_1^2 P dV, \text{ for mechanically reversible, non-flow processes}^5$$

There is thus developed a relationship whereby the work output for a mechanically reversible, non-flow process may be evaluated directly if information is available concerning the exact character of the state change of the fluid as that change is portrayed by the change in the properties pressure and specific volume. Employing the relation in the general energy equation for the non-flow process (Eq. 7 of Art. 34),

$$JQ - \int_1^2 P dV = J(E_2 - E_1), \text{ for} \\ \text{mechanically reversible processes.} \quad (8)$$

By these relations we may analyze further several of the processes of the preceding article. Thus, (a) in a *constant-volume* process the work term disappears, since $dV = 0$; and (b) in a *reversible constant-pressure* process the work term becomes $P(V_1 - V_2)$, since P is constant in the integration and $-\int_1^2 P dV$ simplifies directly to the form $-P \int_1^2 dV = P(V_1 - V_2)$.

⁵ This convention is at variance with that followed by the writers who propose that $\text{Work} = + \int_1^2 P dV$. Following the method used by Planck, it is preferred to consider the opposite viewpoint.

The relation will be further useful for energy analyses of various processes of such character that the equation of the P - V curve for the state change of the fluid may be closely determined. Thus we shall discover that for a *reversible (frictionless) adiabatic process with air* the equation of the state change curve is $P_2 V_2^{1.4} = P_1 V_1^{1.4} = P V^{1.4} = a$ constant. Therefore for such a curve any value of P equals $P_1 V_1^{1.4} V^{-1.4}$ and if the pressure and specific volume of the fluid at any point such as (1) is known the integral $-\int_1^2 P dV$ may be written as $-P_1 V_1^{1.4} \int_1^2 V^{-1.4} dV$ and the definite integral may be readily determined.

(b) *Steady-Flow Processes*. — Restating the steady-flow energy equation (3c) of Art. 31,

$$\frac{U_1^2}{64.34} + J E_1 + P_1 V_1 + W + J Q = \frac{U_2^2}{64.34} + J E_2 + P_2 V_2,$$

let the equation be rearranged in the form

$$J Q + \left[\frac{U_1^2 - U_2^2}{64.34} + (P_1 V_1 - P_2 V_2) + W \right] = J(E_2 - E_1). \quad (3d)$$

The latter arrangement will be observed to be one in which there are grouped together all energy terms which are mechanical in character, that is, the change in kinetic energy, the difference in the flow-work terms and the shaft-work. For convenience let us assign a name to this composite group of mechanical terms, designating their sum as the net **mechanical effects**.⁶ Thus we may write the steady-flow energy equation as

$$J Q + \text{Mechanical Effects} = J(E_2 - E_1). \quad (3e)$$

Scrutinizing this steady-flow relation in conjunction with the non-flow energy equation, $J Q + W = J(E_2 - E_1)$, it is observed that the two are essentially parallel. The second term of the steady-flow equation expresses the composite mechanical effects which may be involved in a steady-flow process; the second term of the non-flow equation represents the sole mechanical effect which is pertinent to a non-flow process. If now it is recalled that for the non-flow process when ideal in the feature of being mechanically reversible the mechanical effect of that process (the external work) may be concretely evaluated by the $-\int_1^2 P dV$, it would seem reasonable to conclude that the same integral might simi-

⁶ Any change in mechanical potential energy, $Z_1 - Z_2$, would also be included in this category.

larly serve to evaluate the net or composite mechanical effects in a mechanically reversible steady-flow process.

To validate this conclusion recourse must be had to a fact which can not be shown positively until later but which seems sufficiently reasonable to permit ready acceptance, namely, that if a given fluid is caused to pass through a specific sequence of states in or by a mechanically reversible process the state change must be accompanied by the same energy transition as heat Q whether the fluid is at rest or in motion during the change.⁷ Since for the given sequence of states $-\int_1^2 P dV$ and $(E_2 - E_1)$ evidently must also have magnitudes which are unique to the given state change whether it occurs under either circumstance, it follows that equation 8 $\left[JQ - \int_1^2 P dV = J(E_2 - E_1) \right]$ as developed for a non-flow process must apply to any given reversible state change whether carried out with the fluid at rest or in motion. It follows further from this relation and equations 3d or 3e above that the composite mechanical effects in a reversible steady-flow process are also evaluated concretely by the $-\int_1^2 P dV$.

To summarize, we may write that

$$-\int_1^2 P dV = \text{the mechanical effects in any mechanically reversible process carried out with a fluid,} \quad (9)$$

$$= \text{the energy in transition as work } W \text{ in a reversible non-flow process, and} \quad (9a)$$

$$= \left[\frac{U_1^2 - U_2^2}{64.34} + (P_1 V_1 - P_2 V_2) + W \right] \text{ in a reversible steady-flow process.} \quad (9b)$$

It should be noted again that, in view of the generality which has thus been ascribed to equation 8, that equation may be taken as a comprehensive energy equation which is applicable to all mechanically reversible processes, or

$$JQ - \int_1^2 P dV = J(E_2 - E_1) \text{ for any mechanically reversible process.} \quad (8)$$

⁷ This statement depends on the fact, shown in Arts. 53 and 54, that for any mechanically reversible process $\int_1^2 d'Q/T$ has a unique and invariable value for any specific state change of a particular fluid. As both $\int_1^2 d'Q/T$ and $f(T)$ have therefore values which are unique to the given sequence of states the value of Q for the reversible process must also be unique to the particular state change.

Expressing the same relation in differential form,

$$J d'Q - P dV = J dE. \quad (8a)$$

Although this relation is valid only for processes which are ideal in the feature of being mechanically reversible, its otherwise generality makes it an outstandingly useful relation.

Example 5. Find V_2 and $-\int_1^2 P dV$ for an expansion of 1 lb. of air from an initial pressure of 100 lb. per sq. in. abs. and a specific volume of 2.07 cu. ft. per lb. to a pressure of 14.7 lb. per sq. in. abs., the state change being of such character that: (a) $PV = \text{constant}$, (b) $PV^{1.4} = \text{constant}$. Plot the state change curve for each, having determined the coördinates at the final state and at least one intermediate state.
 (−57,100 ft-lb; −31,400 ft-lb.)

Example 6. If the state change of example 5b were a frictionless adiabatic one occurring in a properly shaped nozzle, what would be the final velocity of the stream issuing from the nozzle, presuming the initial velocity of the stream entering the nozzle to have been negligible.
 (1680 ft. per sec.)

37. Summary.—Having acquired adequate qualitative notions of the manners in which energy may be stored in a system and by which it may enter or leave a system or region, we are prepared to investigate more concretely the quantitative aspects of the various engineering processes in which such energy storage and transitions occur. The basis of such investigations is necessarily the **Conservation of Energy Principle**; and, when that Principle is formulated as an **Energy Equation**, it also becomes the working tool for the conduct of the investigation.

The following relation would provide a *general* form of equation which conforms to the Conservation Principle and which would serve to account fully for the various energy quantities involved in any process, viz.,

$$\text{Energy entering system} = \text{Energy leaving system} \text{ plus any accumulation or minus any diminution in the amount of energy stored within the system.} \quad (1)$$

This general equation will be of greater practical utility, however, if it is reexpressed in such a manner as to provide individual terms for measurement of the several specific quantities of energy which may be manifested during particular engineering processes. To formulate those more particular equations to better advantage it is necessary first to recognize the two classes of processes which will be encountered most frequently.

These two processes and their distinguishing characteristics are:

(a) *Steady-Flow Processes*, in which a fluid enters (or several fluids

enter), passes through and leaves a region or device at a steady rate, usually with the occurrence of some sort of state change of the fluid while enroute between entrance and exit and frequently with a steady, concurrent entry or departure of energy to or from the region as work or heat or both.

(b) *Non-Flow Processes*, in which a fluid is retained for a time within an enclosure and there undergoes a state change, usually by reason of a concurrent entry or departure of energy to or from the system as work or heat or both.

For a non-flow process the general energy equation is easily particularized by recognizing that the only manners of energy entry or departure will be as work and as heat and that the only manner of change of the energy stored in the system is by change in the internal energy of the fluid (or of the fluid and the container). The energy equation thus becomes

$$W_{\text{in}} + JQ_{\text{in}} = W_{\text{out}} + JQ_{\text{out}} + J(E_2 - E_1) \quad (6)$$

or, combining the several work terms and the several heat terms,

$$W + JQ = J(E_2 - E_1), \text{ ft-lb. per pound of fluid} \quad (7)$$

in which the general work or heat symbols represent the *net* amount of energy entry or departure by those two methods.

From the energy viewpoint the steady-flow process differs from the non-flow process in the absence of any energy accumulation or diminution within the steady-flow system and in the inclusion of several additional modes of energy entry to or departure from the system. These additional items are:

(a) The mechanical potential energy which is associated with the relative elevation of the fluids at sections in the entrance and exit passages, which are symbolized as $M'Z_1$ and $M'Z_2$ (ft-lb. per sec.) respectively but which may almost invariably be neglected in thermodynamic processes because of the insignificant magnitude of their difference.

(b) The mechanical kinetic energy which is associated with the velocity of the fluid at the entrance and exit sections of the region and which equals $M' \frac{U_1^2}{64.34}$ and $M' \frac{U_2^2}{64.34}$ (ft-lb. per sec.) at those respective sections.

(c) The flow-work which is associated with the shoving of the fluid into and out of the region against the pressure existing at the entrance and exit sections and which may be computed in terms of the pressure and specific volume of the fluid at those sections, namely, $M'P_1V_1$ and $M'P_2V_2$ (ft-lb. per sec.) respectively.

Although, in accordance with the defining conditions for steady-flow, there may not be any accumulation or diminution of stored energy within the system, there is nevertheless a steady entry and departure of stored molecular (and perhaps chemical) energy in the flowing fluid itself. Terms will therefore need to be supplied to permit accounting for this internal energy which is in transition by convection.

The general energy equation may thus be particularized for a steady-flow process (with a single fluid) as follows:

$$\begin{aligned} & M'Z_1 + \frac{M'U_1^2}{64.34} + M'P_1V_1 + M'JE_1 + M'W_{\text{in}} + M'JQ_{\text{in}} \\ &= M'Z_2 + \frac{M'U_2^2}{64.34} + M'P_2V_2 + M'JE_2 + M'W_{\text{out}} + M'JQ_{\text{out}} \quad (\text{ft.-lb.} \\ & \quad \text{per sec.}) \end{aligned} \quad (3b)$$

It is evident that in this case of the flow of a single fluid the energy quantities might be expressed quite as well and more conveniently in terms of the energy *per pound of fluid*. Also it has been noted that the mechanical potential energy terms may commonly be neglected in thermodynamic processes. Finally, since E , P and V are all properties of a fluid and thus are fixed invariably by its state, it follows that the combination of properties, $E + PV/J$, is equally a property of a fluid and may advantageously be assigned a name and symbol — for which the name *Enthalpy* and the symbol H (B.t.u. per pound mass of fluid) will be employed. With these simplifications and also following the previous practice of combining the two work terms and the two heat terms equation (3b) may be reexpressed in the following simpler form

$$\frac{U_1^2}{64.34} + JH_1 + W + JQ = \frac{U_2^2}{64.34} + JH_2 \quad (\text{ft.-lb. per lb.}). \quad (5)$$

In the subsequent use of this equation it will frequently be necessary to recognize that the steady-flow process implies an equal mass-rate of entry and of departure for any fluid which is engaged in the process. This rate M' may be expressed in terms of the area A of the particular passage through which it may be flowing, the fluid velocity U at that passage and the concurrent specific volume V of the fluid by the relation $M' = (UA)/V$. It follows that

$$M' = \frac{U_1A_1}{V_1} = \frac{U_2A_2}{V_2} = \frac{U_xA_x}{V_x}.$$

This relation is known as the **Continuity Equation**.

For certain purposes it is illuminating to so rearrange equation (3b) that all terms of a mechanical character shall be grouped together,

that is, in the arrangement

$$JQ + \left[(Z_1 - Z_2) + \frac{U_1^2 - U_2^2}{64.34} + (P_1 V_1 - P_2 V_2) + W \right] \\ = J(E_2 - E_1).$$

The bracketed terms in this relation represent what we shall designate as the **Mechanical Effects** which are involved in a steady-flow process. The equation as so arranged is evidently analogous to the non-flow energy equation

$$JQ + W = J(E_2 - E_1),$$

in which the single term W represents the sole mechanical effect which is involved in a non-flow process.

If any process, whether steady- or non-flow, could be carried out under ideal circumstances in which no mechanical friction, fluid friction nor turbulence should exist (that is, for a mechanically reversible process *and only for such a process*) the Mechanical Effects would depend only on the manner of state change undergone by the fluid. Furthermore if the exact character of that state change should be expressed by a functional relation between the pressure and specific volume of the fluid the Mechanical Effects might be evaluated by the integral

$$- \int_1^2 P \, dV, \text{ or} \\ - \int_1^2 P \, dV = \text{Mechanical Effects in any mechanically} \\ \text{reversible process.} \quad (9)$$

Introducing this relation in the several energy equations, we would have the relation

$$JQ - \int_1^2 P \, dV = J(E_2 - E_1), \quad (8)$$

or in differential form

$$J \, d'Q - P \, dV = J \, dE, \quad (8a)$$

which would apply to any *mechanically reversible process*.

38. Review Questions and Topics. —

1. Define a system.
2. Summarize the several forms of stored energy and transient energy, classifying them with respect to the storage and transition characteristics and with respect to any other coördinating characteristics.
3. (a) State the Principle of the Conservation of Energy and formulate the Principle as a general energy equation.
(b) State the two characteristic conditions under which energy transformations take place in the processes of engineering thermodynamics.
4. Summarize briefly the conditions implied in a steady-flow process.

5. (a) State and explain the Continuity Equation.
- (b) List and write the expressions by which to evaluate the several energy quantities which may enter and depart from a system by reason of the flow of a fluid to and from the system.
- (c) Explain the basis of derivation of the expression by which to evaluate the energy passage as flow-work.
- (d) Illustrate the several other manners in which energy may enter or depart from a steady-flow system.
- (e) Compile the various energy items in a comprehensive energy equation for steady-flow conditions and indicate the several simplifications of the comprehensive equation which may be made immediately.
6. (a) Define enthalpy and justify the introduction of this item as a fluid property and as a term in the steady-flow energy equation.
- (b) State several other names for the enthalpy property which are or have been in current use and discuss them.
- (c) Write the steady-flow energy equation in terms of the enthalpy property.
7. Evolve appropriate adaptations of the steady-flow equation for
 - (a) a steam boiler,
 - (b) a steam engine or turbine,
 - (c) a nozzle or other similar device,
 - (d) a throttle valve,
 - (e) a water-jacketed air compressor,
 - (f) the flow of a relatively incompressible liquid.
8. (a) Indicate the characteristic conditions of a non-flow process and illustrate.
- (b) Evolve the particular form of the general energy equation which will be of more specific utility for the analysis of a non-flow process.
9. Evolve appropriate adaptations of the non-flow energy equation for the case of
 - (a) an adiabatic state change,
 - (b) a constant-volume state change,
 - (c) a constant-pressure state change,
 - (d) an isothermal state change.
10. (a) Show that for a mechanically reversible non-flow process the amount of energy which enters or leaves as work equals algebraically $-\int_1^2 P dV$.
- (b) Explain the graphical interpretation of this integral.
- (c) Define the term Mechanical Effects as it is associated with a steady-flow process and state a method of evaluating the mechanical effects in a mechanically reversible steady-flow process.
- (d) Write an energy equation which will be generally applicable to any mechanically reversible process, whether steady-flow or non-flow.

Symbols and Abbreviations, Chapter III

- A* = area, sq. ft.
E = internal energy, B.t.u. per pound of a fluid.
F = force, lb.
H = enthalpy ($E + PV/J$), B.t.u. per pound of a fluid.
J = Joule's equivalent, 778 ft-lb. per B.t.u.
M' = mass-rate of flow of a fluid, lb. per sec.
P = absolute pressure, lb. per sq. ft.

Q = energy in transition by radiation or (and) conduction (as heat, B.t.u. per pound of a fluid).

s = distance, ft.

U = velocity, ft. per sec.

V = specific volume, cu. ft. per lb.

W = energy in transition as work through a shaft or other medium, ft-lb. per pound of a fluid.

Z = elevation, ft. and (closely) potential energy, ft-lb. per lb.

PART II — THE AVAILABILITY OF ENERGY

39. Foreword. — In the discussions of Part I an endeavor was made to develop certain concrete conceptions of energy as it may be stored in a system or a fluid and as it may be in process of transition from one system to another. Reasoning from the principle of the Conservation of Energy (the First Law of Thermodynamics) there were also developed various energy equations which will be found to be extremely useful in the subsequent analyses of those ideal and practical engineering processes with which we shall be concerned.

Indispensable as the foregoing developments may be, a further scrutiny of the progress made to this point will indicate that certain vitally important information is still lacking. This may be illustrated by considering the simple example of a steam engine or turbine, the energy equation for which was shown to be, neglecting radiation,

$$W = J(H_1 - H_2) \text{ (see Art. 33-b).}$$

This relation states that the work output from a given machine (per pound of fluid passing through) must equal the difference between the enthalpy of the steam entering and that leaving — and that is all that the equation as quoted does state. The conclusion is interesting and important; but of unquestionably greater importance, both theoretical and practical, would be information concerning the minimum attainable magnitude of H_2 for a given value of H_1 and thus concerning *any limitation to the maximum work output which might be attained* per pound of steam and concerning the factors which would influence that output. Neither the energy equation nor any other information which has as yet been considered would offer any assistance in detecting such limitations or, if they do exist, in evaluating them.

Further illustrations might be cited but all would simply emphasize the same inadequacy of our present information. It will therefore be the purpose of Part II (a) to consider certain basic principles and lines of logical reasoning which will disclose the unescapable limitations of our present methods of power generation and will delineate the factors which affect those limitations and (b) to develop the practical scheme which the engineer employs for predicting the optimum performance attainable by those current methods of power production.

In order to accomplish those purposes more effectively it is desirable that initially we should observe several fundamental characteristics of

those contrivances for the industrial production of power which man has been able to devise to date.

First we may recognize that, of all of the forms of energy, shaft-work is the *sine qua non*, the indispensable, in any power generation system. It is the form which inherently the engineer may utilize to greatest advantage and with maximum versatility. Therefore he naturally regards it as the highest *grade* of energy and establishes it as the standard for evaluating the relative grade or *availability* of the other forms, evaluating that grade in terms of the ease and completeness with which they may be converted to shaft-work. It follows that when we subsequently discuss in detail the *availability of energy* of one or another form we are simply endeavoring to ascertain *the degree to which that energy may ideally be transformed to shaft-work*. To illustrate in passing, electrical energy would be regarded as being of a high order of availability in view of the ease and the highly efficient manner in which it may be converted to shaft-work by the electric motor.

A second and a most important characteristic which we must recognize is that, with the exception of wind and water power plants, all commercial processes which have been devised as yet by man for acquiring shaft-work from the energy stores of nature depend (a) on the release of the chemical energy of a *fuel* by the process which we know as *combustion*, and (b) on the concurrent delivery of this energy to the material products of the combustion, whereby they are brought to an *elevated temperature*. It is by direct reason of this latter feature that we shall be justified in directing our subsequent attention to an evaluation of the availability of energy which exists at an elevated temperature level and which possesses utility by reason of its existence at such a level.

In proceeding with the evaluation of the availability of such energy the order of procedure will be first to conceive several idealized processes for the utilization of energy at high temperature, then to recognize the validity of such processes as universal vehicles for measuring the availability of the energy. This will serve to evolve a general method for ascertaining the inherent limitations to the efficiency of our power production machinery and to develop the practical device by which we may readily predict the optimum performance attainable with particular machines operating under particular circumstances and may intelligently analyze those machines with the view of discerning the factors which affect their performance.

CHAPTER IV

REVERSIBLE CYCLES

40. Cycles. — A series of processes which may be repeated over and over in the same order is said to be a **cyclic process** or a **cycle**. The cycles employed in engineering practice are of two general forms. The first are those in which a machine regularly repeats the same series of processes, any simultaneous state of *the machine and its contents* being duplicated in all respects at the same relative instant in each successive cycle. An example is the cycle of an internal-combustion engine, in which the engine returns periodically to any given state as regards the position and motions of its parts, and likewise as regards the state of the fluid it may contain at the instant — but it is to be noted that a different mass of fluid is taken in on each successive cycle and any particular mass does not again return to the same state.

In distinction to this a second type of cycle is *one in which the working fluid undergoes a series of processes of such character that it returns periodically to the same state*. An example is the condensing steam power plant.

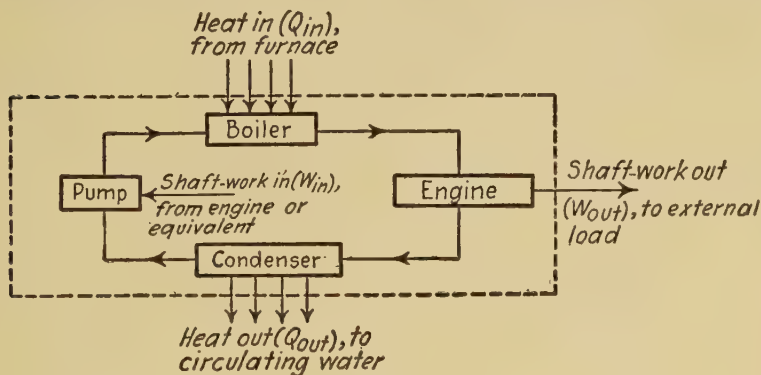


FIG. 6

The diagram, Fig. 6, indicates the major apparatus in such a plant and also shows the direction of flow of the fluid and the character and direction of the energy input and output. This system is steady-flow in character and the cyclic nature of the complete process is apparent. If we regard the system to include the entire series of apparatus and connecting pipes, that is, all parts which are enclosed within the broken

lines of the figure, it is evident that the only energy quantities with which we shall need to be immediately concerned are the energy which enters the system as heat by conduction from the furnace through the boiler tubes and that which leaves as shaft-work from the engine and as heat from the steam space of the condenser (by conduction through the condenser tubes to the circulating water). Thus the energy equation for the cycle becomes

$$JQ_{\text{in}} = W_{\text{out}} + JQ_{\text{out}}, \quad \text{or} \quad J(Q) + (W) = 0, \quad (1)$$

where the brackets indicate that the terms represent the *net energy interchanges for the complete cycle*.

A third type of cycle would be a *non-flow cycle in which a working fluid undergoes a series of processes while retained wholly within a cylinder with piston*, whence the cycle becomes in effect one in which a *fluid cycle* and a *machine cycle* are carried out jointly. Such cycles are not employed in modern engineering practice for the simple reason that in practice the several processes may be performed more advantageously in separate devices which are specifically designed for performing the several distinctive functions. However, though such cycles may be purely academic, we shall find it highly convenient to conceive several hypothetical ones of that character and to analyze at least one of them in some detail. That procedure will be wholly justified not only by its expediency but also by the fact that certain information acquired by the analysis of those cycles will be found to be applicable to any real cycle and thus to be a valuable guide in ascertaining the limitations to the efficiency of the more practical cycles.

The hypothetical non-flow cycles which we shall analyze will properly be those which acquire their energy from some high temperature region. That region we shall term the **source**, and we shall presume for convenience that the energy is transmitted from the source to the fluid within the cylinder by heat conduction and (or) radiation. The cycles are naturally to be ones the function of which is to make available as work at least some portion of the energy which was delivered to the fluid. Anticipating the possibility that not all of the energy received from the source may be delivered as work, it will be presumed that any such unavailable portion will be discarded to some region of lower temperature (such as the atmosphere). That region we shall call the **receiver**. The temperature of the source will be designated as θ_1 , and that of the receiver as θ_2 , both temperatures being taken for the present simply as indices of relative hotness and *without reference to any particular scale of temperature*.

Example 1. In a cyclic process 100 units of energy were delivered to a system from a source as heat and 70 were rejected to some receiver. What external work was done and what was the thermal efficiency of the process? (Thermal efficiency = work output/heat input.)

Example 2. In a condensing steam power plant, 13,000 B.t.u. (per pound of oil fired) were delivered as heat through the boiler tubes to the steam. Twenty-five per cent of this energy was converted into work by the turbines. Neglecting any energy dissipated by radiation in the plant, how many B.t.u. must have been given out from the steam as heat to the condensing water, in B.t.u. per pound of fuel and in B.t.u. per horsepower-hour? What was the oil rate of the plant, in pounds of oil per horsepower-hour? (9,750; 7,640; 0.78.)

Example 3. Represent on P - V coördinates the general character of the state changes undergone by the fluid in the elementary steam power plant cycle of Fig. 6. Indicate on the graph the apparatus in which each state change takes place.

41. Reversible Cycles. — A reversible cycle is *per se* one in which all processes of the cycle are wholly reversible ones. Recalling (Arts. 20–21) that those aspects of reversibility which are germane to the engineering processes are *mechanical* reversibility and *thermal* reversibility, the reversibility of all processes of a cycle and thus of the cycle as a whole would predicate the idealized conditions of (a) the entire absence of friction in the moving parts of the machine in which the cycle is carried out, of fluid friction and of turbulence, and (b) the absence of any sensible temperature difference between any two regions between which energy is to be delivered by conduction or radiation (that is, by heat). The desirability of the former condition in any process is obvious; the equal desirability of the latter in all power generation processes which operate by reason of an energy supply at elevated temperature will become quite apparent in subsequent articles.

It will also be essential that we recall (Art. 19) the fundamental criteria of reversibility: (a) that the sequence of states and events which makes up the process (or cycle) may be exactly retraced in reverse order, and (b) that by such retracing all energy transitions or transformations of the original process (or cycle) may be accomplished in a reverse direction, whereby there shall be a complete restoration of all energy to its original status. The specific implications of these general conditions and criteria will become evident in the analysis of a reversible cycle in the following article.

The several idealized, reversible cycles to which engineers usually give attention are (a) the Carnot cycle, (b) the Stirling cycle, (c) the Ericsson cycle, and (d) the Rankine cycle using saturated steam and regenerative feed-water heating, usually known as the regenerative cycle.

The Carnot cycle is a non-flow cycle and admittedly a rather visionary

one but it is also the one which is the most simple to analyze and is of outstanding historical interest. It will be considered in detail both because of its simplicity and in spite of its impracticability, that impracticability being no objection for the reason that certain conclusions which are deducible from the analysis of *any* reversible cycle, will be found to have universal utility.

The Stirling and Ericsson cycles are the prototypes of actual engines which were built and used commercially during the latter half of the last century. They became obsolete, however, because of various mechanical and metallurgical disadvantages, although test reports indicate that they attained thermal efficiencies (based on indicated horsepower output) well above the efficiencies of high grade modern plants. Because their interest is primarily historical and their analysis is fairly involved they will not be considered further.

The regenerative cycle is the prototype of many modern steam power plant installations but, due to the relative complexity of its analysis, further attention to that cycle will be deferred until the steam power plant is under immediate consideration (see Art. 125).

42. The Carnot Cycle. — Let us conceive a cylinder equipped with piston, rod, et cetera and provided with three alternative cylinder heads, each available for being placed in contact with the cylinder end. Referring to Fig. 7, the cylinder, piston and middle head are to be composed of a material which is perfectly non-conductive of heat and the piston is to be frictionless. The upper head will act as the **source** of energy at the upper temperature θ_1 and for simplicity it will be considered that this source either is of infinite energy capacity or is in some manner maintained at a constant upper temperature even though energy departs from it. The lower head will act as the **receiver** of energy at the lower temperature θ_2 and may likewise be maintained at that temperature. It is considered that any one of the three heads may be applied to the cylinder at will.

Let it also be considered that the cylinder contains a mass of, say, one pound of any compressible fluid at the lower temperature θ_2 . For convenience let the temperature be presumed to be atmospheric temperature. The pressure, specific volume and temperature of the fluid at that initial state we shall designate as P_a , V_a and θ_2 . The state is represented by the point a in the P - V diagrams appearing in the figure above the cylinder. The cycle will be made up of four individual and successive processes carried out as described in the following. These processes are portrayed in the P - V diagrams of Fig. 7 about as they would appear for a gas and for a saturated vapor.

a-b. Since it is specified that energy shall be delivered reversibly

from the source at θ_1 to the fluid within the cylinder it is necessary that before that transition takes place the fluid shall have been brought from the lower to the upper temperature (or to a temperature only infinitesimally below it), and brought there reversibly. This may be done by compressing the fluid under such circumstances that all energy which is

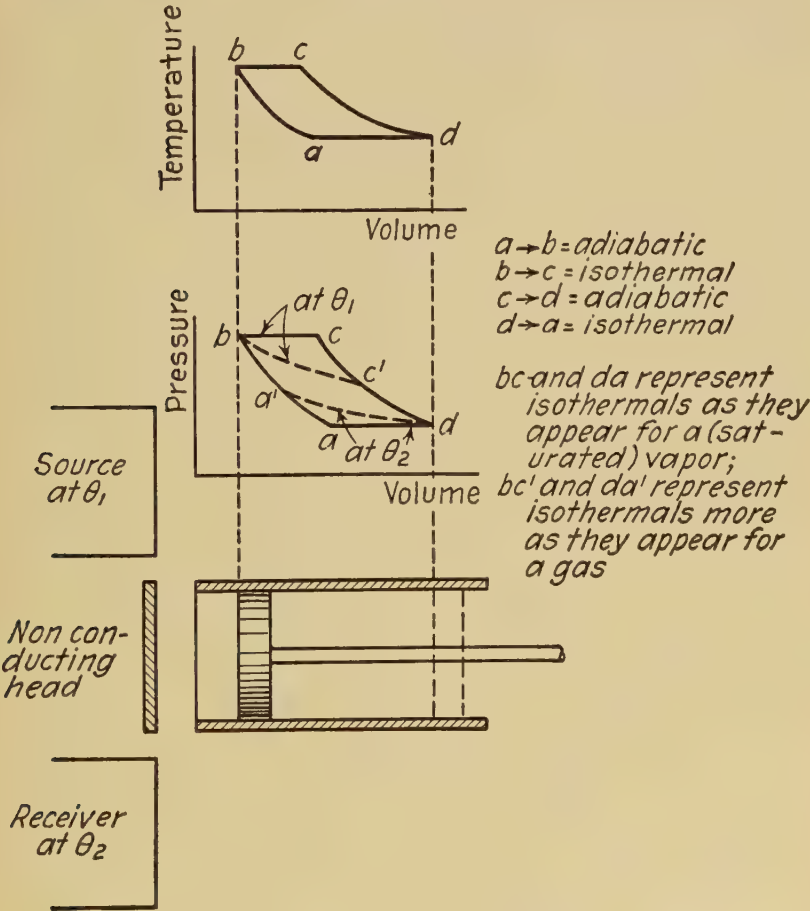


FIG. 7. Carnot Cycle and Engine.

supplied as work for the compression shall be absorbed by the fluid and thus act to increase its temperature, no energy being dissipated in any manner. Specifically this would be accomplished by a *reversible adiabatic compression* with the non-conducting head in place. The process would be accompanied by an entry of energy to the system as work in

the amount $-\int_a^b P dV$ (see Art. 36). It will be recalled that this integral and therefore the work supplied will be represented by the area under the state change curve $a-b$.

$b-c$. With the fluid in state b at (or infinitesimally below) the upper temperature of the cycle the system is prepared for the reversible reception of energy from the source. This may be accomplished by a *reversible isothermal expansion* of the fluid at θ_1 with the hot head in place on the cylinder end. During the expansion energy will be delivered from the source in an amount which we shall designate as Q . Simultaneously, by reason of the reversible expansion, energy will be delivered as work through the piston and rod to some external system in the amount $-\int_b^c P dV$. In view of the maintenance of the fluid at the upper temperature, and thus at a temperature which is invariably higher than that during the preceding adiabatic compression, it may properly be anticipated that at any given volume the pressure of the fluid during the isothermal state change will exceed that during the adiabatic and thus that the state change curve $b-c$ will lie above the curve $a-b$.

$c-d$. Upon completion of the energy reception phase at the upper temperature of the cycle it will be desirable that additional work output be secured if possible. This may be accomplished by continued expansion of the fluid with the non-conducting head again in place, the expansion to be a *reversible adiabatic expansion* in order that no energy shall be dissipated. The process being adiabatic, this additional work output must be secured at the expense of the internal energy of the fluid and thus will undoubtedly be accompanied by a pressure and temperature drop. As the cycle is to be a continuous one operating between the source and receiver temperatures it will be necessary that the expansion be discontinued when the temperature has fallen to (or infinitesimally above) the temperature of the receiver. The state when that occurs is designated by point d . Energy will have been delivered to the external system as work in the amount $-\int_c^d P dV$.

$d-a$. To close the cycle it will be necessary that the fluid which is now at temperature θ_2 but at the specific volume V_d shall be returned reversibly to the initial state (θ_2 and V_a). This may be accomplished by a *reversible isothermal compression* with the receiver head in place. For this compression work must be returned from the external system in the amount $-\int_d^a P dV$. While it is not yet proven it may be

anticipated that some energy departure from the fluid to the receiver will accompany this compression. For any energy so rejected we shall use the symbol Q_R .

The fluid has thus been returned to its original state (in preparation for a repetition of the cycle) and therefore possesses the same amount of internal energy as originally, whence there has been no accumulation nor diminution of the energy stored within the system. It follows that the net amount of energy which has departed from the system as work must equal the difference between the energy supplied from the hot source (Q) and that rejected to the cooler receiver (Q_R). This net work output will equal the algebraic sum of the several values of $-\int P dV$, which we shall represent symbolically as (the "cyclic" integral) $-(\int)P dV$, and will be measured by the area $abcd$ which is enclosed by the cycle as portrayed on P - V coördinates.

It is evident that the machine has acted as an **engine**, or as it is frequently called, a **heat engine**, in that it has *delivered* energy as work. The thermal efficiency of the engine will be the ratio,

$$\frac{\text{Net work output}}{\text{Energy input from source}} = \frac{Q - Q_R}{Q} = \frac{-(\int)P dV/J}{Q}. \quad (1)$$

43. The Reversal of the Carnot Cycle. — Referring to the P - V diagrams of the Carnot cycle as just described, we observe that the cycle was traversed in a clockwise direction. Because each process was specified to be reversible it follows that each might be performed in a reverse or counter-clockwise direction and that the series might also be performed in a reverse order. Starting again with the same initial point of the cycle this reversal would require, briefly, the following sequence of events and energy transitions:

a-d. An *isothermal expansion* at θ_2 , with a return of energy to the system from the prior receiver in the amount Q_R and an energy delivery from the system as work in the amount $-\int_a^d P dV$.

d-c. An *adiabatic compression* from θ_2 to θ_1 , which would require the provision of energy as work from some external system in the amount $-\int_d^c P dV$.

c-b. An *isothermal compression* at θ_1 , which would require a continued external supply of energy as work in the amount $-\int_c^b P dV$

and would be accompanied by an energy return to the prior source in the amount Q .

b-a. An *adiabatic expansion* from θ_1 to θ_2 , with an energy delivery from the system as work in the amount $-\int_b^a P dV$.

Scrutinizing this reversed series of processes we observe that, in conformity with the fundamental criterion of reversibility, in order to drive the machine there has now been required a net supply of energy as work which equals that delivered by the machine when operating as an engine. Also we observe that the energy previously discarded to the lower temperature receiver has been retracted and that this energy plus the work energy supplied has been returned to the original high temperature source, and returned in an amount exactly equal to that previously supplied by the source.

Because of the abstraction of energy (as heat, Q_R) from a lower temperature region which is thus accomplished by this reversed sequence of reversed processes the cycle is known as that of a **heat pump**, in distinction to the direct operation of the cycle as an *engine*. In this feature the heat pump is the prototype of the common refrigerating or ice-making plant, and for that reason the energy abstracted by the heat pump from the lower temperature region is frequently referred to as the **refrigerating effect**.

44. Summary. — In the foregoing analysis there has been developed the general conception of a reversible cycle and a specific conception of the operation of the Carnot engine and of its reversal. It is immaterial that the mechanism by which the cycle functioned was fantastic and impractical or that reversibility is an idealistic condition which is inherently unattainable in all real processes. It may even be of no great import in itself that one should remember the exact state changes of the cycle or their sequence. It is however most essential that one shall have clearly recognized the fundamental objective of the cycle. Let us emphasize that by summarizing it briefly.

The basic purpose of the cycle was, when operating as an engine, to acquire energy from a source at high temperature, to make *available* as work some portion of that energy, to discard to a lower temperature receiver such portion of the energy as might perhaps be *unavailable* and to accomplish all of these objectives in such a manner that by a reversal of the cycle all energy might be retracted from the receiver and from the external work-absorbing system and be wholly restored to the hot source.

Keeping clearly in mind this conception of the objective and essential characteristic of any reversible cycle we are now in a position to recog-

nize the validity and importance of a certain general principle which will enable us to discover "the unescapable limitations of our present methods of power production" (Art. 39) and to determine the conditions for maximum efficiency of power producing plants. The principle is considered in the following chapter.

Example 4. If a reversible engine operating between certain temperatures gave an efficiency of 30 per cent, what would be the ratio between the refrigerating effect and the work required for operating the cycle as a heat pump between the same temperatures? (2.33.)

Example 5. Assume an engine operating on the Carnot cycle with complete reversibility except that 10 per cent of the work is dissipated due to friction in the moving parts. If the efficiency of the reversible cycle were 30 per cent what would be the efficiency of the assumed engine? If the same amount of energy were required to overcome friction if the machine were operated as a heat pump, what would be the ratio between the refrigerating effect and the work required? (27%; 2.12.)

Example 6. Assuming a source and a receiver at 1000°F. and 70°F. respectively, what temperatures must the fluid have during the heat energy reception and rejection phases of a Carnot heat engine cycle operating between the specified source and receiver if a 50°F. temperature difference between the source (or the receiver) and the fluid is necessary in order to effect the heat transfer; what fluid temperatures would be necessary if the fluid were operating on a heat pump cycle between the same source and receiver temperatures? Comment on the reversibility or irreversibility of the fluid cycle.

If it were specified that the fluid cycle should itself be wholly reversible, with an upper temperature of 1000°F. and a lower temperature of 70°F. , what temperatures would the source and the receiver have to have when operating the cycle as a heat engine and what temperatures when operating as a heat pump? Comment on the reversibility or irreversibility of the entire arrangement.

45. Review Questions and Topics. —

1. (a) What character of information is as yet lacking but highly important for the adequate analysis of our power generation processes?

(b) What is implied in the expression, "availability of energy?"

(c) What is a vital characteristic of all processes which have been devised for the production of power from the fuel resources and in what respect should this characteristic influence our subsequent studies?

2. (a) Define a cycle.

(b) Distinguish between a machine cycle and a fluid cycle and indicate how the two might be carried out jointly.

(c) Write the energy equation for a fluid cycle.

3. Review the essential characteristics which must maintain and the basic criteria which must be complied with in order that a cycle might be reversible.

4. (a) Describe the Carnot cycle when operating as a temperature or heat engine.

(b) Write the energy equation for each phase of the Carnot cycle and for the complete cycle.

5. Describe the Carnot cycle when operating as a heat pump and discuss with reference to the operation as a heat engine.

Symbols and Abbreviations, Chapter IV

Q = heat energy entering (or leaving) a reversible engine (or pump) at the upper temperature of the cycle.

Q_R = heat energy rejected (or absorbed) by a reversible engine (or pump) at the lower temperature of the cycle.

θ_1 = hotness of the upper temperature source of the energy for the cycle.

θ_2 = hotness of the lower temperature receiver of the energy for the cycle.

$-(\int)P dV$ = the algebraic summation of the work terms for the various processes of a reversible cycle, that is, the net work output (or input) for the cycle.

CHAPTER V

THE CARNOT PRINCIPLE

46. The Carnot Principle. — In the summary of the last chapter it was indicated that there exists a general principle which will serve as an infallible guide for determining the conditions necessary for maximum efficiency in any engine which operates by reason of an energy supply at high temperature.

This basic principle is that known as the **Carnot Principle**. It was disclosed by a young French army officer and engineer, Sadi Carnot, in a classical essay published in 1824 and entitled: “Réflexions sur la puissance motrice du feu et sur les machines propres à développer cette puissance” (or as popularly paraphrased, “The Motive Power of Heat”).

The Carnot Principle has three general but intimately associated aspects or propositions which may be summarized as follows:

Proposition I. No engine which continuously delivers work output by the reception of energy at high temperature (with rejection of any residue of energy at a lower temperature) can be more efficient than a heat engine operating on a reversible cycle between a source and a receiver at those temperatures.

Proposition II. The efficiency of all reversible heat engine cycles working between the same temperatures is the same, irrespective of the cycle or of the working fluid used in the engine.

Proposition III. The efficiency of a reversible heat engine cycle depends only on the temperature of the source

and receiver and, for a given value of either, depends on the *difference* of the two temperatures.

The validity of the Principle may be shown very clearly by a simple process of logical reasoning. To facilitate that let us refer to Fig. 8, in which there are indicated diagrammatically:

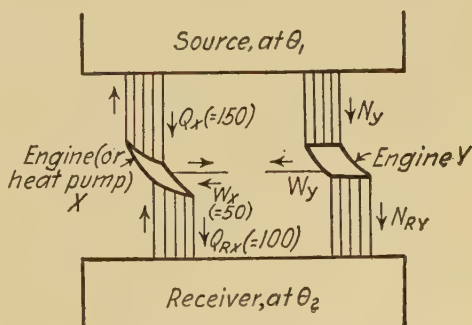


FIG. 8

A source of energy at temperature θ_1 , the temperature being again simply a measure of relative hotness without reference to any particular or arbitrary scale of temperature.

A receiver of energy at a temperature or degree of hotness θ_2 .

A heat engine X operating between the source and receiver on any reversible cycle and with any working substance, receiving energy as heat from the source in the amount Q_X and rejecting energy as heat to the receiver in the amount Q_{RX} .

Any engine Y , whether reversible or otherwise, operating between the same source and receiver, receiving energy in the amount N_Y from the source and rejecting energy in the amount N_{RY} to the receiver.

Quantities of energy appearing as work are similarly represented by the symbols W_X and W_Y .

The method of proof of the several propositions is that known in logic as the "Reductio ad absurdum," in which the opposite of a proposition is assumed and then shown to be impossible, whereby the proposition is proven. The several propositions will be considered in order.

Proposition I. Since heat engine X is a reversible one it may operate either as an engine and thus deliver work, or as a heat pump with work input required to drive it. For present purposes let us presume it to act as a pump and, simply for convenience, let any numerical values be assigned to the energies Q_X , Q_{RX} and W_X , such as respectively 150, 100 and 50 units of energy. As an engine it would thus have had a thermal efficiency of 33 per cent.

Let us now assume that, contrary to Proposition I, engine Y is more efficient than the reversible heat engine X or may have a thermal efficiency of, say, 40 per cent, and let the work output of engine Y be employed to drive engine X as a heat pump. If we stipulate for convenience that engine Y has no other load then its energy input (N_Y) from the source would needs be $(50/0.40 =)$ 125 units and the discard to the receiver (N_{RY}) would be $(125 - 50 =)$ 75 units.

It might thus appear that engine Y and pump X comprise a combination which, when once started, might continue in operation indefinitely. That condition alone would not in any way violate the *First Law of Thermodynamics* (the principle of the Conservation of Energy) but, when the energy quantities are investigated further it is seen (a) that this self-acting combination is delivering continuously to the hotter source an excess of 25 units of energy more than is being withdrawn, (b) that this energy is being secured by the net withdrawal of 25 units of energy from the cooler receiver and (6) that these 25 units are in the form of heat which is thus passing continuously and with no external aid from the cooler receiver to the hotter source.

Here the absurdity is recognized. The common experience of all thoughtful persons indicates that energy will not of itself pass *as heat* continuously from a region of lower temperature to one of higher temperature, and that no self-acting mechanism could accomplish this result. To quote Clausius, the great German contributor to the science of thermodynamics:

“It is impossible for a self-acting machine, unaided by any external agency, to convey heat continuously from one body to another at a higher temperature.”

This thought (or various others which are intrinsically equivalent to it) is known as the **Second Law of Thermodynamics**. The two Laws and the Carnot Principle are the very backbone of the entire science of engineering thermodynamics.

If the assumption that the engine Y can be more efficient than the reversible heat engine X has thus led to an absurd conclusion we are forced to regard the assumption as untenable and to consider Proposition I as proven.

Proposition II. The proof of this proposition is practically identical with that of Proposition I, except that engine X and engine Y are both taken to be reversible heat engines. It is also emphasized that the working fluid in either may be of any character, using perhaps one substance in X and a different substance with different properties in Y , and that the cycles employed may be wholly different so long as both are reversible.

Making again a preliminary assumption, contrary to Proposition II, to the effect that engine Y may be more efficient than engine X , and presuming the engine of less efficiency to be reversed and operated as a heat pump by means of the output of the more efficient engine, then the same line of reasoning as was employed in the consideration of Proposition I will show that Y could not be more efficient than X without violating the Second Law.

Similarly engine X might be assumed to be the more efficient one and the fallacy of that assumption be shown in the same manner. If, therefore, the reversible heat engine Y can not be more efficient than the reversible heat engine X , nor X more efficient than Y , then their efficiencies must be equal and Proposition II is proven.

Proposition III. To investigate this proposition we may conceive of a source and receiver at temperatures θ_1 and θ_2 , and a reversible heat engine X operating between them. However let the single engine Y be replaced by several reversible heat engines so arranged in series that the receiver of the first, at an intermediate temperature θ' , may act as

the source for the second. These two engines in series are represented in Fig. 9 by engines 1 and 2.

If the combined output of engine 1 and engine 2 ($W_1 + W_2$) is employed to drive engine X as a heat pump, then Q_X must equal Q_1 and Q_{RX} must equal Q_{R2} , else a self-acting mechanism would result which again would violate the Second Law. Also, since each of the serial engines is delivering work output, $Q_{R1}(=Q_2)$ must be less than Q_1 but exceed Q_{R2} .

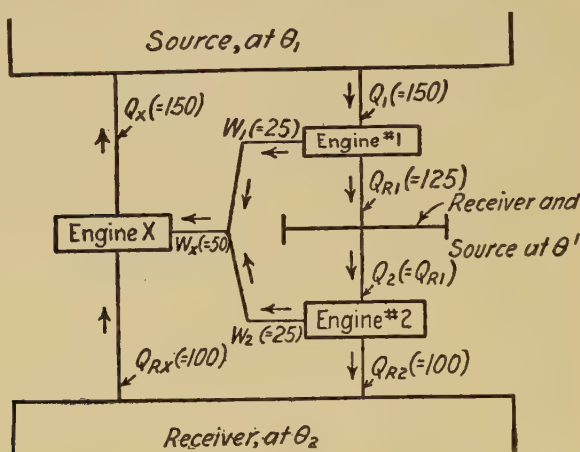


FIG. 9

It may be seen to follow that the efficiency of either engine 1 or engine 2 must therefore be less than that of engine X. To verify this conclusion by concrete numerical values, if again engine X operating as a heat pump is taken to deliver 150 units of energy to the source, to receive 50 units as work for motivation and to abstract 100 units from the receiver, then engine 1 must take 150 units from the source and engine 2 must discard 100 units to the receiver. If it is further taken that engine 1 shall deliver 25 of the 50 units of work which go to drive the pump X and that engine 2 shall supply the remaining 25 units of work, then the energy discard from engine 1 and the energy supply to engine 2 will be 125 units. Investigating the efficiencies of the several engines, that of engine 1 will be $(25/150 =)$ 16.7 per cent, that of engine 2 will be $(25/125 =)$ 20 per cent and that of pump X if operating reversed as an engine between the full temperature range would be $(50/150 =)$ 33.3 per cent.

The significant feature of these results is that the efficiency of either of the two engines which operate between the smaller temperature ranges $(\theta_1 - \theta')$ and $(\theta' - \theta_2)$ must invariably be less than the efficiency

of engine X which operates within the greater range $(\theta_1 - \theta_2)$. The important general conclusions are (1) that for a given source temperature the efficiency of a reversible engine will depend on the receiver temperature, (2) that similarly with a given receiver temperature any increase in the source temperature will increase the efficiency and (3) that finally the reversible heat engine efficiency must therefore depend directly on the temperature of the source and receiver and, for a given value of either, on the difference of the two temperatures.

Since in no case has any limitations been set on the type of reversible cycle employed or on the substance used we may regard Proposition III as proven.

Example 1. Demonstrate Proposition I for the presumed condition that $Q_X = Q_Y$, in distinction to the above presumption that $W_X = W_Y$.

Example 2. Demonstrate Proposition I for the presumed condition that $Q_{RX} = Q_{RY}$. (Hint — use the surplus work from engine Y to drive another heat pump.)

CHAPTER VI

THE FUNDAMENTAL TEMPERATURE SCALE

The third proposition of the Carnot Principle established the fact that, irrespective of the fluid or cycle employed, the efficiency of any reversible heat engine cycle depends solely on the relative hotness of its source and of its receiver. This aspect of the Principle is probably the one of outstanding practical importance in that it will furnish the working tool by which we may evaluate definitely the efficiencies of any reversible cycle and by which we shall subsequently be able to determine the ideal or maximum conceivable efficiencies of our real cycles.

We observe, however, that no concrete functional relation between efficiency and temperature has as yet been formulated, nor has any definite scale or thermometric device been established by which temperature shall be measured. In fact it was emphasized in the foregoing chapter that the temperature of the source and receiver should for the time be regarded simply as indices of relative hotness without reference to any scale or method of measuring that hotness. It will be the purpose of the following articles to develop the conception of a very fundamental scale of temperature and to ascertain by what device any degree of hotness may be definitely allocated on that scale.

47. The Kelvin Scale. — The problem before us is that of formulating a temperature scale which shall express concretely the influence of the source and receiver temperatures on the performance of a reversible heat engine or of any reversible engine which operates by reason of a difference in the temperature at which the engine acquires its energy supply and that at which it discards any unavailable residue of the supplied energy. The scale must evidently be predicated on the third proposition of the Carnot principle.

The most direct procedure in evolving such a scale will be that of conceiving, quite arbitrarily, a scale in which some simple functional relation shall exist between the energy supply and discard of the reversible engine and its source and receiver temperatures. Probably no simpler scale might be conceived nor, as will be evident very shortly, any more fundamental scale than one in which the ratio between the energy discard to the receiver and the energy reception from the source shall equal the ratio between the relative temperature of the receiver and that of the source. Expressed symbolically the relation defining

such an **energy scale of temperature** would be

$$\frac{\theta_2}{\theta_1} = \frac{Q_R}{Q} \tag{1}$$

This relation may perhaps be represented graphically to some advantage. Fig. 10 is one type of representation and Fig. 11 is another. In Fig. 10 vertical distances represent temperatures on this fundamental *energy* scale and horizontal distances represent energy supplied or rejected by the reversible engine. If any values of source and receiver temperature are selected and for those values lines are extended from the vertical axis for distances which measure the energies supplied and rejected, that is, lines such as those to points 1 and 2, then to conform with the requirements of equation (1) the locus of all such points must be a straight line.

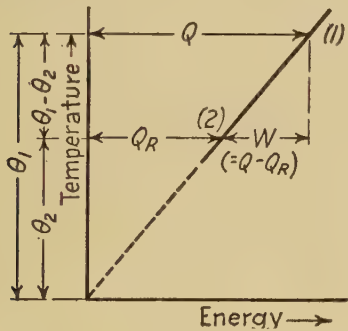


FIG. 10

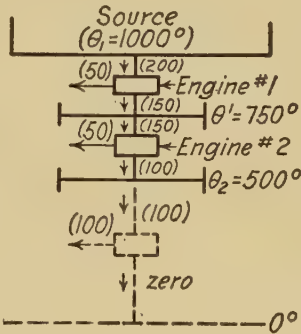


FIG. 11

Fig. 11 is parallel to Fig. 9 of the last chapter in the representation of a source at θ_1 , two engines (1 and 2) in series, with an intermediate receiver-source at θ' , and a receiver at θ_2 . Relative values have been given to the several energy supplies and rejections and then numbers have been assigned to the temperatures of the several regions, the sole limitation in the assignment of numbers being that they shall be ones which will conform to the requirements of equation (1).

Both figures serve to indicate quite obviously the effect of a continued reduction of the receiver temperature, to wit: that if that temperature should be made successively lower and lower a temperature might finally be reached with the use of which the entire energy supply from a source might conceivably be made available as work. That condition would mean, specifically, a 100 per cent thermal efficiency for any reversible engine operating with that receiver temperature. Such an efficiency and therefore *such a minimum temperature is ideally conceiv-*

able, but any lower temperature is utterly inconceivable. It is inconceivable for the reason that with it as a receiver a reversible engine would operate with an efficiency in excess of 100 per cent and would deliver more energy as work than it had received. Such a condition would violate the First Law.

We have thus arrived at a conception of a very fundamental or **absolute zero of temperature** and of a similarly fundamental basis for evaluating temperature or for formulating a temperature scale which depends solely upon energy relations in ideal, reversible heat engine cycles and is wholly independent of the characteristics of any particular substance. This use of the reversible temperature engine as a means for defining an absolute zero of temperature and the fundamental temperature scale was first developed in a general way by Lord Kelvin in 1848. For that reason this *energy* or *thermodynamic* scale is frequently designated as the **Kelvin temperature scale**. The term **Rankine** scale is also used in this connection. The distinction between the two will be stated later (Art. 49). Temperatures measured on this scale are known as **absolute temperatures**.

A significant adjunct of this temperature scale may be made evident by a simple adjustment of equation (1). Subtracting both sides of that equation from unity we have

$$\frac{\theta_1 - \theta_2}{\theta_1} = \frac{Q - Q_R}{Q};$$

but $\frac{Q - Q_R}{Q} = \frac{W/J}{Q}$ = thermal efficiency of the reversible engine, whence we may write the important relation,

$$\text{Thermal efficiency of any reversible heat engine} = \frac{\theta_1 - \theta_2}{\theta_1} \quad (2)$$

where θ_1 and θ_2 are temperatures on the energy or thermodynamic scale of temperature. Frequent reference will be made to this important relation.

The above conception of the energy scale of temperature, important as it may be, has not acted to fix any definite numerical value of the temperature at any certain condition of hotness, although it has closely designated the factor controlling the ratio which shall exist between the numbers assigned to any two "hotnesses." The next problem is therefore the attainment of some practical way of ascertaining the proper relative locations of any given conditions of hotness on the Kelvin or energy scale, this in spite of the fact that recourse to tests of a perfectly reversible engine is in the nature of the case impossible. The problem is considered in the following article.

48. The Realization of the Kelvin Scale in Practical Thermometry. —

The realization of the Kelvin scale in practical thermometry, that is, the contriving of means whereby the Kelvin temperature of a region or substance may be measured, though of utmost ultimate importance to the engineer, is the particular task of the physicist rather than of the engineer. However, for the sake of completeness, there are outlined here as briefly as is consistent the more or less devious lines of reasoning and experimentation by which that realization has been accomplished. It is not regarded as essential that the engineering student shall master the details of this development but it is desirable that it be read through attentively, and it is important that the final conclusions which are presented in the last paragraph of the article should be adequately appreciated.

Recognizing the non-existence of any reversible engine it is evident that whatever method may be devised for realizing the energy-temperature scale it must be an indirect method. Indeed it must be indirect in more senses than one, as is further evident when we recognize that the only methods which man has as yet devised for measuring temperature (whether on the energy scale or on any scale) are wholly inferential methods that rigidly are not temperature measurements at all. For example, in our only available procedures we actually measure properties of substances such as their electrical resistance, electromotive force, pressure, specific volume et cetera — simply properties which we know by experience to change in magnitude with change of hotness of the substance but which in themselves provide only inferential evidence of hotness.

In view of these circumstances it might be anticipated that in attempting to measure temperature on the energy scale it will still be necessary to do so actually by the measurement of various properties other than temperature, albeit ones which depend on and offer evidence of the temperature of a substance. The real problem lies therefore in the selection of a substance which will possess measureable properties that might properly be expected to reflect its temperature and that will also conform to such further requirements as would make it an acceptable substance for indicating temperature *on the Kelvin scale*. In attacking the problem the course is that first of conceiving arbitrarily a hypothetical substance which would possess these necessary characteristics and then of investigating to ascertain if any such substance exists in nature.

Two of the most directly and exactly measurable properties are pressure and specific volume (or density). Also it will be recalled (Arts. 8 and 9) that these properties may be regarded as being very intimately associated with temperature as evidences of the internal molecular state of a substance, the pressure being conceived to be the result of molecular bombardments and to depend in magnitude on the number of molecules in a given space and their individual mass (whence on density or specific volume) and on the molecular kinetic energy (whence on the temperature). Thus it would seem eminently sensible and suitable to designate the pressure or the specific volume or both as the accessory properties which shall be employed as evidences of temperature in the selected thermometric substance.

For reasons which soon will be more evident let there be selected for the hypothetical substance a fluid in the gaseous state and let there be arbitrarily assigned to

the imaginary gas two characteristics which it needs to possess in order that it may serve for indicating temperatures on the energy (Kelvin) scale. With our present information it may easily appear that the two assigned characteristics are somewhat independent ones. On the contrary, by a purely mathematical analysis of the energy equation for a reversible process (which analyses appears in Part V, Art. 200) it may be shown that a fluid which would conform to either characteristic would of necessity conform to the other.

The first assigned characteristic which is ascribed to the proposed gas is that, *for a state change of the gas from one reproducible temperature to a second, the ratio between the initial and the final pressure when the state change is performed at constant volume shall equal the ratio between the initial and final specific volume for a constant pressure state change.* Expressed symbolically,

$$\left(\frac{P_1}{P_2}\right)_V \text{ shall equal } \left(\frac{V_1}{V_2}\right)_P,$$

in which the subscripts V and P indicate the constancy of those properties during the several state changes.

The second but concurrent characteristic is that *the internal molecular energy (E) of the gas shall depend solely on its temperature*, being the same at any particular temperature irrespective of the otherwise condition of the gas but changing of course with change of temperature.

The bare assignment of these two characteristics, though done with reason, has in no way validated the use of the hypothetical gas as a thermometric substance for realizing the *Kelvin* temperature scale. Nevertheless, if one should so wish, it would be quite proper to postulate a temperature scale which should be based solely on the foregoing P - V characteristics of the gas or, in particular, a scale for which the ratio between any two temperatures should be specified to equal the above pressure or volume ratio, that is

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)_V = \left(\frac{V_1}{V_2}\right)_P, \quad (3)$$

in which relation T_1 and T_2 represent temperatures assigned in accordance with data obtained from a constant-volume or a constant-pressure thermometer employing the hypothetical gas. Let such a temperature scale be so postulated and two features of the scale be developed.

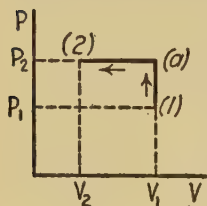


FIG. 12

To develop the first feature observe that if the gas were changed from any one temperature (T_1) to any second temperature (T_2) by a series of two processes, one a constant-volume process and the other a constant-pressure process (as represented by lines 1- a and a -2 in Fig. 12), then for the two processes

$$\frac{T_a}{T_1} = \frac{P_a}{P_1} = \frac{P_2}{P_1}, \quad \text{and} \quad \frac{T_2}{T_a} = \frac{V_2}{V_a} = \frac{V_2}{V_1}.$$

Eliminating T_a by multiplying the first equation by the second,

$$\frac{T_2}{T_1} = \frac{P_2 V_2}{P_1 V_1}, \quad \text{or} \quad \frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1}.$$

Since the two states 1 and 2 were specified to be any states whatsoever, the interpretation of the last relation is that the product of the absolute pressure and the specific volume of the gas divided by the temperature as indicated by the gas thermometer

will give a result which will be the same for any state of the gas. This may be expressed in symbols by the relation,

$$\frac{PV}{T} = \text{a constant } (R). \quad (4)$$

To develop the second and the highly significant feature of this hypothetical gas thermometer let us investigate the property and energy relations for the gas if it were employed in a reversible engine such as the Carnot. The cycle of that engine will be recalled to be made up of two isothermal and two adiabatic state changes (Art. 42). A P - V diagram for the cycle is repeated for convenience in Fig. 13 (see also Fig. 7). Investigating the energy relations of the two adiabatics, since for an adiabatic $Q = \text{zero}$, the equation for either is

$$O = \Delta E + \frac{1}{J} \int P dV.$$

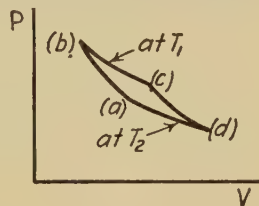


FIG. 13

Since for the postulated gas $P = RT/V$ the equation may be rewritten as

$$O = \Delta E + \frac{R}{J} \int T \frac{dV}{V},$$

or in differential form

$$O = dE + \frac{RT}{J} dV/V.$$

Dividing through by T and transposing,

$$\frac{dE}{T} = -\frac{R}{J} \frac{dV}{V}.$$

Writing this as the definite integral between the upper and the lower temperatures of the cycle,

$$\int_b^a \frac{dE}{T} = -\frac{R}{J} \log_e \frac{V_a}{V_b}, \quad \text{and also} \quad \int_c^d \frac{dE}{T} = -\frac{R}{J} \log_e \frac{V_d}{V_c}.$$

But E and T are mutually dependent variables (from the second characteristic assigned to the gas), and also $T_a = T_2 = T_d$ and $T_b = T_1 = T_c$, whence

$$\int_b^a \frac{dE}{T} = \int_c^d \frac{dE}{T}, \quad \text{and therefore} \quad \frac{V_a}{V_b} = \frac{V_d}{V_c} \quad \text{or} \quad \frac{V_c}{V_b} = \frac{V_d}{V_a}. \quad (5)$$

Investigating the energy relations of the isothermal at T_1 , since for the postulated gas ΔE at constant temperature is zero, the energy equation becomes $Q = \frac{1}{J} \int_b^c P dV$.

But also for the postulated gas $P = RT/V$ and the energy equation may be rewritten as

$$Q = \frac{RT_1}{J} \int_b^c \frac{dV}{V} = \frac{RT_1}{J} \log_e \frac{V_c}{V_b}.$$

By analogy the energy equation for the isothermal at T_2 may be written as

$$Q_R = \frac{RT_2}{J} \log_e \frac{V_d}{V_a}.$$

But equation 5 showed that the volume ratios appearing in these two expressions are equal, whence

$$\frac{Q}{Q_R} = \frac{T_1}{T_2}. \quad (6)$$

Recall now that, from the Carnot Principle and the Second Law, the ratio Q/Q_R for a reversible Carnot cycle operating between a given source and receiver may have *only one value*, irrespective of the scale which might be used to designate the temperatures of those regions, but also note that the above energy relation (Eq. 6) is identical in form with the original equation (Eq. 1, Art. 47) which served to define the Kelvin scale of temperature. It follows that a temperature scale which is postulated upon the $PV/T = \text{constant}$ characteristic of the above designated gas must be identical with the Kelvin or energy scale of temperature and therefore that temperatures measured by employing the gas in a constant-volume or constant-pressure type of thermometer must in truth be, jointly, energy-scale temperatures.

There have thus been discovered and validated the requisite characteristics of a standard thermometric substance for realizing the Kelvin scale. Any gas conforming to those characteristics would be known as a **Perfect Gas**. It only remains to ascertain if any such gas exists.

For the purpose of searching to discover if there exists any real gas which conforms to the standards of the Perfect Gas probably the simplest test is one based on the characteristics formulated in equation 3 of this article. To describe the test more conveniently let that equation be rearranged in the form

$$\left(\frac{P_1 - P_2}{P_2} \right)_V = \frac{T_1 - T_2}{T_2} = \left(\frac{V_1 - V_2}{V_2} \right)_P,$$

or better,

$$\left[\frac{P_1 - P_2}{P_2(T_1 - T_2)} \right]_V = \frac{1}{T_2} = \left[\frac{V_1 - V_2}{V_2(T_1 - T_2)} \right]_P. \quad (7)$$

The immediate importance and utility of equation 7 lies in its indication of the requisite equality between the first and last terms which would have to exist for any gas in order that it might conform to the Perfect Gas requirements and thus might serve as the sought-for thermometric fluid.

To determine these terms concretely for a particular gas it would be necessary to select and have available two regions or conditions of different but exactly reproducible "hotness" and to arbitrarily assign a specific value to the difference in temperature of the two ($T_1 - T_2$). The temperature conditions frequently selected are those of melting ice (T_2) and of water boiling at standard atmospheric pressure (T_1). All are familiar with the fact that the Centigrade scale arbitrarily assigns 100 degrees to this temperature range and the Fahrenheit scale assigns ($212 - 32 =$) 180 degrees to the same range. This range is known in physical parlance as the **fundamental interval**. The values of the two terms,

$$\left[\frac{P_1 - P_2}{P_2(T_1 - T_2)} \right]_V \quad \text{and} \quad \left[\frac{V_1 - V_2}{V_2(T_1 - T_2)} \right]_P,$$

when determined for a given gas and for this interval are known by the physicist as, respectively, the **pressure coefficient** and the **volume coefficient** of the gas.

To proceed with the determination of these coefficients the further steps would be to bring a given mass of the gas to the one temperature and then to the second, maintaining in one instance a constant gas volume and measuring the pressure of the gas at the two temperatures (P_1 and P_2), and maintaining in a second instance a constant gas pressure and measuring the relative volumes occupied by the gas at

the same two temperatures (V_1 and V_2). Evidently such data are sufficient for the computation of the pressure and volume coefficients of the gas, whereby investigation may be made as to their requisite equality.

Physical tests of this character on various of the more permanent gases disclose that none conform exactly to the fundamental criterion of absolute equality of the pressure and volume coefficients, whence none may be said to be a Perfect Gas. Furthermore, such tests disclose that varying values are obtained for either coefficient of a gas as the tests are performed under different imposed conditions. The quest for a suitable real gas by which to realize the Kelvin scale might thus appear to be ineffectual. However, fortunately, the very manner of variation of the coefficients with variation of the imposed pressures offers very strong evidence of a certain condition under which any of the gases would approach "perfection" and also points definitely to a particular gas by the proper use of which the Kelvin scale may in fact be reproduced with an accuracy which is adequate for practically all engineering purposes.

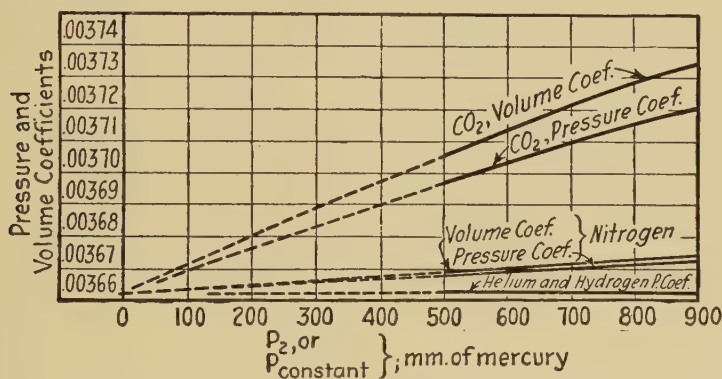


FIG. 14. Graph showing convergence of coefficients with decrease of pressure.

To present this evidence there have been plotted in Fig. 14 (a), as ordinates, the values of the pressure and volume coefficients as found for several gases for which data is available and (b), as abscissas, the values of, respectively, P_2 and the constant pressure at which the coefficients have been obtained. The specific values of the coefficients are based on the use of the 100° of the Centigrade scale for the fundamental interval. The figure portrays clearly the above-noted non-equality and variation of the coefficients. However the extrapolation of the plotted curves affords the significant and important evidence (a) that any of these gases would give mutually identical values for both the pressure and the volume coefficient if the determination were made at zero pressure, (b) that at such a pressure they all would thus conform to the Perfect Gas criterion, and finally (c) that the pressure coefficient of hydrogen or helium gas for this fundamental interval and for P_2 of about atmospheric pressure is virtually what is indicated for all of the gases at zero pressure and for the Perfect Gas, or 0.003662 (closely).

From these and other evidences it has been concluded by the physicist that the *constant volume hydrogen thermometer and helium thermometer* may be taken as valid standard instruments for realizing in practical thermometry the Kelvin or fundamental energy scale of temperature. It follows that any commercial thermometric

device whatsoever, if calibrated by comparison with such a standard instrument, adequately measures temperature on the fundamental scale.¹

Considering the above conclusion that for the Perfect Gas the pressure coefficient would equal 0.003662 when 100 degrees are assigned to the fundamental interval between the ice and steam points, an alternative and useful interpretation of this conclusion may be evolved by a rearrangement of equation 7. Thus, since

$$\left[\frac{P_1 - P_2}{P_2} \times \frac{1}{T_1 - T_2} \right]_V = 0.003662, \text{ at } (T_1 - T_2) = 100,$$

then, for the fundamental interval (and irrespective of the number of degrees assigned to that interval),

$$\left[\frac{P_1 - P_2}{P_2} \right]_V = \left[\frac{P_1}{P_2} \right]_V - 1 = 0.3662$$

and

$$\left[\frac{P_1}{P_2} \right]_V = 1.3662.$$

Finally, recalling that according to the basic P - T relation for the Perfect Gas as expressed in equation 3, i.e. $(P_1/P_2)_V = T_1/T_2$, it follows that

$$\frac{T_1}{T_2} = \frac{T_{\text{steam point}}}{T_{\text{ice point}}} = 1.3662.$$

This relation states that *on the fundamental energy or thermodynamic scale the steam point temperature and the ice point temperature are in the proportion of 1.3662 to 1.00, this irrespective of the number of degrees which may be assigned to the interval.*

Let us summarize briefly the foregoing developments in the arrival at a method for ascertaining the temperature of any region *on the energy or thermodynamic scale*. The several major steps made were as follows:

A. A hypothetical "perfect" gas was conceived which should possess the two interdependent characteristics (1) that the internal molecular energy of the gas should depend only on its hotness and (2) that for a constant volume state change and for a constant pressure state change of the gas between two reproducible levels of hotness the relation $(P_2/P_1)_V = (V_2/V_1)_P$ should exist, whence it developed that a *perfect gas temperature scale* could be conceived in which $\frac{T_2}{T_1} = \frac{P_2 V_2}{P_1 V_1}$ or $\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1} = \text{a constant}$. It was then shown that if such a gas were employed in a reversible heat engine such as the Carnot the relation between the cycle efficiency and the perfect gas temperatures would be the same as the originally selected relation between the efficiency and the Kelvin temperature, whence from the Carnot Principle the identity of the two scales was established.

B. A search was made among the actual gases for one which would conform to the requisite P - V - T relation of the perfect gas, this search being prosecuted by testing for equality of their *pressure coefficient* and *volume coefficient*. None were found to meet the test but evidence was secured which indicated that any of them would conform if they should be tested at zero pressure and that the pressure coefficients of hydrogen and helium are nearly constant over a considerable range of pressure and temperature and are virtually equal to both coefficients of the perfect

¹ At rather high temperatures a physical absorption and perhaps some chemical reaction occurs between hydrogen gas and the walls of the container. For that reason the constant-volume nitrogen thermometer is considered to be more reliable as the standard for higher temperature ranges.

gas. These two gases thus meet to that extent the perfect gas criterion and for that reason they have been selected as fluids which, if employed in a constant-volume gas thermometer, will enable the realization of the thermodynamic scale of temperature.

The final conclusions which the student needs carry from the foregoing developments are:

(1) If the gases hydrogen or helium are enclosed in a *constant-volume* container (such as a constant volume gas thermometer) and if their pressures are measured when the mass of gas is at any two conditions of hotness, the ratio between the absolute pressures when at the two temperature levels will equal the ratio between the Kelvin temperatures at those temperature levels; that is, $(P_2/P_1)_V = T_2/T_1$.

(2) For the fundamental interval of temperature between the temperature of boiling water at normal atmospheric pressure (the "steam point") and the temperature of melting ice at the same pressure (the "ice point") the ratio of the Kelvin temperatures is 1.3662.

(3) In the feature that the *pressure ratio at constant volume* equals the ratio of Kelvin temperatures the above gases conform to one requirement of a Perfect Gas. A complimentary requirement of the **Perfect Gas** is that at any two given temperature levels the *specific volume ratio for a constant pressure state change* would also equal the ratio of the Kelvin temperatures; that is, $(V_2/V_1)_P = T_2/T_1$.

49. Absolute Temperature. — By long established custom commercial thermometers are graduated with scales on which the *ice point* and the *steam point* are called respectively 0° and 100° on *Centigrade* thermometers or 32° and 212° on *Fahrenheit* thermometers. It is to be anticipated that we shall frequently wish to know the relative location of these and other temperatures with reference to the absolute zero of temperatures, which is to say that we shall wish to be able to convert direct thermometer "readings" to temperatures on the fundamental energy scale. Temperatures on that scale are known as **absolute temperatures**. For such temperatures we shall hereafter employ the symbol T , discarding the symbol θ which was introduced to convey the general notion of relative hotness and prior to the evolution of a concrete means for properly evaluating that hotness on the energy scale.

It is evident that if we may ascertain the location of the absolute zero with reference to some point on the ordinary thermometer scale we shall thereby have enabled the ready conversion of a thermometer reading to the corresponding absolute temperature. To ascertain the location of the absolute zero reference needs only to be made to the second of the final conclusions of the preceding article, to the effect that

$$T_1/T_2 = 1.3662,$$

where T_1 = steam point, absolute temperature,
 T_2 = ice point, absolute temperature.

Since in the Centigrade scale ($T_1 - T_2$) is taken arbitrarily as 100 degrees, or $T_1 = T_2 + 100$, then from the above

$$(T_2 + 100)/T_2 = 1.3662 \quad \text{and} \quad T_2 = 273.1^\circ \text{C.}$$

The location of the ice point and thus the zero of the Centigrade scale is therefore 273.1°C. above the absolute zero. It follows that the Kelvin temperature of any region equals the temperature as read on a Centigrade thermometer plus 273.1 , or

$$\text{Absolute temperature, } ^\circ\text{K.} = \text{Temperature, } ^\circ\text{C.} + 273.1. \quad (8)$$

Since similarly in the Fahrenheit scale ($T_1 - T_2$) is taken arbitrarily as 180 degrees, or $T_1 = T_2 + 180$, then for this scale $(T_2 + 180)/T_2 = 1.3662$ and T_2 must equal 491.6 . The location of the ice point is thus 491.6°F. above the absolute zero, and the location of the zero of the Fahrenheit scale is $(491.6 - 32 =) 459.6^\circ \text{F.}$ above the absolute zero.

To permit easy distinction between absolute temperatures expressed with reference to these two common thermometric scales it is convenient and customary to refer to absolute temperatures expressed with reference to the Fahrenheit scale as "degrees Rankine" ($^\circ\text{R.}$). Therefore

$$\text{Absolute temperature, } ^\circ\text{R.} = \text{Temperature, } ^\circ\text{F.} + 459.6. \quad (9)$$

For most engineering computations adequate accuracy is obtained if the absolute zero is taken as 273°C. or 460°F. below the zero of the respective scales.

50. Efficiency of Reversible Temperature Engines or Cycles. — In Art. 47 there was conceived a fundamental energy scale of temperature predicated on the assignment of such temperatures to a source and receiver that the ratio of those temperatures should equal the ratio between the energy supplied by the source (Q) and the energy rejected to the receiver (Q_R) for any reversible heat engine operating between the two. It was also shown that for such a temperature scale (equation 2)

$$\begin{aligned} \text{Thermal efficiency of any reversible heat engine} &= \frac{Q - Q_R}{Q} = \frac{\theta_1 - \theta_2}{\theta_1} \\ &= 1 - \frac{\theta_2}{\theta_1}. \end{aligned} \quad (2)$$

In the two preceding articles it was shown that the absolute temperature T as measured by the ordinary thermometer, if that is properly calibrated by comparison with a constant-volume hydrogen or helium

thermometer, is an adequately exact measure of the temperature θ on the fundamental scale. Therefore we may write that

$$\begin{aligned}\text{Thermal efficiency of any reversible heat engine} &= \frac{T_1 - T_2}{T_1} \\ &= 1 - \frac{T_2}{T_1}.\end{aligned}\tag{10}$$

Four important conclusions are directly evident from this expression for ideal engine efficiency:

(a) That fundamentally the so-called **heat engines** are ones which for their operation depend wholly upon the reception of energy by the working fluid at a temperature which is in excess of that temperature at which the (unavailable) energy must be rejected by the fluid. For this reason it seems quite as appropriate and perhaps more exact to designate the engines under consideration by the term **temperature engines**, thereby permitting the inclusion of engines, such as the internal-combustion engine, which do not receive their energy as heat from an external source but still depend upon temperatures for their operation.

(b) That the ideal efficiency of any temperature engine may be increased by increase of the temperature T_1 at which the working fluid of the cycle receives its energy. The practical recognition of this condition is reflected in the continual trend toward higher steam pressures and consequent higher temperatures in all power plant design and in various refinements in the modern plant. The condition is also evidenced in the greater efficiencies attainable in the internal-combustion engine, in which the working fluid (the products of combustion) receives its energy at distinctly higher temperatures than does the steam in any steam power plant.

(c) That the ideal efficiency of a temperature engine may be increased by decrease of the "exhaust" temperature at which the fluid discards the unavailable energy. Recognition of this condition is reflected in the efforts of the engineer to reduce the pressure and consequent temperature to which the steam expands in an engine and at which it enters the condenser. There is, however, a definite limit for the possible improvement in that direction for the simple reason that man has continuously available in nature no temperature lower than that of the atmosphere or of the water in the streams and seas. This circumstance acts to set a practical lower limit for T_2 at roughly 500 to 540° R. (40 to 80° F.)

(d) That the ideal efficiency of even the most perfect temperature engine could approach 100 per cent only as the temperature T_1 at which the working fluid received its energy from the source should approach infinity or the receiver temperature T_2 should approach abso-

lute zero. Both of these conditions are manifestly impossible. Thus we must conclude that in his power generation processes man must perforce content himself with efficiencies far below 100 per cent unless or until he shall devise means for utilizing the energy stores of nature without imposing a "temperature phase," that is, means which avoid the use of the temperature engine. As yet he knows of no such means.

Example 1. For a reversible cycle operating with a receiver temperature of 70° F. compute and plot the efficiencies for source temperatures ranging from 400° to 3000° F.

Example 2. For a reversible cycle operating with a source temperature of 600° F. compute and plot the efficiencies for receiver temperatures from 220° to 40° F.

51. Summary, Articles 39-50.—At this point let us summarize briefly the progressive line of the thought which has been developed through this Part II to the present point. The outstanding features in this development have been

A. The recognition that shaft-work is the form of energy in transition which is

- (a) the *sine qua non* of any power generation system,
- (b) the form which man may inherently utilize to greatest advantage and considers the highest "grade" of energy, and is
- (c) the standard for evaluating the *grade* or *availability* of the other forms of energy, evaluating that grade in terms of the ease and completeness with which the energy may be converted to shaft-work.

B. The observation in general that all processes devised as yet by man for acquiring shaft-work from the energy stores of nature depend on

- (a) the combustion of fuels and
- (b) the concurrent delivery of the liberated energy to the products of combustion *at an elevated temperature*.

C. An inquiry concerning the availability of the energy associated with a high temperature source, the inquiry being prosecuted by the successive steps of

- (a) the conception of a reversible cycle operating between an elevated temperature source for supplying energy to a fluid and a lower temperature to which the fluid might expand and at which any unavailable energy might be discarded,
- (b) the recognition that with such a high temperature source and lower temperature receiver
 - (1) no cycle or engine could utilize the energy supplied from the source more efficiently than a reversible engine,

- (2) all reversible engines operating between the same source and receiver temperature would have to have the same efficiency, irrespective of the particular fluid or cycle employed,
- (3) the efficiency of such reversible engines would depend only on the temperatures of the source and receiver, being specifically a function of the temperature of either and of their difference in temperature, (which three propositions comprise the Carnot Principle) and
- (c) The recognition of the consequent utility of the reversible cycle as a valid universal device for ascertaining the availability of energy supplied at a high temperature.

D. The conception of a limiting lower temperature or absolute zero of temperature, a temperature lower than which would be wholly inconceivable.

E. The conception of an energy scale of temperature for which the efficiency of a reversible engine would equal $(T_1 - T_2)/T_1$ and the discovery of several fluids (hydrogen and helium at constant volume), which will serve as thermometric substances for closely realizing this energy or Kelvin scale of temperature in practical thermometry.

Having thus established the foregoing general premises we are now in a position to evolve from them the practical device which is employed in evaluating the available and unavailable portions of the energy which may be supplied to a fluid while undergoing any sort of state change or engaging in various sorts of processes. That will be the objective of the following chapter.

52. Review Questions and Topics. —

1. (a) Summarize briefly the series of steps by which the concept of an energy scale of temperature was evolved.

(b) Express symbolically and explain the relation which defines the Kelvin scale.

(c) Express the thermal efficiency of any reversible temperature engine which receives and discards energy at constant temperatures, expressing it in terms of those temperatures when evaluated on the thermodynamic scale.

2. By what practical type of thermometer is the energy scale of temperature realized?

3. (a) What is the numerical value of the ratio between (1) the temperature range from the absolute zero to the ice point and (2) the temperature range from the ice point to the steam point?

(b) Show how the Kelvin temperature is computed from the Centigrade.

(c) Show how the Rankine temperature is computed from the Fahrenheit.

4. (a) Express the thermal efficiency of any reversible temperature engine which receives and discards energy at constant temperatures, expressing it in terms of those temperatures when evaluated on the Fahrenheit scale.

- (b) Discuss the influence of the upper temperature of the cycle.
- (c) Discuss the influence of the lower temperature of the cycle.
- (d) Discuss the efficiency limitations of any temperature engine cycle.

Symbols and Abbreviations, Chapter VI

$^{\circ}C$ = temperature on the Centigrade scale.

E = internal molecular energy (B.t.u. per pound mass).

$^{\circ}F$ = temperature on the Fahrenheit scale.

$^{\circ}K$ = absolute temperature on the Kelvin scale, = $^{\circ}C. + 273.1$.

P = absolute pressure (pound per square foot).

Q = heat energy entering or leaving a reversible cycle at the upper temperature of the cycle.

Q_R = heat energy leaving or entering a reversible cycle at the lower temperature of the cycle.

R = the gas constant (PV/T) for a perfect gas.

$^{\circ}R$ = absolute temperature on the Rankine scale, = $^{\circ}F. + 459.6$.

T = temperature of a region as measured directly or indirectly by a perfect gas thermometer or, synonymously, the absolute temperature of the region on the energy or thermodynamic scale of temperature.

θ = absolute hotness or temperature of a region on the energy scale.

V = specific volume (cubic foot per pound mass).

CHAPTER VII

ENTROPY, THE INDEX OF UNAVAILABILITY

In the foregoing chapters it was shown that, presuming a supply of energy at an elevated temperature and a receiver at a lower temperature (both of which conditions are representative of the conditions of operation of all of our devices for the generation of power from the energy stores in the *fuels*), a reversible cycle or engine is the most efficient device conceivable by which a portion of the energy may be made available as work. Such a cycle thus provides a universal *meter* for ascertaining the inherently available and unavailable portions of the energy supply and, further, the formulation of the efficiency of such a cycle in terms of the Kelvin (or Rankine) temperatures of the source and receiver enables a specific evaluation of the availability or unavailability of the energy. It will be the purpose of the present chapter to indicate a unique relation existing between the unavailable portion of the energy and the receiver temperature and to illustrate the invaluable engineering utility of this relation.

53. $\frac{Q_R}{T_2}$, Change of Entropy. — In Art. 50, equation (10), an expression was finally presented which evaluated the efficiency of any reversible cycle in terms of the (constant) Kelvin or Rankine temperatures of the source and receiver:

$$\text{Efficiency} \left(= \frac{Q - Q_R}{Q} \right) = \frac{T_1 - T_2}{T_1}.$$

For present purposes it will be convenient to rearrange this expression in a form parallel to the original relation upon which the Kelvin temperature scale was predicated, equation (1) of Art. 47, or

$$\frac{Q_R}{Q} = \frac{T_2}{T_1}, \quad \text{or} \quad \frac{Q_R}{T_2} = \frac{Q}{T_1}, \quad \text{or} \quad Q_R = Q \frac{T_2}{T_1}, \quad (1)$$

where

Q = the energy supplied from a source at absolute temperature T_1 .
 Q_R = the inherently *unavailable portion* of the energy Q , that is, the energy which must unescapably be rejected to a receiver at T_2 even by a wholly reversible engine operating between T_1 and T_2 .

This relation is a very useful one for evaluating the inescapably unavailable component of the energy which is supplied to an engine cycle at T_1 and under the condition of energy discard at T_2 . The relation lacks generality however in the feature that it is directly suitable only for the circumstance of a *constant* source and receiver temperature. In many instances we shall wish a more general energy relation which will provide at least for the circumstance of a source which is of finite energy capacity and which therefore must sooner or later fall in temperature as energy is withdrawn from it as heat. An extension of the above equation which will provide for that contingency may easily be developed if one imagines the energy supply of the finite and varying temperature source to be withdrawn in a series of infinitesimally small quantities ($d'Q$), each of such quantities being delivered successively to the reversible engine and utilized in so far as the efficiency of the cycle will permit, any residues being discarded to the receiver. If the symbol T_1 is taken to represent the effective average temperature of the source during each infinitesimal energy withdrawal then for any $d'Q$,

$$\frac{d'Q_R}{T_2} = \frac{d'Q}{T_1}, \quad \text{or} \quad \frac{Q_R}{T_2} = \int_{T_2}^{T_1} \frac{d'Q}{T_1} \quad \text{if } T_2 \text{ is taken as constant}^1. \quad (1a)$$

If a definite functional relation were known between the amount of energy withdrawn from the source as heat and the resulting temperature change of the source the indicated integration might easily be accomplished by calculus. It will be recalled (Art. 24) that the specific heat of a substance expresses exactly that relation, namely, specific heat (c) = $d'Q/dT$. Thus the above summation may be written in the form

$$\frac{Q_R}{T_2} = \int_{T_a}^{T_b} \frac{c \, dT}{T_1}, \quad (2)$$

where T_b and T_a are the limit values of T_1 ,

$$= c \log_e \frac{T_b}{T_a}, \quad (2a)$$

if c should be constant

These several relations provide direct and extremely useful expressions by which one may readily segregate the unavailable and available portions of the energy which may be supplied to or withdrawn from any system. Let us illustrate their use by several examples in which we

¹ As has been observed, the natural receiver for all discarded energy in our power generation cycles is the atmosphere or the streams, lakes or seas. Partly because of the wholly insignificant influence of our energy discards on the general average temperature of those regions and because of their more or less constant temperature and partly for other reasons which will become more evident later we shall ordinarily be justified in limiting our attention to the circumstance of an effectively constant receiver temperature, T_2 .

shall ascertain those portions of the energy which is delivered to or from various common engineering fluids during typical state changes of those fluids. In each example let us presume for the present that the fluid progresses from an initial state to a second state through some specified state change which is performed in a mechanically reversible manner, whereby all energy received by the fluid during the original process is returnable on reversing the process.

Example 1. Let a pound of water at 182 lb. per sq. in. abs. and 374° F. be caused to vaporize to dry steam at that temperature. It may be found from tables of the properties of steam that for this vaporization process an energy supply of 850 B.t.u. is required. It is desired to ascertain the unavailable portion (Q_R) of this energy as so supplied and the available portion ($Q - Q_R$), both with reference to receiver temperatures T_2 of (a) 32° F., (b) 0° F., and (c) 400° R., and also to determine the ratio Q_R/T_2 for each of these receiver temperatures.

Solution. — Two alternative viewpoints might be taken in proceeding to compute the desired results. In one the water at 374° F. and 182 lb. might be conceived to be contained within the cylinder of a reversible engine, to be completely vaporized at that temperature by an energy supply of 850 B.t.u. coming to the fluid in the cylinder from an external source and then to be caused to continue through a reversible cycle of processes such as constitute the Carnot cycle (Art. 42). By evaluating the efficiency of this reversible cycle, in conformity with the requirement that its efficiency must equal $(T_1 - T_2)/T_1$ (Art. 50), the available and unavailable portions of the energy received during the vaporization may be readily determined.

In the other viewpoint the fluid after a vaporization in any sort of container might be conceived to be used in turn as a source for *any sort of reversible heat engine*, condensing at the constant temperature of 374° F. as it emits energy as a source and in its complete condensation emitting the above 850 B.t.u. and thereby motivating the engine. The engine may thus be employed in effect as a *meter* for determining the available and unavailable portions of the energy which had originally been received by the fluid.

This latter viewpoint possesses certain advantages and will therefore be taken. However it is significant that whatever results might be obtained by proceeding from either viewpoint, the same results must obtain with the other, that is, the same available and unavailable portions must result in either case. Otherwise the Carnot Principle would be violated by having one reversible engine more efficient than another when both operate between the same temperature limits.

Proceeding with the solution from the second viewpoint — since the energy is supplied to the metering engine at constant temperature equation (1) above is applicable, $Q_R = Q T_2/T_1$. Thus, tabulating the computations and results,

At receiver temperatures T_2 of	32° F.	0° F.	400° R.
$Q_R = T_2 \frac{Q}{T_1} = T_2 \frac{850}{374 + 460} = T_2 \times 1.02 =$	503 B.t.u.	470	408
Available energy = $Q - Q_R =$	347 B.t.u.	380	442
Efficiency = $(Q - Q_R)/Q =$	0.410	0.447	0.520
$Q_R/T_2 =$	1.020	1.020	1.020

Several significant aspects of this example may well be recognized immediately. These are that:

(1) it constitutes a method of evaluating the unavailable energy which is assignable to a pound of steam as a result of its vaporization at a particular temperature (and pressure), but evaluated with due reference to a particular receiver temperature, and

(2) it emphasizes the important fact that, although for the specified state change of the fluid the unavailable energy depends on the selected receiver temperature, the ratio between the unavailable energy with reference to a given receiver temperature and that temperature (Q_R/T_2) is a constant quantity invariably associated with the specified state change.

Example 2. Let a pound of water at 32° F. be warmed to 374° F. and then act as a source of energy for a reversible engine, letting energy be taken from it as a source in an amount sufficient to cause the water to return in temperature from 374° to 32° F. The specific heat of water through this temperature range varies somewhat but may be taken to have an effective average value of 1.015.

Compute the amount of energy supplied to the water and then delivered from it when acting as the energy source for the engine, the unavailable and available portions of this energy, the ideal efficiency of its utilization, and the ratio Q_R/T_2 , for receiver temperatures of 32° F., 0° F. and 400° R.

Solution. — For the conditions of the problem the source temperature varies, but with an approximately constant specific heat, whence equation 2a may be employed for the solution. Tabulating the computations and results,

	At $T_2 =$	32° F.	0° F.	400° R.
$Q = c \Delta T = 1.015 \times (374 - 32) =$		347 B.t.u.	347	347
$Q_R = T_2 \times 1.015 \times \log_e \frac{374 + 460}{32 + 460} = T_2 \times 0.533 =$		262 B.t.u.	245	213
Available energy = $Q - Q_R =$		85 B.t.u.	102	134
Efficiency = $(Q - Q_R)/Q =$		0.245	0.294	0.385
$Q_R/T_2 =$		0.533	0.533	0.533

Note the identity of Q_R/T_2 for the several receiver temperatures.

Example 3. A pound of air is warmed from 70° to 170° F. at constant volume and then from 170° to 300° F. at constant pressure. Compute the total amount of energy required to accomplish the composite state change, the unavailable and available portions with respect to a receiver at 32° F., and the magnitude of the ratio Q_R/T_2 . Take the specific heat of air at constant volume to be 0.173 and at constant pressure to be 0.241. How would the magnitude of Q_R/T_2 be influenced by variation of the receiver temperature?

Solution.

$$Q = 0.173 (170 - 70) + 0.241 (300 - 170) = 17.3 + 31.3 = 48.6 \text{ B.t.u. per lb.}$$

$$\begin{aligned} Q_R \text{ (by Eq. 2a)} &= (460 + 32) \left(0.173 \log_e \frac{170 + 460}{70 + 460} + 0.241 \log_e \frac{300 + 460}{170 + 460} \right) \\ &= 492 (0.030 + 0.045) \\ &= 14.8 + 22.2 = 37.0 \text{ B.t.u. per lb.} \end{aligned}$$

$$\text{Available energy} = Q - Q_R = 48.6 - 37.0 = 11.6 \text{ B.t.u. per lb.}$$

$$Q_R/T_2 = (14.8 + 22.2)/(32 + 460) = 0.030 + 0.045 = 0.075 \text{ units per lb.}$$

The value of Q_R/T_2 would be the same for any receiver temperature selected since it is computed wholly in terms of the energy supplied during the original temperature rise.

Example 4. A pound of air in the cylinder of a Carnot engine receives energy from a source in the amount of 40 B.t.u. but the air is maintained at a constant temperature of 150° F., during the energy reception by permitting it to expand, that is, the air undergoes a reversible isothermal expansion. Compute the unavailable portion of the energy received by the air, with reference to a 32° F. receiver, and compute the ratio Q_R/T_2 .

Solution.

$$Q_R = (\text{by Eq. 1}) T_2(Q/T_1) = 492 \times 40/(150 + 460) = 492 \times 0.0656 = 32.3;$$

$$Q_R/T_2 = 32.3/492 = 0.0656.$$

Example 5. A pound of a fluid is compressed adiabatically and reversibly in a cylinder by the supplying of 10 B.t.u. of energy as work. What portion of the energy supply was originally unavailable and what portion was unavailable after the reception of the energy by the fluid during the reversible adiabatic compression?

Solution. The original 10 B.t.u. supplied *as work* were wholly available and none unavailable, from the very concept and significance of "available" energy (Art. 39). After the reversible adiabatic compression the whole of this energy has been delivered to and stored in the fluid as internal energy because, by the definition of an adiabatic process, no energy was transferred as heat (i.e., $W = JE_2 - JE_1$). However, since the process was specified to be reversible, the available energy which was furnished during the compression may be regained as work upon a reversible adiabatic expansion of the fluid and in an amount identical with that supplied (Art. 20). Therefore the energy which was stored in the fluid during the reversible adiabatic compression was quite as completely available as was the original work and none became unavailable during the process.

This conclusion holds for *all reversible adiabatic processes* but it is important to recognize that it will hold *only* for a *reversible* adiabatic for the reason that the existence of any fluid friction or turbulence (internal mechanical irreversibility) would of necessity prevent the complete recovery of the original available energy and would induce a corresponding degradation of a portion of the available to unavailable energy (Q_R), in an amount depending on the degree of irreversibility.

The same conclusion might be reached by inspection of equation (1a), which will be recalled as a relation based upon reversible processes. In that equation,

since $d'Q$ and thus $\int \frac{d'Q}{T}$ is zero for an adiabatic process, Q_R or Q_R/T_2 must likewise be zero for a reversible adiabatic process. Again, however, it is important to recognize that a process may be adiabatic ($d'Q = 0$) but at the same time incur an *increase* of unavailability (ΔQ_R) if internal mechanical irreversibility exists.

The foregoing examples have involved a variety of state changes of several distinct types of engineering fluids. However they jointly will serve to make manifest and further emphasize the several common but important characteristics of all processes with such fluids, to wit:

(A) When energy is supplied as heat to a fluid in such a manner as to produce a given state change (or when the energy is withdrawn in like amount on reversing the process) *a certain portion of the energy received by*

the fluid during the state change becomes inherently and unescapably unavailable for work production even if the fluid or the energy is employed in the most efficient device conceivable for the transformation of the energy to work, that is, with a reversible engine.

(B) The amount of the inherently unavailable energy thus chargeable to any unit mass of the fluid while passing through a given state change will vary with the receiver temperature selected for the reversible engine but *the quotient of the unavailable energy with reference to any particular receiver temperature divided by that temperature (Q_R/T_2) is independent of the receiver temperature and is thus a fixed and invariable quantity assignable to the given state change of a unit mass of the fluid.*

It should thus become evident that the quantity Q_R/T_2 is a valuable function of any state change of a fluid. By reason of its tremendous importance and utility in both the theoretical and practical work associated with any phase of thermodynamics scientists have assigned it a name, the **change of entropy** of the fluid. Also, if a state change of a fluid is one from some *standard* reference state to a second state, the fluid in the second state is said to have an **entropy** which equals the change of entropy associated with a state change from the reference state to the second.

To illustrate the use of the term observe that in example 1 of this article the steam during the reception of energy and vaporization would be said to undergo an *increase* of entropy of 1.02 units² per pound, or to *decrease* in entropy by 1.02 units per pound during the condensation, the increase or decrease being associated respectively with the reception of energy by the fluid or its departure from the fluid and the corresponding increase or decrease in the amount of unavailable energy thus "chargeable" to the fluid. Similarly the *change* (increase or decrease) *of the entropy* of the water between 32° and 374° F. (Ex. 2) was 0.533 and also, because water at 32° F. is commonly taken as the standard reference state for any water or steam product, the water at 374° F. would be said to have an *entropy* of 0.533 per pound. The change of entropy of the air during the composite constant-volume and constant-pressure state change of example 3 was $(0.030 + 0.045 =) 0.075$ per pound and during the isothermal state change of example 4 was 0.0656 per pound. During the reversible adiabatic state change of example 5 there was no change of entropy. Such a process would therefore be known as an **isentropic process** (a process of constant entropy).

² No name is assigned to the *unit* of entropy. In this connection it is interesting to note that if, from the energy concept of temperature, the absolute temperature may properly be regarded as having the dimension of energy, then the ratio Q_R/T_2 is dimensionless and an abstract number.

It has thus been shown that there is a fixed and definite change of entropy associated with any particular manner of *reversible* state change of a given (or unit) mass of a fluid. A still more useful characteristic of entropy change arises from the fact that the magnitude of the change will be the same for *any* manner of state change whatsoever by which a fluid may be brought from one given state to a second. That will be shown in the following article but prior to that let us formulate verbal and symbolical definitions of the change of entropy of a fluid and also set down the several relations by which the change may be readily computed.

In line with the foregoing, *the change of entropy of a fluid which accompanies any (reversible or irreversible) state change of the fluid may be defined as the change in the unavailable energy assignable or chargeable to the fluid divided by the absolute temperature of the receiver with reference to which the unavailable energy has been computed, or symbolically,*

$$\Delta S = Q_R/T_2; \quad dS = d'Q_R/T_2, \quad (3)$$

where ΔS or dS = change of entropy (per pound of fluid).

Although this definition expresses the intrinsic significance of entropy change it does not afford so direct a means of evaluating that change as is afforded by recalling that *for a reversible state change* which is accomplished by the delivery of energy to or from a fluid as heat

$$\frac{Q_R}{T_2} = \frac{Q}{T_1} = \int \frac{d'Q}{T_1} \quad (1 \text{ and } 1a)$$

$$\text{or} \quad = \int_{T_a}^{T_b} \frac{c \, dT}{T_1} \quad (2)$$

$$\text{or} \quad = c \log_e (T_b/T_a) \text{ if "c" is constant,} \quad (2a)$$

where T_1 is a constant or variable upper temperature and T_a and T_b are respectively the initial and final values of a variable upper temperature at which the fluid is receiving the energy, and where c = the specific heat for a variable temperature process. Thus, *for a reversible process,*

$$\Delta S = \frac{Q_R}{T_2} = \frac{Q}{T_1}, \text{ if } T_1 \text{ is constant} \quad (3a)$$

$$\text{or} \quad = \int \frac{d'Q}{T_1} = \int_a^b \frac{c \, dT}{T_1}, \text{ if } T_1 \text{ is variable} \quad (3b)$$

$$\text{or} \quad = c \log_e (T_b/T_a), \text{ if "c" is constant.} \quad (3c)$$

Example 6. Taking the mean specific heat of water between 32° and 366° F. to be 1.01 and the energy required for complete vaporization at that temperature to be 857 B.t.u. per pound, compute the increase of entropy per pound of fluid during the warming of the water from 32° to 366° F. and during the vaporization at that temperature. Also compute the unavailable portions of the energy received, with reference to a 32° receiver. Compare the computed values of the entropy increase

with the tabular values of the "entropy of water" and the "entropy of vaporization" as listed in any modern Steam Table for an absolute pressure of 165 lb. per sq. in. and a temperature of 366° F. (0.525; 1.038; 258 + 510 = 768.)

Example 7. Compute the increase of entropy of one pound of air resulting from an isothermal expansion during which 10 B.t.u. are supplied at 400° F., followed by a reversible adiabatic expansion to 150° F. (0.0116.)

54. Entropy, a State Function or Property of a Fluid. — In the examples of the preceding article we considered various engineering fluids in progress from one state to a second by certain specific mechanically reversible processes, employing the examples to convey the general notion of entropy change as an indirect but very useful measure of the increase or decrease of the unavailable energy assignable to the fluid as a consequence of the process. It now remains to be shown that *the increase or decrease of the entropy assignable to the fluid in progressing from one particular state to a second is the same for any process whatsoever by which the fluid may pass from the first to the second state*, wholly irrespective of the path or the character of the process. In that feature the entropy change would thus become a function of the initial and final states and may be regarded as a *property change*.

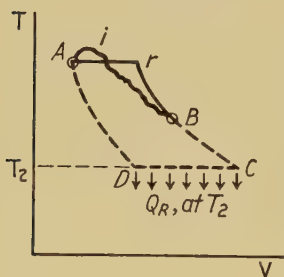


FIG. 15

For verifying these conclusions it is desirable first to recognize that a fluid may conceivably be caused to pass from any initial state to any second state in a wholly reversible manner. Thus taking any two states *A* and *B* as depicted (on *T-V* coordinates) in Fig. 15 the fluid may conceivably proceed from the one to the other via the path *ArB*

which shall consist of a reversible isothermal process *Ar* accomplished by heat energy reception and a reversible adiabatic process *rB*.

Selecting a receiver temperature T_2 the unavailable portion of the energy supplied during the state change may be evaluated by considering the fluid to proceed through a reversible cycle such as the Carnot cycle *ArBCDA*. The unavailable energy Q_R associated with the original state change *ArB* must be that rejected along the isothermal *CD* at T_2 . The entropy change for the process *ArB* is evaluated by the ratio Q_R/T_2 .

Now let the fluid pass from *A* to *B* by some wholly different path such as, say, an irreversible one which is indicated in the figure by path *AiB*. Again to evaluate the unavailable energy associated with this state change the fluid may be caused to proceed about a cycle by passing along the reversible paths *BC*, *CD* and *DA*. Through these three latter paths the fluid passes through the same states as in the previous cycle

and the energy quantities involved in these several state changes are thus the same. Therefore the energy rejection Q_R at T_2 is the same, even though the sequence of states and the energy quantities involved in path AiB may differ completely from those associated with path ArB . It follows that the entropy changes Q_R/T_2 for the two paths are identical. As path AiB was taken as representative of *any* path whatsoever between A and B it follows further that the entropy change for *all* paths between A and B must be the same.

The immediate and major significance of this identity of the entropy change of a fluid in passing from one state to a second irrespective of the manner by which the passage may be accomplished is two-fold, to wit:

(A) Entropy change, being thus a quantity which depends only on the initial and final state of a fluid, becomes in that respect an item which is wholly parallel to any *property* of the fluid, such as its pressure, temperature, specific volume, internal energy or enthalpy. We are therefore justified in regarding entropy change as a property change and in assigning to a fluid in any particular state a particular magnitude of its entropy (with respect to any selected standard or reference state). To this property various symbols are assigned by various writers, such as S , ϕ or N . We have adopted the first of these, S , or the symbol ΔS for change of entropy, implying in all cases the entropy or change of entropy per pound mass of the substance.

(B) Since the entropy change is the same for any manner of process by which a fluid may proceed from one state to a second the computation of the numerical magnitude of the change may be based on any sort of process which may offer the most ease or convenience for the purposes of computation. It will be found that some ideal, reversible process is usually the most convenient. Also, because for such a (reversible) process the entropy change Q_R/T_2 is *numerically equal* to the quantity Q/T_1 , or to $\int d'Q/T_1$ if T_1 is variable (see equations 1 or 2 of Art. 53), it results that the easiest and most frequently employed procedure in determining the entropy change corresponding to a specific state change of a fluid is the computation of the Q/T_1 (or $\int d'Q/T$) for some reversible process between the same end states.³

³ As a result of this circumstance entropy is at times defined, in thermodynamics literature, simply by the ratio Q/T_1 , or $\int d'Q/T_1$. This definition becomes inadequate for any mechanically irreversible state change, and as such irreversibility is rather the rule than the exception the definition is basically inadequate for most real processes. In distinction the concept of entropy as an index of unavailability is one that is both invariably valid and preëminently significant.

To summarize, it has thus been shown (1) that there is a fixed and definite change in the amount of unavailable energy (with reference to any given receiver temperature) which is chargeable to a fluid for a change of that fluid from one specific state to a second, (2) that the change of entropy between the two states is thus independent of the manner of accomplishment of the state change, (3) that in this respect entropy change is essentially a *property* change and thus a particular magnitude of the entropy (with respect to any selected reference state) may be assigned to any particular state of a fluid, and (4) that the change of entropy (ΔS) may be and usually will be ascertained for a conveniently analyzed reversible state change from the first to the second state.

Example 8. Presume one pound of water at 32° F. and 182 lb. per sq. in. to be changed to dry steam at that pressure and a temperature of 374° F. (see examples 2 and 1, Art. 53) by (a) stirring vigorously with a mechanically actuated paddle wheel, (b) by means of an electrical resistance coil, or (c) by forcing it into a boiler under 500 lb. pressure, there vaporizing it by supplying energy as heat, delivering it to a turbine engine at that pressure, and expanding it through the turbine. What would be the difference of entropy between the final and initial state for each manner of accomplishing the state change? Drawing conclusions from its availability or unavailability, what would you regard as the "entropy of the energy" supplied as work in actuating the paddle wheel of method "a"?

55. Entropy as a Coördinate. — Since entropy is thus a property or state function of a fluid it evidently may be employed as a coördinate, in connection with various of the other properties, in the graphical representation of state changes. The more common diagrams in which entropy is so employed are: (a) the temperature-entropy (T - S) diagram in which absolute temperature is the ordinate and entropy per pound of fluid is the abscissa, and (b) the "Mollier" chart (named after its inventor) in which enthalpy ($E + PV/J$, or H) is the ordinate and entropy again the abscissa. The latter, as may be anticipated, is of more particular convenience in connection with the portrayal of processes in which steady-flow occurs. Its consequent wide utility will later become quite apparent but it will not be considered in further detail until more information shall have been secured concerning the manner of variation of the enthalpy in certain sorts of processes with various particular fluids.

The temperature-entropy diagram acquires a particular utility from the relationships between those very properties as they enter into the definition and the common manner of evaluation of the change of entropy of a fluid, or respectively (Art. 53),

$$Q_R/T_2 = \Delta S, \text{ or } Q_R = T_2 \Delta S, \text{ for any process, and} \quad (3)$$

$$Q_R/T_2 = Q/T_1 = \Delta S, \text{ or}$$

$$Q = T_1 \Delta S, \quad (3a)$$

$$= \int_a^b T_1 dS, \text{ for a reversible process.} \quad (3b)$$

To illustrate the use of these expressions in connection with the T - S diagram let the line ab in Fig. 16 be a graphic record of the simultaneous values of the absolute temperature and the entropy of a fluid during any assumed process and state change, and let the horizontal line mn be a contour line of constant temperature at absolute temperature T_2 . From equation (3) the change in the unavailable energy chargeable to the fluid as a result of the process is represented by the cross-hatched area below the line mn (the area $mnb'a'm$) since that area is the product of T_2 times ΔS . Furthermore, from equation 3b, if the process were one without fluid friction or other internal irreversibility and were accomplished solely by the reception of energy by the fluid (as heat), then the area between the line ab and the S axis (the area $abb'a'a$) would represent the amount of energy received by and chargeable to the fluid as a result of the process, since that area represents the summation of all

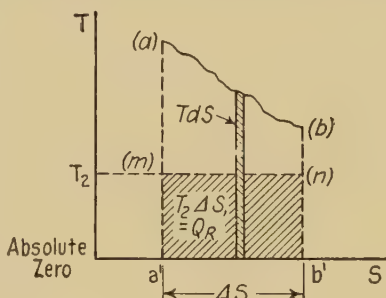


FIG. 16

infinitesimal products of T and dS between the limits of a and b , or equals

$$\left(\int_a^b T dS \right).$$

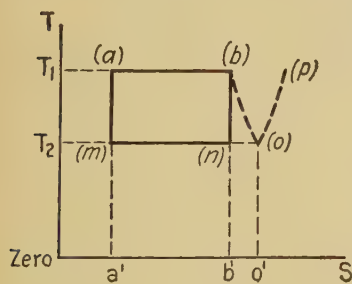


FIG. 17

It is apparent that any constant temperature (isothermal) state change would be represented by a straight line parallel to the S -axis. Also a *reversible adiabatic*, being a *constant entropy* or *isentropic process* (Art. 53, Ex. 5), would be a straight line parallel to the T -axis. It follows that for any fluid undergoing the sequence of reversible processes of the Carnot cycle the state changes would be represented by the sequence of lines forming the rectangular figure $abnma$ of Fig. 17. In that figure the area $a'abb'a'$ again represents and is a scalar measure of the (heat) energy received

by the fluid during the isothermal expansion at T_1 , the area $a'mnb'a'$ measures the unavailable energy rejected during the isothermal compression at T_2 and also, since the work output of the cycle is the difference between the energy supplied and that rejected, the area $mabnm$ measures the available portion of the energy supplied. The thermal efficiency of the cycle is thus the ratio $\frac{mabnm}{a'abb'a'}$, which obviously equals the ratio $\frac{T_1 - T_2}{T_1}$ upon which the absolute temperature scale was predicated. The advantageous effect of increased T_1 and of decreased T_2 is thus portrayed graphically.

Aside from the isothermal and the reversible adiabatic the sorts of state changes which we shall have more frequent occasion to portray on T - S coördinates are the irreversible adiabatic and certain internally reversible state changes in which the specific heat ($d'Q/dT$) during the process is either actually or approximately constant.

For the *irreversible adiabatic*, due to the increase of unavailable energy (Q_R) which is inherent to any such process, the essential and invariable characteristic is an *increase in the entropy* of the fluid as a result of the process, this irrespective of whether the temperature may tend to decrease (as in an expansion) or to increase (as in a compression). Thus an irreversible adiabatic expansion from T_1 to T_2 would be represented by a line such as bo in Fig. 17 and a like compression by a line such as op . For the expansion the increase of unavailable energy with respect to a receiver at T_2 would be represented by the area $b'noo'b'$.

In this connection it may be helpful again to conceive a fluid to proceed through the series of state changes which characterize the Carnot cycle but with the exceptions that the sequence is not carried out in a single visionary device such as the Carnot engine (Art. 42). Instead let it be presumed that the reversible adiabatic compression is accomplished during the passage of the fluid through a pump or compressor, that the isothermal energy reception at T_1 takes place in some device such as a heater or boiler, that the adiabatic expansion takes place during the passage of the fluid through a turbine engine, that this expansion does not proceed reversibly (as it does in the Carnot engine) but is accompanied by such fluid friction and turbulence as to be internally irreversible, and finally that the isothermal discarding of the unavailable energy at T_2 takes place in a cooler or condenser, the fluid having been caused to flow successively through the series of devices. Summarizing, the Carnot cycle has been carried out but in a more practicable manner than in the Carnot engine and with an exception

in the irreversibility which occurs in the adiabatic expansion in the turbine engine. The cycle would be one such as that represented by the sequence of lines forming the figure *mabom* of Fig. 17. Again the energy received at T_1 would be represented by the area $a'abb'a'$ but that discarded at T_2 would be represented by the area $a'moo'a'$. The energy made available (delivered as work) must equal that received less that rejected and thus, from the geometry of the figure, would equal area *mabnm* minus area $b'noo'b'$ or, verbally, the ideally available energy less that made unavailable by reason of the irreversibility occurring in the turbine along path *bo*.

Returning to a consideration of internally reversible state changes in which the specific heat ($d'Q/dT$) during the process is actually or approximately constant — such processes would be typified by the warming of a liquid or a gas at constant pressure (see examples 2 and 3, Art. 53) and of a gas at constant volume or during any of a myriad of so-called “polytropic” processes in which circumstances so combine as to effect a state change with effectually constant specific heat (see also example 13, Art. 24). The appearance of such state changes when shown on T - S coördinates will become evident on rewriting the general relation for entropy change during energy transition at variable temperature, or (Art. 53)

$$dS = \frac{d'Q}{T} = \frac{c dT}{T}, \quad (3b)$$

whence we may write either

$$\frac{dT}{dS} = \frac{T}{c}, \quad \text{or} \quad (3c)$$

$$S_b - S_a = c \log_e (T_b/T_a). \quad (3d)$$

Equation 3d shows the method for computing the total entropy changes in terms of the specific heat and the initial and final temperatures. Equation 3c indicates that a line representing the state change on T - S coördinates will have a varying slope (dT/dS) which increases with increase of the temperature and which depends inversely on the specific heat of the process. To illustrate, the warming of water from 32° F. (492° R.) to 374° F. (834° R.) at an average specific heat of 1.01 (example 2, Art. 53) would be represented by a line such as *ab* in Fig. 18. The water at 32° F. is taken as a standard reference state of zero relative entropy.

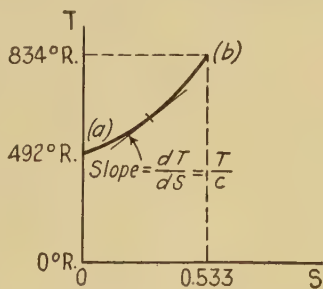


FIG. 18

Example 9. Taking the data of example 6, Art. 53, plot to scale on T - S coördinates the path representing the warming of the water from 32° to 366° F., plotting intermediate points at 100° and 250° F., and also the path for the vaporization at 366° F. Use scales of 1 in. = 200° F. and of 1 in. = 0.5 units of entropy per pound. Measure approximately the area (sq. in.) under the two paths, evaluate these areas in B.t.u. per pound and compare the results with the actual energy quantities required for the state changes.

Example 10. Plot on the diagram of example 9 the following cycle which is approximately representative of that of a simple steam power plant, the cycle consisting of

- (a) the warming of feed water at 165 lb. per sq. in. from 100° F. to 366° F. (see example 6);
- (b) the complete vaporization of the water to steam in the boiler at 366° F., and 165 lb. per sq. in. (see example 6);
- (c) an irreversible adiabatic expansion of the steam through a turbine from 366° F. and 165 lb. to 100° F. and 0.95 lb., with an increase of entropy from 1.563 (as at the turbine entrance, example 6) to 1.750 because of the irreversibility;
- (d) the condensation of the exhaust steam in a condenser from steam at 100° F., 0.95 lb. and entropy of 1.75 to water at the same temperature and pressure;
- (e) an adiabatic compression of the water in a feed pump from 0.95 lb. to 165 lb. per sq. in. (Note — This feed pump action on the water takes place with a wholly negligible increase of temperature or of entropy.)

Compute from the diagram or by any means (see also the data of example 6) (a) the amount of energy supplied in processes *a* and *b*, (b) the minimum unavailable and maximum available portions of the total energy supply and the corresponding ideal or maximum thermal efficiency with which the energy supply might have been utilized by the cycle if reversible, (c) the energy actually discarded in the condenser and thus actually unavailable, (d) the remaining portion of the energy supply which was actually available for delivery as work by the plant, and (e) the corresponding thermal efficiency of the actual cycle.

Example 11. Taking the data of example 3, Art. 53, plot to scale on T - S coördinates the two successive state changes of the air, locating at least three points on each curve. (Note — Only changes of entropy need be represented and therefore the curve may be started at any convenient point.)

56. The Entropy of Energy. — In the foregoing articles entropy change has been regarded as a measure of the increase or decrease of the unavailable energy *which is assignable to a mass of a fluid* as the result of a state change of the fluid. This aspect of the entropy concept is perhaps the one of major utility in the working computations of engineering thermodynamics but it is as well a limited one. From a more fundamental and more comprehensive viewpoint entropy should be regarded as a measure of *the unavailability of energy itself*. The significance of this viewpoint may be developed to advantage by several illustrative examples which portray the increase of the entropy of a given quantity of energy as the result of various irreversible processes

in which energy undergoes transformation or transposition. The examples are so selected as to illustrate the effects of (a) *external thermal irreversibility*, as in heat transmission from a higher to a lower temperature region (Art. 21), (b) *internal thermal irreversibility*, as in the mixing of a warmer and a cooler fluid (Art. 21), (c) *external mechanical irreversibility*, as resulting from friction between the moving parts of a machine (Art. 20), and (d) *internal mechanical irreversibility*, as resulting from fluid friction or turbulence accompanying flow (Art. 20).

(a) *External Thermal Irreversibility*. — All heat transmission is inherently irreversible and can be conceived only to *approach* reversibility, but the degree of irreversibility of some processes obviously exceeds that of others. An example of pronounced irreversibility is the warming and vaporization of water in a boiler by exposure of the boiler heating surfaces to radiation from the furnace or to contact with hot products of combustion which carry energy from the furnace by convection (Art. 17).

Example 12. In examples (1) and (2) of Art. 53 it was noted that for warming one pound of water at 182 lb. abs. from 32° to 374° F. and for completely vaporizing it at that temperature a total of $(347 + 850 =)$ 1197 B.t.u. must be received by the pound of fluid. Of this energy $(262 + 503 =)$ 765 B.t.u. were shown to be unavailable with respect to a 32° F. receiver and $(1197 - 765 =)$ 432 B.t.u. available. The entropy of the water was therefore said to increase by $(765/492 \text{ or } 0.534 + 1.020 =)$ 1.554 units per pound.

Presuming that the 1197 B.t.u. of energy had been supplied by radiation from a furnace in which the temperature was maintained at 2500° F., what was the entropy of the energy as delivered by the furnace, the unavailable energy with respect to a 32° F. receiver, the increase of entropy of the energy by reason of the thermally irreversible heat transmission and the corresponding increase in unavailable energy (with respect to a 32° F. receiver)?

Solution. For each 1197 B.t.u. delivered by the furnace at 2500° F., if it were utilized reversibly,

$$\frac{Q_R}{T_2} = \frac{Q}{T_1} = \frac{1197}{2500 + 460} = 0.404 = \text{entropy of original energy as supplied from furnace.}$$

$$Q_R(32^\circ \text{ F.}) = 0.404 \times 492 = 199 = \text{energy originally unavailable.}$$

$$1.554 - 0.404 = 1.150 = \text{increase of entropy of the energy as a result of the irreversible heat transmission.}$$

$$1.150 \times 492 = 566 \text{ B.t.u.} = \text{corresponding increase of unavailable portion of the 1197 B.t.u. supplied (or } = 765 - 199 = 566).$$

(b) *Internal Thermal Irreversibility*. — The increase of entropy and unavailability occasioned by irreversible mixing is illustrated in the following example.

Example 13. Five (5) lb. of air at 2600° F. and atmospheric pressure are permitted to mix with 25 lb. of air at 80° F. and the same pressure. Compute (1) the temperature

(t_x) of the resulting mixture, assuming a constant specific heat of 0.24 for air at constant pressure, (2) the amount and entropy of the energy imparted by the hot air, (3) the entropy of the same amount of energy after delivery to the mixture, and (4) the increase of entropy of the energy and the increase of unavailable energy (with reference to a 32° F. receiver) by reason of the irreversible mixing process.

Solution.

$$(1) 5 \times 0.24 \times (2,600 - t_x) = 25 \times 0.24 \times (t_x - 80), \quad t_x = 500^\circ \text{ F.}$$

$$(2) Q = 5 \times 0.24 \times (2,600 - 500) = 2,520 \text{ B.t.u.}$$

Original entropy of the 2520 B.t.u., if withdrawn reversibly,

$$= M \times c \times \log_e \frac{T_a}{T_b} = 5 \times 0.24 \times \log_e \frac{2600 + 460}{500 + 460} = 1.393.$$

$$(3) \text{ On withdrawing 2520 B.t.u. from the 30 lb. of mixture initially at } 500^\circ \text{ F. the}$$

$$\text{final temperature would be } \left(500 - \frac{2520}{30 \times 0.24} \right) = 150^\circ \text{ F.}$$

$$\text{Entropy of this energy} = 30 \times 0.24 \times \log_e \frac{500 + 460}{150 + 460} = 3.265.$$

$$(4) \text{ Increase of entropy} = 3.265 - 1.393 = 1.872.$$

$$\text{Increase of unavailable energy (with respect to } 32^\circ \text{ F. receiver)} = 1.872 \times 492 = 922 \text{ B.t.u.}$$

These examples not only serve to illustrate the fundamental notion of entropy as a measure of the unavailability of energy itself but also point an obvious "moral" as regards the undesirable result of thermal irreversibility and the corresponding desideratum of utilizing energy at the maximum practicable temperature level or "head."⁴ Such utilization is in fact accomplished in the internal combustion engine, whence its markedly higher thermal efficiency even though handicapped by the discarding of its unavailable energy at unfavorably high temperatures (in the hot exhaust gases). In the steam plant a very considerable wastage of temperature "head" is unavoidable, between the hot furnace or furnace gases and the steam, but the trend of modern practice is toward a minimization of this loss by the increase of steam pressure and temperature.

Thermal irreversibility has the same unfavorable influence on general efficiency wheresoever it may occur in the power plant. For many years this was inadequately recognized, particularly in the use of relatively high temperature steam for the warming or vaporization of relatively low temperature water (as in feed-water heaters, evaporators, etc.), or in other processes in which only low temperature energy was

⁴ The term "head" is used here in analogy with its use in the science of Hydraulics. Quite as it would be manifestly wasteful to permit the water supply of a hydraulic power plant simply to drain from a reservoir at high elevation or "head" to a low head reservoir without utilizing the original potential energy of the high head source, so it constitutes an irrecoverable wastage of available energy to permit the energy of a high temperature source to pass irreversibly to a low temperature.

necessary. Again, modern practice minimizes this condition by providing for at least a partial expansion of the high temperature steam through an efficient engine prior to the utilization of the steam for any heating purposes, thereby realizing some portion of the available energy. Modern steam power plant cycles which so reduce thermal irreversibility are considered in a subsequent chapter.

(c) *External Mechanical Irreversibility.* —

Example 14. In a given reciprocating engine energy was being delivered as work at the face of the engine piston at the rate of 3,300,000 ft-lb. per minute but 10 per cent of this energy was continuously diverted to overcoming mechanical friction between the moving parts of the engine, this frictional energy being concurrently dissipated as heat to the surrounding atmosphere (at 80° F.). Compute the unavailable portion of the frictional energy (with reference to an 80° F. receiver) and the increase of entropy of the work energy per minute.

Solution. The work energy supplied to the piston face was wholly and inherently available and of zero entropy. Of that energy ($3,300,000 \times 0.10/778 =$) 424 B.t.u. per minute were dissipated to the atmosphere and became wholly unavailable with respect to a receiver at atmospheric temperature. The corresponding increase of entropy of the energy was ($424/540 =$) 0.79 units per minute.

(d) *Internal Mechanical Irreversibility.* —

Example 15. If the 1197 B.t.u. required to warm and vaporize the water of example 12 had been supplied as work in driving an impeller which was immersed in and violently agitated the water, what was the increase of the entropy of the energy as a result of the process?

Solution. The energy when supplied as work was wholly available energy and of zero entropy. After delivery to the water the entropy of the energy received by the water was 1.554 units (see example 12), which therefore measures the increase of the entropy of the original energy.

The phenomenon of entropy increase resulting from mechanical friction (external mechanical irreversibility, example 14) is simply a technical substantiation of the quite obvious undesirability of such friction. Example 15 illustrates an extreme case of irreversibility and entropy increase resulting from fluid friction and turbulence. Less extreme and more usual instances are typified in the case of flow of steam through a turbine (see example 10c) or of air through a centrifugal compressor. In fact a primary objective in the design of such machines is a minimizing of the shock, turbulence and friction accompanying flow and a corresponding minimizing of entropy increase. Various practical examples which will involve those flow phenomena will be considered after we have acquired more definite information concerning the properties of specific fluids.

57. Summary. — Recognizing that, for circumstances in which energy is supplied at an elevated temperature, the reversible temperature en-

gine is the most efficient device conceivable whereby a portion of the energy might be made available as work, and recognizing further that such an engine thus affords a universal meter for ascertaining the inherently available and unavailable portion of the energy supply, it follows that the formulation of the efficiency of a reversible cycle in terms of the absolute temperature of the source and receiver will enable a direct evaluation of the availability or unavailability of the energy.

The expression for reversible cycle efficiency when the source temperature and receiver temperature are both constant was shown to be (Art. 50),

$$\text{Thermal efficiency} = \frac{Q - Q_R}{Q} = \frac{T_1 - T_2}{T_1}.$$

Rearranging this relation we may write

$$\frac{Q_R}{T_2} = \frac{Q}{T_1}, \text{ for a reversible process,} \quad (1)$$

or, if it is desired to extend this relation to accommodate the circumstances of a source the temperature of which (T_1) is caused to vary as energy enters or leaves, we may write further that, again for a reversible process,

$$\begin{aligned} \frac{Q_R}{T_2} &= \int_{T_a}^{T_b} \frac{d'Q}{T_1} = \int_{T_a}^{T_b} \frac{c dT}{T_1} \text{ or} \\ &= c \log_e (T_b/T_a), \end{aligned} \quad (2)$$

if the specific heat (c) is constant. (2a)

It will have been observed that each of these is an expression for the ratio Q_R/T_2 , or *the quotient of the unavailable portion (Q_R) of the energy with respect to a receiver at an absolute temperature T_2 , divided by that temperature*. This ratio is of such great practical significance that scientists have assigned it a name, the **entropy** of the energy, or, in referring to a state change of a fluid wherein the amount of unavailable energy assignable to a unit mass of the fluid may increase or decrease, the **change of entropy of the fluid**.

If a given state change of a fluid were accomplished through the withdrawal of energy from the fluid as heat, whereby the fluid might be conceived to act as a source of energy for a reversible temperature or heat engine, the fluid would be said to decrease in entropy in an amount which is measured by the ratio Q_R/T_2 , where, as above, Q_R is the inherently unavailable portion of the energy given up by the fluid (to the engine) and T_2 is the receiver temperature (of the engine). Conversely, if the reverse state change were effected by the supplying of energy to the fluid it would be said to increase in entropy by an equal

amount. However, if a fluid should undergo a state change in which energy entered or left only as work and should this work be received or delivered without the existence of flow friction or turbulence (the fluid should undergo a reversible adiabatic compression or expansion), the available energy supplied as work during compression would be wholly recoverable as work during expansion, or vice versa, whereby a reversible adiabatic state change of a fluid would involve no increase or decrease of the unavailable energy assignable to the fluid and thus no change of entropy. The process would therefore be said to be **isentropic**.

The above general expression for Q_R/T_2 , and thus for the entropy change of a fluid, would be found sufficient for the computation of that change during a wide variety of processes and state changes. It may further be shown that whatsoever the process or combination of processes by which a particular fluid might pass from one specific state to some second specific state the net change of entropy of the fluid would be the same for each. It follows that the entropy change of the fluid depends solely on its initial and final states and therefore that change of entropy may be regarded as a property change. In view of this, if some particular state of a fluid is selected as a standard reference state to which a relative value of zero entropy is arbitrarily assigned a particular value of its entropy may be ascribed to any other particular state of the fluid. The magnitude of the entropy so ascribed is that per pound mass of the fluid.

Since the entropy change of a fluid may thus be regarded as a property change and a certain magnitude per pound may be ascribed to each state of a fluid, entropy may properly and conveniently be employed as a coördinate in the graphic representation of fluid state changes. It is more commonly so employed with either temperature or enthalpy as the second coördinate or property. The enthalpy-entropy (H - S) or "Mollier" chart is of particular utility in connection with state changes which take place conjointly with flow. The temperature-entropy (T - S) diagram has a certain utility because, since $\int dS = \int d'Q/T$ or $\int d'Q = Q = \int T dS$ for a reversible process and $\int dS = Q_R/T_2$ or $Q_R = \int T_2 dS$ for any process, the heat added or withdrawn in any reversible process and the corresponding unavailable energy increase or decrease in any process may be represented by areas on T - S coördinates. On either H - S or T - S coördinates a reversible adiabatic process, or isentropic process, will be represented by a straight

line perpendicular to the entropy axis and an irreversible adiabatic by a line of increasing entropy.

The concept of entropy as a property of a fluid is perhaps the one of major utility in the working computations of engineering thermodynamics but from a more fundamental viewpoint entropy is rather to be regarded as the measure of the relative unavailability of energy itself. So regarded, it may be shown that the existence of any manner or amount of irreversibility (whether thermal or mechanical) in any process whatsoever will invariably predicate an increase of unavailable energy and of the entropy of the energy which is transferred or transformed during the process. This second and broader significance of the entropy concept is much the more enlightening one as a measure of the degree to which the processes of nature or the man-devised processes of power generation approach or digress from the ideal.⁵

In this connection it may not be amiss again to note that our processes of power generation which depend on the combustion of fuels are in one respect unfortunate because they involve the transition of the stored chemical energy into (primarily) the molecular form, with attendant high temperature. We have seen that, *after having put the energy into the molecular form*, the higher the temperature which is attained and utilized the greater the thermal efficiency which is attainable, but we have also seen that the energies associated with temperature (molecular energy and heat) are inherently relatively unavailable forms. In a sense the objective of the science of engineering thermodynamics is a study whereby this admittedly unfortunate condition may be recognized and means for alleviating it may be devised.

58. Review Questions and Topics. —

1. What device is available which will serve as a universal meter for ascertaining the availability and unavailability of the energy stored in or emitted by any high temperature region or fluid?

2. (a) Define the change of entropy of a fluid during a state change.

(b) State and illustrate the use of the relation by which entropy change may be computed for an isothermal state change of a fluid.

(c) State the relation by which entropy change may be computed for a state change of a fluid during which the temperature of the fluid changes, and illustrate this relation for the case in which the specific heat of the fluid during the process is constant.

⁵ In this sense all actual processes, being irreversible in a greater or less degree, must result in an increase of entropy and of the unavailability of nature's stores of energy. For that reason the science of Thermodynamics is regarded as predicting an eventual running down of our universe and therefore (along with the science of Economics and its Laws of Malthus) has been termed a "dismal science." Perhaps it is. At least man's experience has led him to believe that a gain in availability of energy (decrease of entropy) in any actual process is so highly improbable as to be practically inconceivable.

(d) Define an isentropic state change of a fluid and designate what particular type of fluid state change would come under that category. Is an adiabatic process necessarily an isentropic process?

3. What characteristic of the entropy change of a unit mass of a fluid marks entropy as a state function or a property of the fluid?

4. (a) State, prove, and illustrate the particular utility of a graphic representation of a state change of a fluid on coördinates of absolute temperature (T) and entropy per pound (S).

(b) Show an isothermal, a reversible adiabatic, and an irreversible adiabatic state change on T - S coördinates.

(c) Show the cycle of a Carnot engine on T - S coördinates and depict the heat taken in from the source and that rejected to the receiver.

(d) Depict on T - S coördinates a state change of a fluid during which the specific heat is constant.

5. (a) What is meant by the entropy of energy?

(b) State and illustrate in general terms the influence of external thermal irreversibility in any heat transmission process and the influence of internal thermal irreversibility. Discuss any instances of thermal irreversibility occurring in the processes of heat-power plants, giving attention to their influence on over-all plant efficiency.

(c) State and illustrate in general terms the influence of mechanical irreversibility.

Symbols and Abbreviations, Chapter VII

c = specific heat of a fluid during a state change.

$d'Q$ = a small quantity of energy supplied or rejected (as heat).

Q = the energy supplied (as heat) at an upper temperature in accomplishing a state change of a unit mass of a fluid.

Q_R = the unavailable portion of the energy supply, Q .

T = absolute temperature.

T_1 = the absolute temperature of any region which acts as a source of energy supply.

T_2 = the absolute temperature of any region which acts as the receiver of unavailable energy.

S = the change of entropy per unit mass of a fluid between some specific state and a standard reference state.

ΔS = change of entropy per unit mass of a fluid between any two specific states.

PART III — PROPERTIES OF FLUIDS

59. Foreword. — In Parts I and II there have been developed the fundamental methods and principles which will provide the working tools for the subsequent thermodynamic analyses of practical power, compression, and refrigeration processes. Before those analyses may actually be made it will, however, be necessary that information be available concerning the various properties and property relations of the fluids which are encountered in the study of such engineering processes. It is the purpose of Part III to supply that information.

It will be recalled that adequate concepts of the following properties have been developed in the foregoing:

Part I	Part II
Internal energy (E)	Absolute temperature (T)
Pressure (P)	Entropy (S)
Specific volume (V)	
Enthalpy ($H, = E + PV/J$)	

For the purposes of general thermodynamics it would be necessary that several additional properties be available for use¹ but for the purposes of engineering thermodynamics these are sufficient.

The fluids with which we shall be concerned are broadly divisible, according to the usual classification, into **liquids, vapors and gases**. For engineering purposes the gases may be further divided into (a) the so-called *permanent gases* such as hydrogen, oxygen, nitrogen, et cetera, the characteristics of which approach those of the hypothetical Perfect Gas (Art. 48) sufficiently closely that practical computations may reasonably be made upon the perfect gas basis; and (b) the *imperfect gases*, the property relations of which depart more materially from those of the perfect gas and perhaps tend more in character toward a vapor. In various practical circumstances we shall also be concerned with mixtures of these fluids, or, specifically, with mixtures of a vapor and its liquid, mixtures of gases, and mixtures of a vapor and a gas.

In the discussions of the properties and property relations of these various fluids the order of presentation will be

- (a) Liquids, Vapors and Liquid-Vapor Mixtures (Chapter VIII).
- (b) Perfect Gases (Chapter IX).
- (c) Mixtures of Gases and Gas-Vapor Mixtures (Chapter X).

¹ See Art. 195.

It is not regarded as practicable to consider the Imperfect Gases in detail in this material. It may be remarked that the order of consideration of the Vapors and of the Perfect Gases is more one of convenience than one of any significance or necessity. If the reader should so desire, the material on Mixtures (Chapter X) might be deferred without disadvantage until just prior to a consideration of Combustion, (Chapter XV).

CHAPTER VIII

LIQUIDS, VAPORS AND LIQUID-VAPOR MIXTURES; PROPERTIES AND PROPERTY RELATIONS

60. A Liquid and its Vapor. — It is desirable to recall briefly the physical concepts concerning a liquid and its vapor when the two are contiguous and in equilibrium in any container or region. To do so let us consider a vessel into which a liquid has been introduced and exists at any specified temperature (below its "critical" temperature, see Art. 64). The molecules of the liquid possess a motion and average kinetic energy which depend on the temperature (Art. 10) but there is sound evidence of a tendency for individual molecules to possess momentarily abnormally high velocities whereby certain ones which chance to be at or near the liquid surface are enabled to escape the attraction of the liquid, enter the free space and exist there in the vapor phase. With the progressive increase in the number of molecules in the vapor phase and space there is a correspondingly increasing probability that some which chance to have momentarily less than the average kinetic energy corresponding to the temperature will again be entrapped in the liquid. As a result there is an effective tendency for the number of molecules per unit space in the vapor to arrive at an equilibrium condition and, as a further result, for an equilibrium pressure thus to be established in the vapor space which will depend solely on the molecular nature of the fluid and its temperature, but which increases with the joint temperature of the liquid and the vapor. There thus exists for any particular fluid a definite relation between the temperature and pressure of a contiguous liquid and vapor. The curve of Fig. 19 portrays this equilibrium temperature-pressure relation as it is found to exist for water and its vapor, steam.

The foregoing considered more particularly a vessel or region in which the liquid and vapor are existing with neither flow nor energy transition to or from the fluid. The more usual circumstance in which the engineer is concerned is one in which either energy transition or flow or both exist. Representative of those conditions would be the boiler, evaporator or condenser.

If water were contained in a boiler to which energy was being supplied (as by heat flow) but no steam vapor were permitted to escape the rising temperature of the water would effect a progressive rise of pressure of the

water and steam exactly according to the curve of Fig. 19. If however the vapor were permitted to depart from the steam space at a rate equal to that at which evaporation was occurring (the latter rate being established by the rate at which the energy was being delivered to the water) an equilibrium condition would again be established in which the above temperature-pressure relation is maintained.

In engineering parlance, any temperature or pressure thus co-related would be known as the **saturation temperature** of the fluid corresponding to a specified pressure or the **saturation pressure** corresponding to the temperature. Further, any liquid, such as water, which is under any given pressure and is at the saturation temperature corresponding to that pressure would be referred to as **saturated liquid**.

If the liquid were at any temperature less than the saturation temperature corresponding to its pressure it would be said to be **subcooled**. Similarly, vapor which is being formed or delivered at the saturation temperature corresponding to the vapor pressure is known as **saturated vapor** (as, for example, "saturated steam"). This is in distinction to a vapor which, *after being removed from contiguity with its liquid*, may be brought to a higher temperature but without increase of pressure, this being possible if the vapor is permitted to expand adequately during the supplying of the energy that effects the temperature rise. Such vapor which is at a temperature in excess of the saturation temperature corresponding to its pressure is known as **superheated vapor** and the excess of its temperature over the saturation temperature is known as the number of **degrees of superheat**.

Perhaps the condition of vapor production (or of its condensation) with which the engineer is most frequently concerned is one in which a fluid flows steadily to, through and from a device and remains at virtually the same pressure en route. Thus in a boiler the feed-water is delivered to the boiler at or about the boiler pressure, but usually as subcooled water, is warmed to the saturation temperature corresponding to the boiler pressure, and is vaporized (both of these by reason of energy reception from the furnace through the boiler heating surfaces) and then departs as saturated steam or perhaps as a mixture of saturated steam with some droplets or fog of saturated but unvaporized water which are

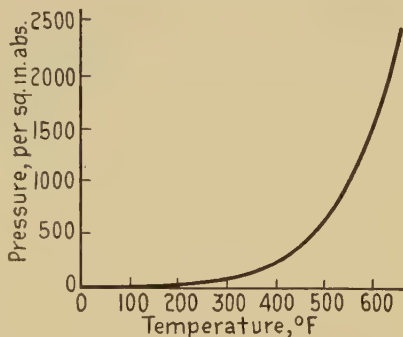


FIG. 19. P - T diagram, saturated water vapor.

mechanically entrained with the steam vapor. Such a mixture would be known as **wet steam**. The proportion of the mixture (by weight) which is saturated vapor is called the **quality** or **dryness fraction** and is usually designated by the symbol x . Thus if a wet steam mixture were one with two (2) per cent of moisture it would be said to have a "98 per cent quality."

After leaving the boiler as wet or saturated steam the flowing fluid might be directed through a superheater (still at or about boiler pressure) in which superheating might be accomplished by further supplying of energy from the furnace through the superheater surfaces.

Summarizing the foregoing, we have recognized the following typical conditions or phases of a fluid, such as water, in its liquid and vapor forms, to wit:¹

Subcooled water	Saturated steam
Saturated water	Wet steam
	Superheated steam

Also, associated with those were the terms:

Saturation temperature	Quality or dryness (per cent)
Saturation pressure	Degrees of superheat.

61. Properties of Saturated Liquids and Vapors. — It has previously been indicated that by a property of a fluid is here meant any characteristic of the fluid which has a certain invariable magnitude at any certain state, the state thus serving to fix definitely the value of all of the properties, or, conversely, any specific combination of property magnitudes serving to fix or designate the state. It follows that for any particular fluid which may progress through a continuous sequence of states there must exist a continuous and definite sequence of property magnitudes and therefore some invariable relation between the properties. Expressed mathematically, the properties of a fluid are mutually dependent variables and are therefore connected by some *functional relation*. Such a relation is known as the **characteristic equation** or **equation of state** of the fluid.

For the gases these functional relations are found to be expressible with adequate accuracy by relatively simple mathematical equations. As a result the practical predictions of the property changes accompanying state changes of a gas are usually accomplished by computations based on those mathematical relations. For the liquids and vapors, however, the characteristic equations are found to be entirely too

¹ It should be remarked that these are the "stable" or "equilibrium" states. Various transient and unstable conditions may be encountered. See Art. 115.

complex in form for direct use in practical work. As a consequence it is the custom of the engineer to depend upon the physicist for the primary determinations of the functional relations (from extensive experimentation) and to use for practical computations tabular statements of the concurrent properties of the liquids and vapors at a large variety of states. It is therefore our purpose simply to explain the typical arrangement and the use of such **tables of vapor properties**.

Among the engineering fluids for which adequately complete and reliable tables of properties are available are steam and the refrigerants ammonia (NH_3) and carbon dioxide (CO_2); and less complete tables exist for a number of other vapors. These tables are usually arranged in sections which give separately the properties of saturated liquid and vapor and of superheated vapor, usually with these arranged variously with pressure or temperature or perhaps entropy as the major argument. An excerpt from the saturated steam section of the latest American tables of the properties of steam appears below.² The symbols are those which at this writing are accepted as standard and which we shall use throughout this material. It will be observed that the subscripts

TABLE IV
PROPERTIES OF SATURATED STEAM

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Absolute pressure, lb. per sq. in.	Temperature, deg. F.	Specific volume, cu. ft. per lb.			Enthalpy, B.t.u. per lb.			Entropy		
		Saturated liquid	Evapora- tion	Saturated vapor	Saturated liquid	Evapora- tion	Saturated vapor	Saturated liquid	Evapora- tion	Saturated vapor
p	t	V_f	V_{fg}	V_g	H_f	H_{fg}	H_g	S_f	S_{fg}	S_g
99	327.10	0.01770	4.450	4.468	297.57	888.8	1186.4	0.4732	1.1298	1.6030
100	327.83	0.01771	4.408	4.426	298.33	888.2	1186.6	0.4742	1.1280	1.6022
101	328.55	0.01772	4.367	4.385	299.08	887.7	1186.8	0.4751	1.1262	1.6014

² The more recent American Steam Tables are, in the order of their publication: Marks and Davis, 1909; Goodenough, 1917; and Keenan (A.S.M.E.), 1930. The excerpts and tabular arrangements used here are from the latter tables. These represent the results of very extensive tests conducted under the auspices of the American Society of Mechanical Engineers and compiled by Professor J. H. Keenan. The A.S.M.E. tables cover a much more extensive range of pressures and temperatures than do the other tables.

f and g are used to denote properties at the saturated liquid state and at the saturated vapor state respectively, and that the subscript fg denotes the change of a property between the same two states.

The columns of Table IV are numbered for convenience in subsequent reference. The properties given in the columns are:

Columns 1 and 2 state the saturation pressure of water and steam corresponding to the temperature, and vice versa. The published tables always present for convenience two tables that are identical in form except that one employs even units of the pressure as the major argument and the other even degrees of temperature. The pressures are invariably *absolute* pressures, in pounds per square inch, and *not* gage pressures. The temperatures are degrees Fahrenheit. The symbols indicated are the usual ones.

Columns 3, 4 and 5 give, respectively, the specific volume of saturated water (V_f), the change (increase) of specific volume during evaporation (V_{fg}), and the specific volume of (dry) saturated steam (V_g). The general symbol V or v has been used regularly for specific volume, but other methods have been employed for denoting individually the liquid and vapor volumes and their difference.

Column 6 records the enthalpy (or *total heat* or *heat content*), H_f , of saturated water with respect to an arbitrarily assigned zero of enthalpy for saturated water at 32° F. (and 0.0887 lb. per sq. in.).³ As the PV/J item in the enthalpy function, $E + PV/J$, is only 0.00026 B.t.u. per lb. for saturated water at 32° F. it follows that the placing of a zero enthalpy at that temperature is essentially equivalent to regarding the internal energy of water at 32° F. as also relatively zero. The symbols h , q and i' , as well as other symbols, have also been employed for this item.

Column 7, which is headed Enthalpy of Evaporation, H_{fg} , measures the increase of enthalpy during evaporation and thus in fact the energy supply per pound (normally supplied as heat) which is required to accomplish the evaporation. In addition to the symbol as given the symbols r and L have also been used. The item is frequently known as the *Latent Heat of Evaporation*.

Column 8, headed Enthalpy of Saturated Vapor, H_g , is evidently the sum of the items on columns 6 and 7. Alternatively it is the sum, $E_g + PV_g/J$. Again it records the enthalpy of the vapor with respect to an arbitrarily assigned zero of enthalpy for saturated water at 32° F. For the item the terms *Total Heat of Vapor* or *Heat Content of Vapor* are also employed. Although their use is regarded as confusing and misleading (see Art. 32) it is perhaps justified in a degree by circumstances parallel to those noted in the footnote discussing column 6. Instead of the symbol indicated, the symbols H and i'' have also been used.

Column 9 gives the entropy ($\int^c dT/T$, Art. 53) of saturated water with reference to a zero of entropy for water at the customary reference temperature 32° F. Instead of the symbol S_f the symbols n and θ have also been used.

³ The use of the terms *total heat of the liquid* or *heat content of the liquid* in designation of this item is regarded as something of a misnomer but is quite current. Its use was introduced in earlier technical literature and is perhaps in some degree justified by the circumstance that the item *nearly* equals the amount of energy, usually supplied as heat, which is necessary to raise the temperature of water en route through a boiler at constant pressure from a feed-water temperature of 32° F. to the saturation temperature corresponding to the pressure.

Column 10 gives the change of entropy (H_{fg}/T) during evaporation at constant pressure and temperature. The symbols r/T or L/T have also been used. They indicate the method of computation of the item (see Art. 53).

Column 11 is the total entropy of the saturated vapor and must equal the sum of the items of columns 9 and 10, that is, $S_g = S_f + S_{fg}$. The symbols N and ϕ have also been used.

In the saturated steam section of the Goodenough Tables of the Properties of Steam and Ammonia and of the Marks and Davis Steam Tables virtually the same items appear as in the Keenan Tables, except that they include direct data on the internal energy of evaporation, E_{fg} , and the internal energy of saturated vapor, E_g . Table V below illustrates those tables, the values presented being however those corresponding to the Keenan data. It is also to be remarked that for values of V_f , and other associated data, the Goodenough and the Marks and Davis Tables provide a separate tabulation.

TABLE V
PROPERTIES OF SATURATED STEAM

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Absolute pressure, lb. per sq. in.	Temperature, deg. F.	Specific volume, sat. vapor	Density, sat. vapor	Enthalpy, B.t.u. per lb.			Internal energy, B.t.u. per lb.		Entropy		
				Saturated liquid	Evapora- tion	Saturated vapor	Evapora- tion	Saturated vapor	Saturated liquid	Evapora- tion	Saturated vapor
p	t	V_g	$1/V_g$	H_f	H_{fg}	H_g	E_{fg}	E_g	S_f	S_{fg}	S_g
100	327.8	4.43	0.226	298.3	888.2	1186.6	806.6	1104.6	0.474	1.128	1.602

The special items E_{fg} and E_g which appear in columns 8 and 9 of Table V may be computed from the data of Table IV quite directly and easily. Thus $E_{fg} = H_{fg} - PV_{fg}/J$, and $E_g = H_g - PV_g/J$. All data necessary for their ready computation are thus provided in columns 7, 4 and 1 and columns 8, 5 and 1 of Table IV. The symbols ρ and l have frequently been employed instead of the current symbol E_{fg} , and the symbols E and u have similarly been used for the item E_g . The above quantity PV_{fg}/J has at times been termed the *external latent heat of evaporation*. It will be recognized as that portion of the energy supplied during evaporation which under conditions of steady-flow

furnishes the excess of flow-work energy for delivery of the steam *from* a boiler over that entering in delivery of the water *to* the boiler.

Example 1. For 100 lb. per sq. in. check column 8 in Table IV from columns 7 and 6, check column 11 from columns 9 and 10, find the mean effective specific heat of water between 32° F. and 327.8° F. by the data of column 9 and equation 2*a* of Art. 53, check column 10 from columns 7 and 2, compare column 6 with a value obtained by subtracting 32 from column 2 and discuss and find the mean specific heat of water between 32° F. and 327.8° F. from columns 6 and 2; check column 8 in Table V from columns 7, 1 and 4 of Table IV and check column 9 in Table V from columns 8, 1 and 5 of Table IV, compute the internal energy of saturated water both from columns 6, 3 and 1 of Table IV and from columns 8 and 9 of Table V.

Example 2. From the ammonia tables of Goodenough's Properties of Steam and Ammonia check in "Table 10" and "Table 7" at 50 lb. per sq. in. pressure all items as called for in example 1. Note that the enthalpy and entropy values quoted are again ones with respect to a zero at 32° F. Explain any negative values.

Example 3. The Keenan, Goodenough, and Marks and Davis tables are each based on different experimental data. Compare values by making parallel tabulations from each table, doing so at 14.7 lb. per sq. in. and at 470° F. Compare the tables as regards the upper pressure limit to which they extend.

62. Properties of Superheated Vapor.—The published tables also provide extensive tabular data on the properties of superheated steam. In such tabulations, because a vapor which is not contiguous with its liquid may be superheated to any temperature above the saturation temperature corresponding to the pressure, the properties are given for a wide range of temperatures or of degrees of superheat at each pressure. An abridged excerpt from the superheated steam section of the Keenan table appears in Table VI.

TABLE VI
PROPERTIES OF SUPERHEATED STEAM

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Abs. pressure, lb. per sq. in. (Sat. temp.)		Sat. water	Sat. steam	Temperature, deg. F.					
				350	400	500	600	800	1000
100 (327.83)	<i>V</i>	0.018	4.426	4.594	4.934	5.585	6.215	7.439	8.644
	<i>H</i>	298.3	1186.6	1199.7	1226.9	1278.0	1327.9	1429.2	1533.1
	<i>S</i>	0.4742	1.6022	1.6185	1.6512	1.7075	1.7569	1.8445	1.9209

The properties listed are seen to be the specific volume, enthalpy and entropy. For convenience there is repeated directly below the

pressure item the corresponding saturation temperature; and in columns 3 and 4 there are repeated the several properties for saturated water and saturated steam at the given pressure. The remaining columns evidently give the properties at the temperatures which are noted at the head of the column.

Example 4. From Table VI find V , H and S for steam at 100 lb. per sq. in. and 72.2° F. superheat; also find the mean effective specific heat of superheated steam for the temperature range, basing computations both on the enthalpy change and the entropy change from the saturated steam state. (0.56.)

Example 5. Compare values from the Keenan, Goodenough, and Marks and Davis tables by making parallel tabulations from each for 250 lb. at 500° F. and at 600° F., and for 600 lb. at 600° F. Compare the tables as regards the upper pressure and temperature limits to which they extend.

Example 6. Find the temperature, specific volume and enthalpy for steam at atmospheric pressure and an entropy of 1.8550. (367.3; 33.30; 1223.7.)

63. Properties of Subcooled Liquid and Wet Vapor. — The tables described have provided means for the direct determination of all of the necessary properties of saturated liquid, saturated vapor and superheated vapor. The two additional conditions of fluids for which data are frequently required are those of **subcooled liquid** and **wet vapor**. It will be found that the table for the saturated condition supplies the requisite primary data for determination of the properties at these latter conditions but some computations may be necessary. The following describes the procedures to be followed.

Subcooled Liquid. — Subcooled liquid was defined in Art. 60 as liquid at a temperature less than the saturation temperature corresponding to the pressure under which it may be placed. An equivalent but alternative and frequently useful definition is that of a *liquid under an externally imposed pressure which exceeds the saturated vapor pressure corresponding to the temperature*. Simple illustrative conditions would be that of water in an open stream under atmospheric pressure but at 70° F. instead of at the saturation temperature of 212° F. which corresponds to the pressure, or that of boiler feed-water at 328° F. (see Table IV) but under a pump pressure of 400 lb. per sq. in. instead of the saturation pressure of 100 lb. per sq. in. which corresponds to the temperature. The state might well be designated as that of *super-pressure* liquid but the above term is currently employed.

As regards the specific volume, internal energy and entropy of subcooled water, those items are predominantly dependent on and influenced by the temperature. Consequently, although the specific volume and internal energy are unquestionably influenced slightly by super-pressure, that influence is customarily neglected and these three properties of

subcooled water are regarded as the same as those of saturated water at the specified temperature (*not pressure*) of the subcooled water.

As regards the enthalpy of subcooled water the situation differs, for the reason that the enthalpy property ($E + PV/J$) contains directly the pressure property as an inherent component and so must be definitely influenced by any material super-pressure. For computation of the enthalpy of subcooled (super-pressure) water at a given temperature perhaps the simplest procedure is that of adding to the value of the enthalpy of saturated water (H_f), as quoted in the saturated vapor table at the specified temperature (*not pressure*), the excess PV/J occasioned by the super-pressure. More specifically,

$$H_{\text{subcooled liquid at } t \text{ and } P_x} = H_{f, \text{ at } t} + (P_x - P_f)V_f/J. \quad (1)$$

In many practical instances in which P_x does not greatly exceed P_f the item $(P_x - P_f)V_f/J$ may be found to be relatively minor in magnitude and with adequate accuracy the enthalpy of the subcooled liquid may be taken as that of saturated liquid at the temperature (*not pressure*) of the subcooled.

Example 7. Find the values of V , PV/J , E , H and S for boiler feed-water at 400 lb. abs. and 200° F. Compare the value of H with that of H_f at 200° F., and also with the value of H_f at 400 lb., and discuss.

(0.01663, 1.23, 167.91, 169.14, 0.2938.)

Wet Vapor. — Wet vapor is a physical mixture of saturated vapor with droplets or fog of saturated but unvaporized liquid which is mechanically entrained with the vapor. There are three aspects from which such a mixture may be regarded, each of which enables one to write directly an expression for any property which is of a "specific" character, that is, a property of a certain magnitude per unit total mass of mixture. Thus a mixture of quality x (Art. 60) may be considered as

(a) one containing $(1-x)$ pound of saturated liquid and x pound of saturated vapor, or

(b) one in which an entire pound of liquid has been brought to the saturation temperature and x pound has been vaporized, or

(c) one in which, from an original pound of saturated vapor, $(1-x)$ pound has been condensed.

Corresponding expressions evaluating V_x , PV_x , H_x , S_x in three alternative manners are quoted below. Simple analysis will show that all of any group are algebraically equivalent. The second and third of each are the ones that find the most practical utility.

$$V_x = (1-x)V_f + xV_g^4 = V_f + xV_{fg} = V_g - (1-x)V_{fg}. \quad (2)$$

$$PV_x = (1-x)PV_f + xPV_g = P(V_f + xV_{fg}) = P[V_g - (1-x)V_{fg}]. \quad (3)$$

$$H_x = (1-x)H_f + xH_g = H_f + xH_{fg} = H_g - (1-x)H_{fg}. \quad (4)$$

$$S_x = (1-x)S_f + xS_g = S_f + xS_{fg} = S_g - (1-x)S_{fg}. \quad (5)$$

$$E_x = H_x - PV_x/J. \quad (6)$$

Example 8. Compute the properties V , PV/J , E , H and S of wet steam at 250 lb. per sq. in. and (a) $x = 0.10$ and (b) $x = 0.97$. Draw conclusions as to which manner of computation is practically the more accurate for these two characteristic conditions of low and high quality.

Example 9. For steam at a pressure of 250 lb. per sq. in. and $S = 1.430$ find the other properties.

Suggestion. — First find the quality.

Example 10. Complete the following tables, entering either numerical results or dashes in each block as the item may or may not be pertinent.

State No.	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Mass, pounds.....	3	4	...	2	...	...	...	1
Total vol. cu. ft.....	...	..	40	4	626	340	50	...
Specific volume.....	...	...	...	...	200	...	...	...
Pressure, gage.....	...	90	...	...	...	...	100	...
Pressure, absolute.....	85	..	200	200	...	...	...	250
Temperature, °F.....	...	...	500	...	...	185	...	...
Superheat, °F.....	...	80	...	...	...	...	...	...
Quality (dryness, x)	0.80	..	...	...	1.00	1.00	0.80	0.01
Per cent of moisture.....	...	...	...	...	...	...	...	...
Entropy, per lb.....	...	..	...	...	...	...	...	...

The foregoing several articles have described the character and manner of use of the conventional saturated and superheated vapor tables. A further manner of tabulation which finds marked utility, and more emphatically so for certain characteristic processes, is one in which pressure and entropy are used jointly as primary arguments, just as pressure and temperature are used jointly in the superheated steam tables. Listings are made of the quality of the steam (if wet) or its temperature (if superheated), of the specific volume and of the enthalpy at each concurrent value of the pressure and entropy. Such a Pressure-Entropy table appears for steam in the Goodenough Tables but has not been arranged in those of the other Steam Table authors.

64. Graphical Representation of Vapor Properties.—In Fig. 19 there was portrayed graphically the relation between the temperature and pressure of a liquid and vapor at saturation. In a parallel manner one may choose as coördinates any other pair of properties of a fluid and depict graphically the manner of their joint variation as the fluid

⁴ Frequently, for high quality steam, $V_x = xV_g$ with adequate accuracy.

progresses through any characteristic sequence of states. Of the various diagrams which might be so devised those which have been found to be most generally useful are the P - V diagram, the T - S diagram and the H - S or Mollier diagram. These are described and discussed in the following paragraphs.

P - V Diagram.— In the P - V diagram, as represented for a typical fluid in Fig. 20, the absolute pressure is employed as the ordinate and the specific volume as the abscissa. In the diagram the

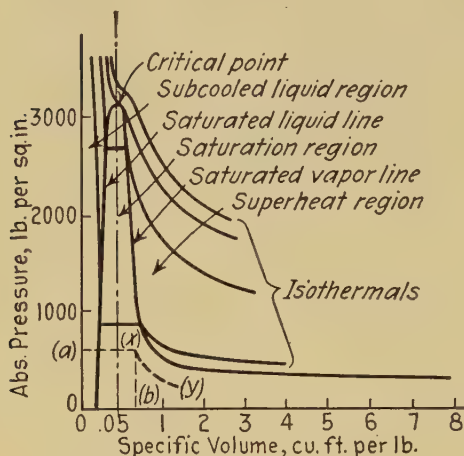


FIG. 20. P - V diagram, water and water vapor.

slightly oblique and curved line on the left which is labeled the **saturated liquid line** shows the manner of variation of the specific volume of saturated liquid as the pressure (and temperature) is progressively increased. The state of a subcooled liquid at any given pressure (but a temperature less than saturation) would be represented by a point to the left of the saturated liquid line, this by reason of the lesser specific volume at the lesser temperature. Consequently

the general region which lies to the left of the saturated liquid line is known as the **subcooled liquid region**.

The sloping line labeled the **saturated vapor line** similarly shows the relation between the pressure and specific volume of (dry) saturated vapor. The length of any horizontal line intercepted by the liquid and vapor lines thus represents the increase of specific volume during vaporization at constant pressure (V_{fg}). Therefore, since by equation (2) $V_x = V_f + x V_{fg}$, the state of a wet vapor mixture of quality x and at any given pressure would be represented by a point on that pressure line and lying at a distance $x V_{fg}$ to the right of the liquid line. The region between the two saturation lines is known as the **saturation region**.

Since the superheating of a vapor at a given pressure effects an increase of the specific volume the state of a superheated vapor would be represented by a point to the right of the saturated vapor line, whence the region to the right of that line is known as the **superheat region**.

The saturated liquid and saturated vapor lines are observed to join at a **critical pressure**, which for water is at 3226 lb. per sq. in. and at

which pressure the saturation temperature of water is 706.1°F. ⁵ The saturation temperature at the critical pressure is similarly known as the **critical temperature**. The diagram indicates that a liquid under that pressure would on warming pass imperceptibly into a dense superheated vapor state without the appearance of the characteristic phase of vaporization at constant temperature.

The **isothermal lines** which appear in the figure are contour lines of constant temperature and indicate the specific volume of a fluid at any given temperature and pressure or, alternatively, the pressure corresponding to a given state as designated by the temperature and specific volume. Since the temperature is constant during vaporization at constant pressure the isothermals become straight horizontal lines within the saturation region. In the subcooled liquid region the isothermals are very nearly vertical, showing the very slight change (decrease) of volume of a liquid as it is compressed at constant temperature. In the superheat region the isothermals are approximately hyperbolic curves, approaching that curve more exactly at very low pressures or considerable superheats. This last characteristic will later be seen to reflect an approach of low pressure or highly superheated vapors to the characteristics of a perfect gas.

For temperatures which are above the critical a vapor may not be condensed at any finite pressure. Some writers consider the critical temperature isothermal to provide an arbitrary line of demarcation between the **gas phase** (above the line) and the **vapor phase** (below the line) of a substance.

In addition to the isothermal lines one might draw contour lines of *constant quality*, *constant superheat*, *constant internal energy*, *constant enthalpy* or *constant entropy*, which lines might be useful for various particular purposes. Also any sort of state change whatsoever may be represented by a suitable curve or line on the diagram.

In connection with the P - V diagram it should be remarked that the product PV corresponding to any state of a fluid is represented by the rectangular area bounded by the coördinates, by a constant pressure line through the state point and by a constant volume line through the point, that is, by an area such as $oaxb$ for state " x ." Also for any manner of state change as represented by the line " xy ," the $\int_x^y P dV$ (Art. 36) is represented by the area between the line and the V -axis (that is, by the area *below* the line) and the $\int_x^y V dP$ is represented

⁵ See the various engineering hand books for the critical pressure and temperature of other substances.

by the area between the line and the P -axis (the area *back of* the line).

T-S Diagram. — In the T - S diagram (Fig. 21) *absolute* temperature is the ordinate and entropy per pound mass (relative to liquid at 32° F. or 492° R.) is the abscissa. The liquid line, the critical point and the saturated vapor line have the same significance as in the P - V diagram,

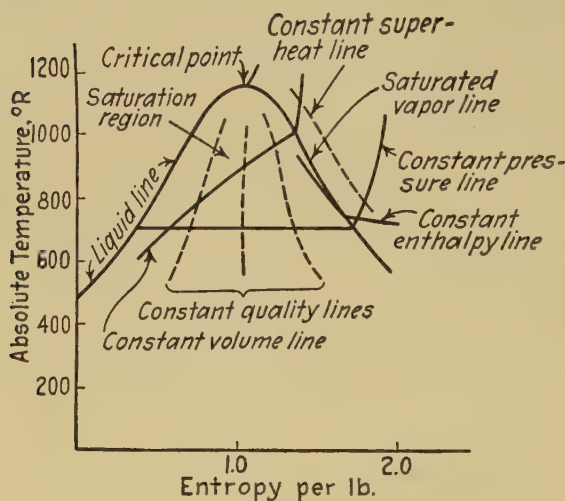


FIG. 21. T - S diagram, water and water vapor.

with the exception that the liquid line is the virtual locus of points not only for the saturated liquid states but for subcooled as well,⁶ whence there is no subcooled liquid region as there is on P - V coordinates.

As a result of the foregoing, all constant pressure contour lines virtually coincide along the liquid line up to the saturation temperature corresponding to a particular

pressure, from which point that pressure line proceeds across the saturation region at the saturation temperature of vaporization.

At any temperature (or pressure) the horizontal intercept between the liquid and saturated vapor lines measures the increase of entropy S_{fg} during vaporization and, since for a wet vapor mixture (equation 5) $S_x = S_f + xS_{fg}$, the state of a wet vapor of quality x is represented by a point on the pressure line at a distance xS_{fg} to the right of the liquid line. Contour lines of constant quality will appear as in the diagram.

Typical contour lines of constant specific volume, constant enthalpy, and constant superheat also appear in the diagram.

The principal utility of the T - S diagram will be recalled (Art. 55) to arise from the facts that for any process

$$Q_R = T_2 \Delta S,$$

⁶ This is the result of the very minute and negligible temperature rise and the absence of entropy change accompanying a reversible adiabatic compression of a liquid.

and for any reversible process

$$Q = \int T dS.$$

Thus for any state change of a fluid, as plotted on T - S coördinates, the change of the unavailable energy (Q_R) with reference to a receiver temperature T_2 and chargeable to a pound mass of the fluid is represented by the area bounded by the S -axis below, by the T_2 line above and on the sides by the initial and final entropy lines. Likewise the heat energy reception or departure which accompanies a reversible state change is represented by the area between the state change curve and the S -axis (*below* the curve). Recall also that a reversible adiabatic process is an isentropic (constant entropy) one and thus represented by a straight vertical line.

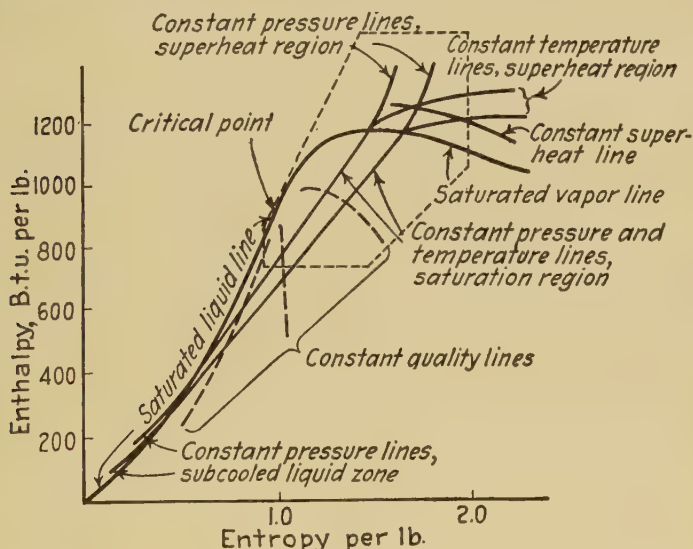


FIG. 22. H - S diagram, water and water vapor.

H - S ("Mollier") Diagram.—The coördinates of the Mollier diagram are the enthalpy and entropy of the fluid, each per pound mass and each with reference to zero relative values of those properties for a liquid at 32° F. By reason of the intimate utility of the enthalpy property for all circumstances in which steady-flow occurs (see Art. 32) this diagram is of particular convenience in connection with the portrayal of steady-flow processes. A portion of the diagram is invariably incorporated with the formal tables of vapor properties.

Referring to Fig. 22, the saturated liquid line, the critical point, and

the saturated vapor line again have the same significance as in the P - V and T - S diagrams. The subcooled liquid region is practically restricted to a rather narrow band immediately above the saturated liquid line. Refer to Art. 63, equation 1, for recognition and computation of the enthalpy of subcooled liquid.

Constant pressure lines in this subcooled liquid band approach the saturated liquid line as the excess of their pressure over the saturation pressure approaches zero. Within the saturation region the constant pressure lines are straight lines and obviously are also constant temperature contours. The vertical distance between the intersections of a constant pressure line with the saturated liquid and saturated vapor lines measures the increase of enthalpy during vaporization and the horizontal distance measures the entropy of vaporization. That the constant pressure lines must be straight lines in the saturation region follows from the relations (Art. 63),

$$\begin{aligned} H_x &= H_f + xH_{fg} \quad \text{and} \\ S_x &= S_f + xS_{fg}, \end{aligned}$$

whence, solving for x ,

$$\frac{H_x - H_f}{H_{fg}} = \frac{S_x - S_f}{S_{fg}}.$$

On entering the superheat region the constant pressure lines veer upward, and the constant temperature contours approach constant H lines with increase of superheat (which is again an evidence of the approach of a superheated vapor to conformity with perfect gas relations; see Art. 77).

Contour lines of constant quality and of constant superheat are also drawn in Fig. 22 and other contour lines might be shown. However, the two noted are usually the ones which appear. It may be remarked that the usual Mollier chart, as used for many practical purposes, shows to larger scale only a small region around the saturated vapor line, such as that enclosed by dotted lines in the upper portion of Fig. 22.

Example 11. Plot to scale, from Steam Table data, a complete T - S diagram showing the liquid and saturated vapor lines, 15, 200, and 600 lb. per sq. in. (abs.) constant pressure lines (extending to 650° F.), lines of constant 20 and 80 per cent quality, a line of constant H at 1150 B.t.u. per lb. (from the 600 lb. line to $S = 1.9$), a line of constant specific volume of 2.8 cu. ft., a line of constant 100° F. superheat. (Note—Use a scale of 1 in. = 200 degrees and 0.4 units of entropy. Locate at least three points for each line.)

Example 12. From the diagram of example 11, find by area computation the enthalpy of vaporization at 15, 200, and 600 lb. per sq. in. and check with the steam table values.

Example 13. Plot to scale, from Steam Table data, a Mollier diagram showing the saturated liquid and saturated vapor lines, 15, 200, and 600 lb. per sq. in. (abs.) constant pressure lines extending from a temperature of 60° F. to an enthalpy of 1350 B.t.u., lines of constant 20 and 80 per cent quality, an isothermal at 382° F., a line of constant 100° F. superheat. (*Note* — Use a scale of 1 in. = 200 B.t.u. and 0.4 units of entropy. Locate at least three points for each line.) Outline the portion of this diagram which is included in the steam table Mollier Chart.

65. Simple State Changes and Processes. — In the consideration of the various actual processes which will be encountered in engineering thermodynamics certain classes of state changes of fluids are referred to with marked frequency, whence it is desirable that we should now familiarize ourselves with the general principles and methods regularly employed in the analysis of those typical state changes. It is therefore the purpose of the succeeding paragraphs to give attention to several such which involve state changes of a liquid or vapor. However, before proceeding to the analysis of specific state changes it is advantageous to observe several features which will be found to be common to all.

In the first place it is obvious that for the analysis of any process it is an inherent requisite that the initial state of the fluid must be adequately specified. Such a specification may evidently be made in terms of a quantitative statement of the properties of the fluid at that state but it may not as yet be evident how many or what properties will suffice for adequate designation of the state. For the present we shall depend on physical experience for that information. Such experience indicates that *for the fluids and the states with which we shall be concerned in engineering thermodynamics information concerning any two properties is, with two exceptions, sufficient for designation of the state.* These exceptions are (a) that for a saturated liquid or a (dry) saturated vapor a single property is adequate, the fact of saturation being in a sense equivalent to the specification of a second property, and (b) that for a mixture of two fluids or a mixture of a fluid in the saturated liquid state with the same fluid in the saturated vapor state (as in wet steam) information concerning the relative proportions of each in the mixture is also necessary. There is no further limitation to the number and character of the requisite properties.⁷ However, in practice it is commonly found that one or more of the specified properties will be those which we are best equipped to measure readily, that is, the pressure or temperature or perhaps the specific volume.

A second feature of common concern is the manner of specification of the character of the state change. In this connection it will be found that *many of the actual processes are characterized by the actual or effective*

⁷ This physical experience is confirmed by Gibb's "phase rule," which is established wholly from the laws of thermodynamics.

constancy of some property of the fluid during the state change. To illustrate, a state change taking place in a fluid which is retained within a closed and non-extensible vessel is one in which the specific volume remains constant or is a *constant volume* process; one taking place during the passage of a fluid through a device such as a boiler or superheater may closely approach *constant pressure* conditions and be taken as effectively such; the ideal state change taking place during the expansion of a fluid in an engine or turbine is the reversible adiabatic, which we have seen to be characterized by a constant entropy of the fluid or thus to be an *isentropic* state change; et cetera.

As a third feature it will be observed that, as the result of such constancy of one property of the fluid during a change of its state and thus a change of its other properties, this particular property is therefore known at the cessation of the process. Therefore, *to designate the final state of the fluid and thus to complete the description of the process, it will in general be necessary to specify the magnitude of one additional property at the final state.* With sufficient information thus available for fixing that state, all the other properties may then be determined.

Various other typical processes are not characterized by a constancy of any one property but may perhaps be described in terms of some mathematical relationship existing between a pair of properties during the progress of the process. Such circumstances are, however, rather more typical of state changes of gases and detailed consideration of them will be deferred until the following chapter.

A fourth feature of most process analyses is *the desirability or necessity for determining the energy transformations and transitions accompanying the processes.* For the energy analyses the equations of Arts. 29, 31, 32 or 34 will be freely employed (see also Art. 36). In certain of the simpler state changes the energy analysis will be limited to that of mechanically reversible processes, this in spite of their non-actuality and because of their ideality.

The following representative processes with steam vapor will be analyzed: (a) constant volume process, (b) constant pressure process, (c) reversible adiabatic process, (d) irreversible adiabatic process and (e) throttling process. Each analysis is illustrated and to a large extent developed by the use of an example.

66. Constant Volume Process. — The simplest circumstance under which a constant volume process may take place is that in which a fluid is retained within a closed container while the state change is effected through the transition of energy to or from the system as heat conducted through the walls of the container from or to the surrounding region.

Example 14. A tank of 20 cu. ft. capacity contains steam at 200 lb. per sq. in. abs. and 500° F. The tank and contents are cooled to a temperature of 326° F. What will be the final pressure and quality of the steam, how much energy was emitted by the steam in cooling, and what was the change of enthalpy of the steam? At what pressure and temperature did the steam become (dry) saturated?

Solution. Saturated steam at 200 lb. abs. has a temperature of 382° F., whence at state (1) the steam had a superheat of 118° F. From Superheated Steam Tables at 200 lb. and 500° F., $V_1 = 2.72$ cu. ft. per lb., whence the mass of steam in the tank was

$$M = 20/2.72 = 7.35 \text{ lb.}$$

After cooling to 326° F. at constant volume the specific volume V_2 must be the same. From the Saturated Steam Tables the specific volume of (dry) saturated steam at 326° F. is 4.534 cu. ft. per lb., whence a portion of the steam must have condensed and the fluid in the tank become a mixture of saturated liquid and saturated vapor (wet steam). The pressure of this saturated mixture at 326° F. must be 97.5 lb. per sq. in. abs. (from Saturated Steam Tables).

To find the quality x_2 from equation 2, Art. 63,

$$x_2 = \frac{V - V_{f,2}}{V_{fg,2}} = \frac{2.72 - 0.0177}{4.517} = 0.60.$$

The energy emitted as heat in cooling must have come wholly from the internal energy of the steam since no energy emission as shaft-work, flow-work or kinetic energy was possible. Therefore, for each pound of fluid,

$$\begin{aligned} Q_{\text{out}} &= E_1 - E_2 \\ &= [H_1 - P_1 V_1/J] - [(H_{f,2} + x_2 H_{fg,2}) - P_2 V_{x,2}/J] \\ &= \left[1267.9 - \frac{200 \times 2.72}{5.4} \right] \\ &\quad - \left[(296.4 + 0.60 \times 889.7) - \frac{97.5 \times 2.72}{5.4} \right] \\ &= 1167.0 - 780.1 = 386.9 \text{ B.t.u. per lb.} \end{aligned}$$

Total energy emitted, as heat, $= 7.35 \times 386.9 = 2,844$ B.t.u.

Change of enthalpy $= H_2 - H_1$

$$\begin{aligned} &= (H_{f,2} + x_2 H_{fg,2}) - H_1 \\ &= 829.3 - 1267.9 = -438.6 \text{ B.t.u. per lb.} \end{aligned}$$

To find the pressure and temperature at which the steam became (dry) saturated it is necessary to search the Saturated Steam tables for the state at which $V_g = 2.72$. This is found at a pressure of 166.7 lb. per sq. in. abs. and a temperature of 366.8° F.

Note that in this non-flow, constant volume circumstance only the change of internal energy ($E_2 - E_1$) measures or denotes an energy quantity, the change of PV becoming simply a change in that product of properties, *without an energy significance* (see also footnote, Art. 32).

Example 15. Presuming a boiler to have a normal water space of 200 cu. ft. and a steam space of 50 cu. ft., these containing saturated water and vapor respectively at a temperature of 212° F., compute (a) the mass of water and vapor in their respective spaces at this original temperature, (b) the mass and volume of water and of vapor after sufficient energy has been supplied to bring the boiler up to a steaming pressure of 285.3 lb. per sq. in. gage (presuming no change in the total volume and total mass of fluid) and (c) the amount of energy supply as heat which has been required to bring the boiler contents up to the steaming pressure and temperature.

($M_{w,1} = 11,975$, $M_{v,1} = 1.865$; $M_{w,2} = 11,962$, $M_{v,2} = 15.35$; $Q = 2,558,000$ B.t.u.)

67. Constant Pressure Process. — A constant pressure process may readily be conceived to occur (a) without flow, as in a cylinder with piston, or (b) with steady-flow, as in any vessel such as a steadily steaming boiler, a steadily operating condenser et cetera, the state change being effected in either case by energy transition to or from the fluid, this usually as heat.

Example 16. A pound of water *in a cylinder (with piston)* is initially at 200° F. and 235.3 lb. per sq. in. gage and by energy reception as heat is transformed to steam of 98 per cent quality at the same pressure, the constancy of pressure being attained by permitting the piston to retreat as the fluid expands.

Compute the amount of energy required to effect the process and also that portion of this energy which is stored internally in the fluid.

Solution.

From equation 7 of Art. 34,

$$\begin{aligned} JQ &= J(E_2 - E_1) - W \\ &= J(E_2 - E_1) + W_{\text{out}}. \end{aligned}$$

In this device the work output at the piston face equals $\int_1^2 P \, dV = P(V_2 - V_1)$, since the pressure P is constant (see also Art. 36a). Therefore

$$\begin{aligned} Q_{\text{in}} &= (E_2 - E_1) + P(V_2 - V_1)/J \\ &= (E_2 + PV_2/J) - (E_1 + PV_1/J) \\ &= H_2 - H_1, \text{ for a non-flow constant-pressure process.} \end{aligned}$$

For the specific data of the problem;

$$\begin{aligned} H_1 &= 167.9 + \frac{(250 - 11.5) \times 144 \times 0.0166}{778} \\ &= 168.7 \text{ (exactly)} = 168.0 \text{ (approx., see Art. 63).} \\ H_2 &= 376.0 + 0.98 \times 824.5 = 1184.0 \\ Q = \Delta H &= 1184.0 - 168.7 = 1015.3 \text{ B.t.u. per lb.} \\ E_1 &= 167.9 - \frac{11.5 \times 144 \times 0.0166}{778} = 167.9 \\ E_2 &= 1184.0 - \frac{250 \times 0.98 \times 1.841}{5.4} = 1100.5 \\ \Delta E &= 1100.5 - 167.9 = 932.6 \text{ B.t.u. per lb.} \end{aligned}$$

The difference between the energy supplied (ΔH) and that stored as molecular energy in the steam (ΔE) is transferred directly via the fluid to the piston face and is there delivered as work [$P(V_2 - V_1)$, = 1015.3 - 932.6 = 82.7].

The foregoing example has had to do with a non-flow constant pressure process. For the circumstance of steady-flow at constant pressure through a device such as a boiler, from equation 5, Art. 32, since there is no shaft-work,

$$Q_{\text{in}} = (H_2 - H_1) + (U_2^2 - U_1^2)/64.34 \, J.$$

As the change of kinetic energy in the usual device of this character would very rarely indeed exceed one B.t.u. per lb. and would usually be much less, this in comparison with a change of enthalpy which commonly would be some thousand B.t.u. in magnitude, the kinetic energy term becomes effectively negligible and may be omitted. Thus for both a non-flow and a steady-flow constant pressure process the energy received by the fluid is virtually measured by its change of enthalpy.

Example 17. A boiler is operating steadily with feed-water at 235.3 lb. gage and 200° F. and is delivering steam of 97 per cent quality at the same pressure. Neglecting any difference of the velocities in the feed and steam lines, compute the amount of energy received by the water (per pound) and also compute the per cent error in neglecting the kinetic energy terms if the velocity in the feed line were 8 ft. per sec. and that in the steam line were 1200 ft. per min. ($\Delta H = 1007$.)

If the steam leaving the boiler were delivered at the same pressure through a superheater and there received 115 B.t.u. additional energy per pound, what would be the state of the steam leaving the superheater? ($t_3 = 551^\circ \text{ F.}$)

Example 18. Check the results of example 17 as far as is possible by use only of a Mollier diagram.

68. Reversible Adiabatic Process. — Recalling (Art. 35a) that, by definition, *any process whatsoever in which no energy enters or departs from the system as heat is designated as an adiabatic process*, we may immediately write the following general energy equations for an adiabatic non-flow and an adiabatic steady-flow process:

$W = J(E_2 - E_1)$, for a non-flow adiabatic process (by Art. 34, equation 7);

$W + (U_1^2 - U_2^2)/64.34 = J(H_2 - H_1)$, for a steady-flow adiabatic process (by Art. 32, equation 5).

These equations are universally applicable whether the adiabatic process may be ideal and reversible or actual and thus irreversible. For the further condition of a *reversible* adiabatic process there is introduced the additional feature that the entropy remains constant during the process (see Art. 53, example 5 et sequi), or the process is isentropic. It may well be remarked that the outstanding practical utility of the entropy property lies in this very feature of its constancy during a reversible adiabatic process. To denote such constancy we shall hereafter employ the letter S as a subscript attached to any change of properties of a fluid during an isentropic process. Thus, rewriting the above equations but now in the special form which is applicable to the reversible adiabatic,

$W = J(E_2 - E_1)_S$, for a non-flow reversible adiabatic;

$W + (U_1^2 - U_2^2)/64.34 = J(H_2 - H_1)_S$, for a steady-flow reversible adiabatic.

For an easy and accurate determination of the fluid properties after isentropic expansion from a given initial state to a specified final pressure (or temperature or quality or volume) the P - S tabulation mentioned at the end of Art. 63 is extremely convenient. Lacking such a tabulation the H - S (Mollier) chart obviously is useful. Also the final quality after expansion to a given pressure or temperature *within the saturation region* may be computed directly by equation 5 of Art. 63:

$$x_2 = \frac{S - S_{f,2}}{S_{fg,2}} = 1 - \frac{S_{g,2} - S}{S_{fg,2}}, \quad (7)$$

where S is the initial and constant entropy. Knowing the final quality the other properties may be computed as usual from equations 2, 3 or 4. Alternatively, the following more direct relations may easily be developed:

$$H_2 = H_{g,2} - (1 - x_2) H_{fg,2} \quad (8)$$

$$\begin{aligned} &= H_{g,2} - \frac{S_{g,2} - S}{S_{fg,2}} H_{fg,2}, \\ &= H_{g,2} - (S_{g,2} - S) T_2 \end{aligned} \quad (8a)$$

and

$$V_2 = V_{g,2} - \frac{S_{g,2} - S}{S_{fg,2}} V_{fg,2}. \quad (9)$$

Example 19. Steam at 200 lb. per sq. in. abs. and a quality of 0.995 is delivered to an ideal steam turbine, expands adiabatically and reversibly therein to a pressure of 14.7 lb. per sq. in. abs. and is discharged at that pressure.

Compute the entropy, quality and enthalpy of the steam leaving the turbine and the shaft-work which would ideally be obtainable per pound of steam used, presuming negligible kinetic energies entering and leaving the turbine.

Solution.

$$S_1 = S_{g,1} - (1 - x_1) S_{fg,1} = 1.5450 - 0.005 \times 1.0012 = 1.5400.$$

$$S_2 = S_1 = 1.5400.$$

$$x_2 = \frac{1.5400 - 0.3119}{1.4446} = 0.850.$$

$$H_2 = 1150.2 - 0.15 \times 970.2 = 1004.7.$$

$$H_1 = 1197.8 - 0.005 \times 842.4 = 1193.6.$$

$$W = J(H_1 - H_2)_s = J(1193.6 - 1004.7) = 778 \times 188.9 = 147,000 \text{ ft-lb.}$$

Example 20. Check as far as possible the results of example 19, using only a Mollier diagram.

Example 21. Steam at 180 lb. per sq. in. abs. and 450° F. is supplied to an ideal nozzle and expands therein reversibly and adiabatically to a pressure of 65 lb. per sq. in. abs. Compute by use of the steam tables the quality, enthalpy and velocity of the departing steam and check the results by use of the Mollier diagram.

$$(U_2 = 2080 \text{ ft. per sec.})$$

Example 22. In the ideal reciprocating steam engine the sequence of events is (a) the entry of high pressure steam to the cylinder through the admission valve during a portion of the "power" stroke, thereby causing a forward movement of

the piston, (b) closure of the admission valve and a reversible adiabatic and non-flow expansion of the entrapped steam to some lower pressure, causing continued forward movement of the piston to the end of the power stroke, (c) a return of the piston on the "exhaust" stroke to its original position, the returning piston accomplishing a delivery of the lower pressure steam from the cylinder through the exhaust valve against the exhaust pressure.

Presuming a steam supply at the same state as that of example 19 and a like isentropic expansion to 14.7 lb. abs., compute the work obtained during admission, the work obtained during the non-flow expansion in the cylinder, the work returned to effect the delivery during exhaust, and the consequent *net* work of the cycle, all per pound of steam. Check the final result by comparing with the work output computed for the turbine of example 19. (65,500; 129,800; 48,300 ft.-lb.)

69. Irreversible Adiabatic Process. — The essential characteristic in which the irreversible adiabatic differs from the reversible is the increase of entropy which results from the irreversibility of the process. Typical of the processes in which such irreversibility occurs is the actual expansion through a turbine. The following example is indicative of the method of procedure.

Example 23. The high pressure steam of example 19 is delivered to and expands adiabatically through a steam turbine but with an increase of entropy of 0.0800 units per lb. and is delivered thence, the expansion being to a pressure of 14.7 lb. abs. What would be the entropy, quality, and enthalpy of the steam leaving the turbine and the shaft-work deliverable (presuming negligible difference of kinetic energies and heat loss)? Compute the results by use of the Steam Tables, and check by use of the Mollier diagram. (0.9055; 1058.6; 105,000 ft.-lb.)

70. Throttling Process. — Technically a pure throttling process is one in which a fluid expands adiabatically but thoroughly irreversibly through a labyrinthal or porous obstruction in a line, the arrangement being such that the velocities of the stream approaching, through, and departing from the obstruction shall be very small and their differences wholly negligible. Obviously no shaft-work is performed.

This process is approached closely in practice in the "wire-drawing" of a vapor (or gas) through a constriction in a line such as a partially closed valve or small orifice. In such devices there is unquestionably an accretion of velocity and kinetic energy directly in the constriction but this momentary unidirectional velocity is immediately dissipated by reason of the high degree of turbulence existing in the stream directly after the constriction, whereby the kinetic energy in the stream finally departing from the region ($U_2^2/64.34$) is negligible or differs negligibly from that in the stream approaching the region ($U_1^2/64.34$). (See Fig. 23.)

The process is also approached, but less exactly (due to energy dissi-

pation as heat), in the flow of a vapor through a long pipe line in which there is a distinct pressure drop due to friction.

To discern the working scheme for analyzing the throttling process



FIG. 23. Throttling process.

note that in fitting the general steady-flow energy equation (equation 5, Art. 32) to the conditions of the process the Q and W terms disappear and the kinetic energy terms are either negligible or effectively canceled, whence (see also Art. 33d)

$$H_1 = H_2.$$

This equality of the initial and final enthalpy is the outstanding characteristic of the throttling process.

Example 24. Steam at 250 lb. abs. and 99 per cent quality is throttled to a pressure of 200 lb. abs. What is the state of the steam after throttling and what is the loss of available energy, presuming that the steam were being supplied to a reversible engine operating with a 70° F. exhaust?

Solution.

By Eq. 4, Art. 63, $H_1 = 1192.3 = H_2$.

To ascertain the second state, since H_g at 200 lb. abs. is 1197.8 the fluid must still be a slightly wet mixture. To ascertain the quality,

$$H_2 = 1192.3 = H_{f,2} + x_2 H_{fg,2} = 355.3 + x_2 842.4, \quad x_2 = 0.9935.$$

To ascertain the increase of unavailable energy,

$$S_1(\text{at 250 lb. and 99\%}) = 1.5165; \quad S_2(\text{at 200 lb. and 99.35\%}) = 1.5385.$$

$$\Delta S = 0.0220; \quad \Delta Q_R = T_2 \Delta S = (70 + 460) \times 0.0220 = 11.7 \text{ B.t.u. per lb.}$$

Example 25. Check the loss of available energy as determined for the throttling process of example 24, doing so by individual computations of the isentropic change of enthalpy and the available energy for expansion to 70° F. from (a) the initial state at 250 lb. and (b) the second state, after throttling to 200 lb. The difference is the loss of available energy. Depict the two complete processes on H - S coordinates and indicate the available energy loss.

71. The Throttling Calorimeter. — Both by reference to example 24 or to a Mollier diagram (see Fig. 24) it becomes apparent that if a wet vapor passes through a throttling process the requisite equality of the initial and final enthalpies will in general produce an increase in the quality of the vapor.⁸ Also if the initial quality of the vapor is sufficiently high and the pressure drop sufficiently great, some degree of superheat may well

⁸ This statement must be made with some qualification. To illustrate, inspection of the Mollier diagram of steam (Keenan tables) shows that with initial pressures in excess of about 500 lb. per sq. in. a throttling through a moderate pressure range may in fact act to increase the moisture content.

exist in the exit or lower pressure vapor. By reason of the latter circumstance and the additional facts (a) that *in the superheat region the pressure and temperature will suffice to determine the state and enthalpy of the fluid* and (b) that these P and T properties are both directly measurable ones, the throttling phenomenon may be and is regularly employed in a device which is known as the **throttling calorimeter** and is used for ascertaining the enthalpy and quality of steam which contains a limited amount of moisture.

Referring to Fig. 25, a simple arrangement of such a device is there indicated. In the figure A is a steam line through which is passing the steam the enthalpy and quality of which are desired. Pipe B is a *sampling tube* through which it is endeavored to draw a true sample of the steam in A . Gage C or thermometer D determines the saturation

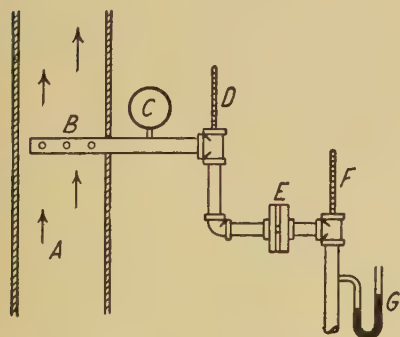


FIG. 25. Throttling calorimeter.

pressure or temperature of the steam supply. These obviously should "check" if the steam is saturated. Between the flanges at E is located a plate with a small (1/16 in. to 1/8 in. diameter) orifice through which the sample is throttled to the discharge pressure, which is usually about atmospheric. Thermometer, F and mercury manometer G , together with data on the barometric pressure, provide the requisite data on the temperature and pressure of

the throttled fluid by which its state may be ascertained. Thus the enthalpy of the leaving steam and thereby that of the entering steam is determinable.

The quality of the steam sample is found by solving for x in the expression for the enthalpy of wet steam, equation 4 of Art. 63, or

$$H_2 = H_1 = H_{f,1} + x_1 H_{fg,1} \quad \text{or} \quad x_1 = (H_2 - H_{f,1}) / H_{fg,1}.$$

An alternative method is to enter the Mollier chart at the intersection of the P_2 and t_2 lines, pass across at constant H to P_1 and read the quality directly.

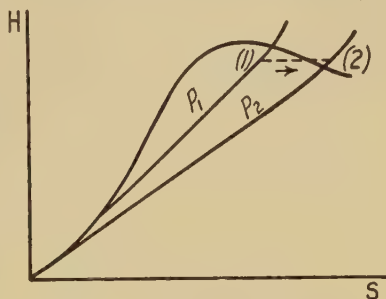


FIG. 24. H - S diagram of throttling process.

This device if properly constructed and operated will give the enthalpy and quality of the sample with good accuracy, although a slight correction for radiation may be desirable. A serious practical difficulty lies in the securing of a truly representative sample of the steam in line A. Also its use is limited to fairly dry and fairly high pressure steam since an excess of moisture or low steam pressure will prevent the superheating, and *its whole utility depends on the possibility of fixing the state of superheated steam from knowledge of its pressure and temperature.*

Example 26. Steam at 200 lb. abs. is throttled through a calorimeter to a pressure of 16 lb. abs. The thermometer in the throttled steam shows a temperature of 260° F. Compute the enthalpy and quality of the steam supply and check by use of the Mollier chart. Under the given pressure conditions what would be the lowest or limiting quality of the supply steam which would permit use of the calorimeter? (0.971; 0.945 +.)

Example 27. Water at 100° F. is discharged by a feed pump through a feed-water heater and into a boiler which is operating at a pressure of 232.3 lb. per sq. in. gage. In the heater, where the pressure may be taken to be virtually the same as in the boiler, the temperature of the water was raised to 240° F. The steam leaves the boiler with a quality of 0.97 and then at virtually the same pressure flows through a superheater, where it attains a superheat of 200° F.

The steam leaving the superheater is delivered to a turbine where it expands adiabatically to atmospheric pressure with an increase of entropy of 0.14 units per lb. From the turbine the steam passes through a steam heating system which is at atmospheric pressure, where it condenses and cools to 100° F. and from which it returns to the feed tank.

- (a) Determine the values of V , PV , H , E and S at the exit from each device in the whole system.
- (b) Make a diagram indicating each successive device and below the diagram record in tabular form the values of t , p , V , PV , H , E and S at the exit from each.
- (c) Draw to approximate scale a diagram representing the cycle on P - V coordinates and designate on the diagram the areas which represent the quantity PV for the fluid as it leaves each device. Would either the area *under* or that *back of* the adiabatic expansion curve through the turbine correctly measure the work output in that device?
- (d) Draw to approximate scale a diagram representing the cycle on T - S coordinates and designate the areas on the diagram which represent the heat energy received by the fluid in the boiler and in the superheater and that departing from the fluid in the heating system.
- (e) Draw to approximate scale a diagram representing the cycle on H - S coordinates and designate on the diagram the vertical distances measuring the change of enthalpy in each device, indicating which enthalpy changes represent work energy and which heat energy transition.
- (f) Tabulate for each device the amount of energy entering as heat, internal energy, flow-work or shaft-work, and likewise the amount and nature of energy leaving. The energy account for each device should balance.
- (g) Summate the energy entering and leaving the system as a whole as heat and as shaft-work, and check to see if the account balances.

72. Summary.— Having developed a formulation of the energy relations which must hold in all processes (Part I) and an understanding of the several properties (E , P , V , H , T and S) of any of the substances employed in those processes (Parts I and II) it is necessary to ascertain how those properties may be evaluated for any state or state change of a particular substance. In the present chapter such information has been given for those liquids and vapors for which tables of properties are available. The state changes considered (and the energy relations for those changes) have been limited to the constant volume process, the constant pressure process and the adiabatic processes, both reversible and irreversible.

The several distinctive stable states of a liquid and its vapor are those of

- (a) *saturated liquid*, which is at the boiling or *saturation* temperature corresponding to the pressure on the liquid;
- (b) *subcooled liquid*, in which the liquid is at a temperature less than the saturation temperature corresponding to the pressure, or, conversely, the imposed pressure is greater than the saturation pressure corresponding to the temperature of the liquid;
- (c) *saturated vapor*, which is (dry) vapor at the saturation temperature corresponding to the pressure;
- (d) *wet vapor*, which is a physical mixture of saturated vapor and saturated liquid, the ratio between the mass of vapor and the mass of mixture being known as the *quality* of the mixture; and
- (e) *superheated vapor*, in which state the vapor is necessarily not in contiguity with its liquid and its temperature is above the saturation temperature corresponding to the pressure, the excess of temperature being known as the *degrees of superheat*.

The tables of properties usually provide data on

- (a) the saturation temperatures for a full range of pressures, and *vica versa*;
- (b) the specific volumes of saturated liquid (V_f) and of (dry) saturated vapor (V_g) and the change of volume during evaporation (V_{fg});
- (c) the enthalpy per pound of saturated liquid (H_f) and of (dry) saturated vapor (H_g), both relative to liquid at 32° F. as a datum condition, and the change of enthalpy during evaporation (H_{fg});

- (d) the entropy per pound of liquid (S_f) and of (dry) saturated vapor (S_g), both relative to that of liquid at 32° F., and the increase of entropy during vaporization, S_{fg} ; and
- (e) the values of V , H , and S for a range of pressures and of superheats (or superheat temperatures) at those pressures.

For saturated liquid, saturated vapor, and superheated vapor most of the commonly needed properties may be taken directly from the tables. For subcooled liquid the specific volume, internal (molecular) energy, and entropy are practically those of saturated liquid *at the temperature* of the subcooled liquid. The enthalpy may differ more materially from H_f at the given temperature if the pressure materially exceeds the saturation pressure corresponding to the temperature, or specifically,

$$H - H_f = (P - P_f)V/J, \quad (1)$$

where P and P_f are respectively the actual and the saturation pressures.

For wet saturated vapor mixtures,

$$V_x = V_f + xV_{fg} = V_g - (1 - x)V_{fg}, \text{ or approx., } xV_g; \quad (2)$$

$$PV_x = P(V_f + xV_{fg}) = P[V_g - (1 - x)V_{fg}] \quad (3)$$

$$H_x = H_f + xH_{fg} = H_g - (1 - x)H_{fg} \quad (4)$$

$$S_x = S_f + xS_{fg} = S_g - (1 - x)S_{fg} \quad (5)$$

$$E_x = H_x - PV_x/J \quad (6)$$

The properties of a liquid and its vapor as well as any state change of the fluid may with advantage be shown graphically on P - V , T - S or H - S coördinates. On these diagrams contour lines are also frequently employed to indicate the manner of change of the coördinate properties for constant values of some other property. The P - V diagram has particular utility because the mechanical effects during a mechanically reversible process are measured by areas on the diagram. Also on the T - S diagram the heat energy reception or departure which is required to accomplish a reversible state change is represented by an area, as is also the change of unavailable energy during any state change. The H - S diagram has particular value for processes involving steady-flow, this by reason of the intimate utility of the enthalpy property in all such processes.

For the analysis of any process which involves a state change of a fluid it is necessary (a) that the initial state of the fluid shall be adequately designated, (b) that the final state also shall be suitably specified and (c) that the manner of progress of the fluid from the initial to the final state shall be known. It has been found by experience that for the adequate designation of any specific state a quantitative knowledge of any two properties will in general be sufficient, with the exceptions

that one is sufficient if the fluid is known to be a saturated liquid or (dry) saturated vapor and that for a mixture of saturated liquid and saturated vapor it is also necessary to know the relative proportions of each in the mixture. As regards the description or characterization of a process, it will frequently be found that in actual processes one property is either actually or effectively constant. This circumstance thus serves to designate one property at the cessation of the process, in which event it becomes necessary to specify only one additional property at the final state. For the case of processes which are not characterized by the actual or virtual constancy of any property special methods of handling will need be devised for any special case.

Very frequently in the analysis of a process or state change it is necessary to determine the energy transformations and transitions accompanying the process. The general energy equations of Chapter III have been devised for exactly that purpose and should only require a fitting to the particular circumstances of the process in order to enable an evaluation of the energy relations.

73. Review Questions and Topics. —

1. (a) Describe the phenomenon which establishes for any fluid a definite relation between the temperature and pressure of a contiguous liquid and vapor.

(b) Define saturation temperature, saturation pressure, subcooled liquid, saturated liquid, saturated vapor, superheated vapor.

(c) Define wet vapor, quality or dryness fraction, degrees of superheat.

2. (a) May the characteristic relations between the various properties of a liquid or of a vapor be expressed by easily solvable mathematical equations? If not, what practical procedure is followed in order to enable the engineer to determine readily the properties of such fluids at any selected state?

(b) List the headings of the columns as ordinarily found in a saturated vapor table and explain each.

3. List the properties usually presented in a superheated vapor table.

4. (a) Describe the method of procedure for ascertaining each of the properties of a subcooled liquid.

(b) Describe the method of procedure for ascertaining each of the properties of a wet vapor.

5. (a) Show the typical appearance of the saturation lines for a liquid and its vapor as those would appear on P - V coordinates. Indicate the subcooled liquid region, the critical point, the saturation region and the superheat region. Show a typical constant temperature line, several constant quality lines and a reversible adiabatic expansion line, the latter starting from an initial state on the saturated vapor line. Indicate the $\int P dV$ and $\int V dP$ for the latter state change.

(b) Show the typical appearance of the saturation lines for a liquid and its vapor as those would appear on T - S coordinates. Indicate the critical point, the saturation region, and the superheat region and explain the absence of a subcooled liquid region. Show a typical constant pressure state change from that of a subcooled liquid to a superheated vapor, a typical constant volume line, several constant

quality lines, and a constant enthalpy line. Indicate the areas $\int T \, dS$ and $\int T_2 \, dS$ for the constant pressure line and explain the significance of each.

(c) Show the typical appearance of the saturation lines for a liquid and its vapor as those would appear on H - S coordinates. Indicate the critical point, the subcooled zone, the saturation region, and the superheat region. Show a typical constant pressure state change from that of a subcooled liquid to a superheated vapor, several constant quality lines and a constant temperature line in the superheat region. Indicate the portion of the entire figure which commonly appears as the Mollier chart of a steam table.

6. (a) How many properties are in general required in order to determine the state of an engineering fluid, how many for a saturated fluid?

(b) List various typical sorts of state changes.

(c) If one property should remain constant during a state change what minimum number of additional properties must be specified in order to designate the final state at the end of the process?

7. What property change measures the energy entry or departure as heat in a constant volume process?

8. What property change measures the energy entry or departure as heat in a constant pressure process?

9. What property is constant in a reversible adiabatic state change, what property change measures the energy entry or departure as work in a non-flow reversible adiabatic process, and what property change measures the sum of the energy transition as work and the change of kinetic energy (or either alone if the other is negligible) in a steady-flow reversible adiabatic process?

10. What are the essential characteristics of an irreversible adiabatic process?

11. Describe a throttling process and indicate the essential property and energy relations for the process.

12. Describe the throttling calorimeter, indicate its principles of action and note its limitations.

Symbols and Abbreviations, Chapter VIII

f = a subscript attached to any property of a saturated liquid.

g = a subscript attached to any property of a (dry) saturated vapor.

fg = a subscript employed to designate the change of any property between the saturated liquid and saturated vapor states, at constant pressure.

E = molecular (internal) energy, B.t.u. per pound.

H = enthalpy ($E + PV/J$), B.t.u. per pound.

J = Joule's equivalent (= 778 ft.-lb. per B.t.u.).

p = pressure, pound per square inch (absolute).

P = pressure, pound per square foot (absolute).

Q = energy transferred as heat (by conduction or radiation, B.t.u. per pound.

S = entropy, per pound.

t = temperature, degree Fahrenheit.

T = temperature, degree Rankine (Fahrenheit absolute).

U = velocity, feet per second.

V = specific volume, cubic feet per pound mass.

W = shaft-work, foot-pounds per pound mass.

x = quality of a vapor-liquid mixture; also a subscript to any property to characterize it as that of a vapor-liquid mixture.

CHAPTER IX

THE PERFECT GAS; PROPERTY RELATIONS AND SIMPLE PROCESSES

The perfect gas was introduced in Chapter VI as an hypothetical substance which would serve as an exact medium for ascertaining the location of the absolute zero on the fundamental thermodynamic (energy) scale of temperature. Search among a number of the real gases revealed that none conformed exactly to the perfect gas characteristics but that they did not deviate from conformity in great degree. Partially by reason of this relatively close approach of the real gases (and likewise of very low pressure vapors) to the perfect gas characteristics, and partially because of the lack of exact tables of properties of these gases, it has become the common custom among engineers to employ the property relations of the hypothetical perfect gas in practical computations of the relations for a real gas. For that reason we are justified in a full development and consideration of the property relations of the admittedly non-existent perfect gas.

74. P - V - T Relation for the Perfect Gas. — The original property relations upon which the perfect gas conception was based were (a) that if the absolute pressures are varied by means of temperature variation while the specific volume is maintained constant the ratio of the absolute pressures must equal the ratio of the absolute temperatures, and (b) that if the specific volumes are similarly varied while the pressure is maintained constant the ratio of the specific volumes must also equal the ratio of the absolute temperatures. Stated symbolically,

$$\left(\frac{P_2}{P_1}\right)_V = \frac{T_2}{T_1} \quad \text{and} \quad \left(\frac{V_2}{V_1}\right)_P = \frac{T_2}{T_1}.$$

To combine these two requirements into a more useful *single* relation let us conceive a unit mass of a gas to be brought from any initial state 1 to any second state 2 by two successive processes which shall be a constant volume change to an intermediate state a at which the pressure is that of the second state, this followed by a constant pressure change from the intermediate to the final state. A graphical representation of these successive processes appeared in Fig. 12 of Art. 48. The property relations for the two processes becomes

$$\frac{T_a}{T_1} = \frac{P_a}{P_1} = \frac{P_2}{P_1} \quad \text{and} \quad \frac{T_2}{T_a} = \frac{V_2}{V_a} = \frac{V_2}{V_1}.$$

These relations may be combined by solving each for T_a , equating and thus eliminating T_a , or

$$T_a = T_1 \frac{P_2}{P_1} \quad \text{and} \quad = T_2 \frac{V_1}{V_2}, \quad \text{whence}$$

$$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1}.$$

Since the two states were specified to be *any* states whatsoever the interpretation of the last relation must be that the product of the absolute pressure and the specific volume of a perfect gas divided by its absolute temperature can not be changed by any process whatsoever through which the gas may pass, or that this functional relation between the three properties is a constant. This constant is generally symbolized by the letter R . Thus we may write the following important **characteristic equation, or equation of state**, between the three properties:

$$\frac{PV}{T} = R, \text{ a constant; or } PV = RT. \quad (1)$$

For various purposes it will be convenient to have this characteristic equation in differential form, or, differentiating,

$$P dV + V dP = R dT. \quad (1a)$$

There is no implied limitation or specification as to the magnitude of the constant, R . As may be anticipated, physical tests of the real gases and very low pressure vapors show that it will differ for different fluids, also (as may again be anticipated) that it is not strictly constant for any actual gas. This is considered in further detail in Art. 81.

A simple but highly important feature of this characteristic equation is its utility for computing directly any one of the three properties, P , T or V of a gas, given the other two and the gas constant of the gas in question. Thus if the constant R of a given gas has been ascertained by physical test of any character whatsoever, then the specific volume of that gas would be directly determinable from the characteristic equation by noting that $V = RT/P$, or the density ($=1/V$) $= P/RT$, or the pressure exerted by the gas when at any particular specific volume and temperature must invariably equal RT/V , or again the only temperature at which the gas could be when at a given pressure and specific volume will be established by the quantity PV/R .

Directly parallel would be the computation for M lb. of a gas, which would occupy a total volume of MV cu. ft. and for which the characteristic relation becomes

$$P(MV) = MRT. \quad (1b)$$

Example 1. A portion of a surface which would represent graphically the equation $PV/T = \text{constant}$ is shown in Fig. 26. If point x lies on this surface, show a constant pressure, a constant volume and a constant temperature state change through x and the projections of those changes on the P - V plane.

Example 2. What size tank must be supplied to hold 20 lb. of air at 200 lb. per sq. in. abs. and 70° F. ? (R for air = 53.3.) To what values would the pressure rise if the temperature were brought to 150° F. ? (19.6 cu. ft.; 230.)

Example 3. At 14.7 lb. abs. and 32° F. helium weighs 0.01114 lb. per cu. ft. What is the value of its gas constant? (386.)

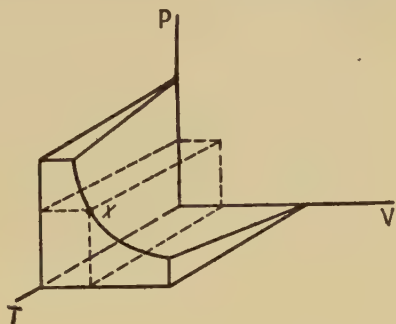


FIG. 26

A relation has thus been developed between the three directly measurable properties, P , V and T . It still remains to evolve relations whereby the additional major properties E , H and S may be correlated with the foregoing properties. The internal energy will be considered first.

75. Internal Energy of the Perfect Gas. — Although there is no way of evaluating absolutely the amount of internal energy stored in the molecular system of a gas it will be essential that means be provided for ascertaining the *change* in that energy (ΔE) which accompanies any state change of the gas.

By a mathematical treatment of thermodynamics,¹ it may be shown that for any substance which obeys the perfect gas relation $PV/T = R$ (a constant), *the internal energy must depend solely on the absolute temperature*, which would require that it be wholly independent of the pressure, specific volume or any other state function. More specifically this means that for a given gas at a specified temperature the internal energy per pound is the same whether the pressure be high, low or indeterminate. Expressing this statement symbolically and in mathematical parlance,

$$\Delta E = f(\Delta T).$$

This relation is known as **Joule's Law**. It will be our purpose to establish the functional relation concretely in the following article.

The above conclusion does not seem unreasonable when we recall (Art. 5, last paragraph) that in the permanent gases the intermolecular attractions appear to be practically eliminated, with no consequent opportunity for change in their store of molecular potential energy,

¹ See Part V, Chapter XIX, Art. 200.

and (Art. 10) that the temperature of a substance bears a very close dependence on its average molecular kinetic energy. Change in temperature of a gas would thus be the only cause for a change in its internal energy store.

The conclusion may also be closely verified by experiment. If we could perform an experiment in which there is a pronounced state change as regards the pressure and specific volume of a gas, but in which there is no change of the internal energy of the gas ($\Delta E = 0$), and if the experimental evidence showed also that no temperature change occurs ($\Delta T = 0$), then we could agree that the results of the experiment are at least in accordance with Joule's Law and to that extent substantiate the Law.

The classical experiment of this character is that known as the **Joule experiment**, in which two receivers are connected by means of a pipe with closed valve (see Fig. 27). One receiver contains a compressed gas and the other receiver is exhausted. The whole apparatus is immersed in a well-insulated water bath in which the water may be adequately circulated and its mean temperature accurately measured. Upon opening the valve between the receivers the compressed gas is permitted to expand and occupy both receivers, with consequent material change of state as regards both volume and pressure.

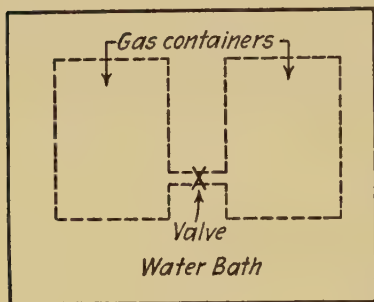


FIG. 27

Let us analyze the energy relations for this process. Since the apparatus is so arranged that no external work is done and there is no flow of the gas into or out of the apparatus the energy equation becomes

$$Q = E_2 - E_1.$$

But we recognize that any addition or rejection of energy from the mass of gas as heat would necessarily be evidenced by a change in the mean temperature of the water bath. With the "permanent" gases such as oxygen, nitrogen, hydrogen, helium, et cetera, which we know to more closely approach the perfect gas, only the very slightest change of temperature of the bath is observed. This would indicate three things:

- (a) that Q is practically zero, else the bath temperature would change;
- (b) that therefore (from the energy equation) the internal energy of the gas can not have changed, this in spite of the very

material change of state as regards pressure and specific volume; and

- (c) that the temperature of the gas can not have changed, else there would perforce have been an energy transition between the gas and water as heat and a corresponding change in the temperature of the water bath.

We have thus a state change as regards both pressure and specific volume but without significant change of internal energy or of temperature — a condition which would have to exist to satisfy Joule's Law.

These results do not serve to *prove* the Law, and as remarked before its proof is best made mathematically, but they do provide distinct evidence of its validity in the case of these nearly perfect gases. It is significant that experiments of this character on other gases show increasing change of temperature as the substance deviates more and more in other respects from the perfect gas criterion.²

76. ΔE as a Function of ΔT . — In order to ascertain a concrete relationship between change of the internal energy of a gas and its change of temperature it will be necessary to recognize two consequences of the Joule Law. These are that

(a) The change of internal energy resulting from a given change of temperature of a gas must be exactly the same for any process whatsoever by which the specified temperature change may be effected; and that

(b) One is thereby privileged to evaluate the energy change for some convenient but particular manner of passage from one temperature to a second and acclaim the result as true for all processes by which the same temperature change might be effected.

Bearing these considerations in mind let us investigate a non-flow constant volume process. Such a process is *per se* one in which no energy transition to or from the system as work can take place, whence its energy equation is (see also Art. 35),

$$Q_v = E_2 - E_1 = \Delta E,$$

in which the subscript v designates the constancy of the volume. But let it be recalled that by definition (Art. 24) the ratio between the energy

² An experiment which permits a much more exact and a quantitative measurement of the deviation of the actual gases from the perfect gas is that known as the Joule-Thompson experiment. In this the gas is caused to flow slowly and steadily through a porous plug and temperatures and pressures are measured on each side of the plug. It will shortly be apparent that for such a steady-flow, throttling process the perfect gas would exhibit no change of temperature. See Arts. 77 and 89, also Arts. 199 and 200.

interchange as heat in a constant volume process (Q_v) and the accompanying temperature change (ΔT) is known as the *specific heat at constant volume* (c_v). Thus $c_v = Q_v/\Delta T$, or $Q_v = c_v \Delta T$. Introducing this expression for Q_v in the above,

$$\begin{aligned}\Delta E &= c_v \Delta T, \text{ or in differential form,} \\ &= \int c_v dT. \end{aligned} \quad (2)$$

Since c_v is a characteristic of a gas which may be readily determined by physical experiment we have thus evaluated the internal energy change in terms of the temperature change and a readily ascertainable characteristic of a gas, c_v . In this connection observe that, in the light of the first paragraph of this article, the internal energy change accompanying a given temperature change of a gas may be computed by equation 2 for any process whatsoever by which the temperature change may be effected. The utility of the equation is *not* limited to a constant volume process.

Since the internal energy is a function solely of the temperature it follows from equation 2 that c_v must either be a constant or, if variable, must depend only on the temperature of the gas. It is found by experiment to vary with the temperature to an appreciable extent and thus should be expressed in terms of the temperature, although in computations involving moderate temperature changes its variation is frequently neglected, and this with no serious inaccuracy. However for greater temperature ranges its variation must usually be considered. It is for that reason that equation 2 was also written in the differential form, because it is necessary that such a form should be employed when the variation is to be accounted for. Further information as to the character of the variation is given in Art. 82.

Example 4. Air is compressed from atmospheric pressure and 70° F. to a pressure of 100 lb. per sq. in. gage in such a manner that for the process $PV^{1.267} = \text{a constant}$, it is then cooled and is finally throttled to a pressure of 97 lb. gage. The final temperature is 200° F. What was the net change of internal energy per pound of air if $c_v = 0.173$?

77. Enthalpy Changes for a Perfect Gas. — By definition, $H = E + PV/J$. Writing for PV its equivalent RT (by equation 1), $H = E + RT/J$. Since E has been shown to be dependent on temperature only and since R and J are both constants it is evident that the enthalpy of a gas is likewise dependent solely on its temperature, or

$$\Delta H = f(\Delta T).$$

To evaluate concretely this functional relation it will again be necessary to recognize a pair of conditions which parallel those considered in the preceding article; (a) that the change of enthalpy in any given change of temperature of a gas will be the same for any process whereby a change from the first to the second temperature may be effected, and (b) that the enthalpy change as evaluated for *any* particular process between those temperatures will therefore constitute a perfectly general evaluation.

Investigating a non-flow constant-pressure reversible process, such as the constant pressure expansion of a gas in a cylinder with (frictionless) piston, the general energy equation for such a process is (from Art. 35c)

$$Q = E_2 - E_1 - W/J.$$

But for this particular expansion the work at the piston face equals $-\int_1^2 P dV = -P(V_2 - V_1)$, since the pressure is constant. Therefore, employing the subscript p to designate constancy of pressure,

$$\begin{aligned} Q_p &= E_2 - E_1 + PV_2/J - PV_1/J \\ &= H_2 - H_1 = \Delta H. \end{aligned}$$

Recall now that by definition (Art. 24) $Q_p/\Delta T$ equals the *specific heat at constant pressure* (c_p) or $Q_p = c_p \Delta T$. Introducing this expression for Q_p in the above,

$$\begin{aligned} \Delta H &= c_p \Delta T, \text{ or in differential form,} \\ &= \int c_p dT. \end{aligned} \tag{3}$$

It is important to recognize that this relation correctly evaluates the enthalpy change for any process whatsoever, whether one at constant pressure or otherwise.

Since c_p is also a characteristic which may be determined by experiment the enthalpy change of a gas is thereby evaluated for *any* process in terms of the temperature change and the ascertainable characteristic of the gas, c_p .

In order that the enthalpy change shall be a function only of the temperature change it is necessary that c_p should either be constant or a variable depending only on the temperature. It is the latter, but, like c_v , may be taken as effectively constant for computations involving temperature changes of moderate range.

Example 5. Find the net change of H for the process of example 4, if c_p for air = 0.241. (+ 31.3 B.t.u.)

Example 6. In 1842 Dr. Mayer found that the heat required to raise the temperature of 1 lb. of air from 32° F. to 523° F. at constant atmospheric pressure was 116.7 B.t.u., during which process the air expanded from 12.33 to 24.66 cu. ft. per lb. For the same temperature rise at constant volume he found that 83 B.t.u. were required. From these data he computed Joule's Equivalent to be 774 ft.-lb. per B.t.u. Check his value from the data given.

78. Relation between c_p and c_v ; between dH and dE . — The ratio c_p/c_v will be found to occur so very frequently and to be of such utility that it is customary to assign to it a symbol, as k or γ (gamma). We shall adopt the former and write

$$\frac{c_p}{c_v} = k, \quad \text{or} \quad c_p = kc_v. \quad (4a)$$

An equally useful relation between c_p and c_v is one expressing their difference ($c_p - c_v$). To determine this recall that $H = E + PV/J = E + RT/J$, or $dH = dE + (R/J) dT$. But $dH = c_p dT$ and $dE = c_v dT$, whence $c_p dT = c_v dT + (R/J) dT$, or

$$c_p - c_v = R/J. \quad (4b)$$

From this relation it appears that for a perfect gas the difference between c_p and c_v is constant even though each individually may be variable. It follows that the ratio between the two, k , is variable if each individually is variable.

Useful expressions for c_v and c_p in terms of R and k may also be developed from equation 4b. Thus, since $c_p = kc_v$, that equation may be rewritten as $kc_v - c_v = R/J$, whence

$$c_v = \frac{R/J}{k - 1}, \quad \text{and} \quad c_p = \frac{kR/J}{k - 1}. \quad (4c)$$

Since $dE = c_v dT$ and $dH = c_p dT$,

$dH = k dE$, or for constant specific heats,

$$H_2 - H_1 = k(E_2 - E_1). \quad (4d)$$

79. E and H as Functions of P , V and k . — Very frequently it will be of considerable convenience to have E and H directly in terms of P , V and k , although the practical application of the relations will be limited to the condition of constant specific heats and thus of constant k .

If $E_2 - E_1 = c_v(T_2 - T_1)$, then by Equations (4c) and (1)

$$E_2 - E_1 = \frac{1}{J(k - 1)} (P_2 V_2 - P_1 V_1), \quad \text{and} \quad (2a)$$

$$H_2 - H_1 = \frac{k}{J(k - 1)} (P_2 V_2 - P_1 V_1). \quad (3a)$$

80. Entropy of the Perfect Gas. — Because entropy is a property or state function the change of entropy in passing from any one state of a gas to a second state will depend only on the initial and final states and will be independent of the process whereby the state change is accomplished. Consequently we may be wholly unhampered (except as expediency is considered) in the selection of the process which we shall analyze for the purpose of expressing the entropy change for a given state change. Let us therefore presume any two states and a passage from the one to the other by any reversible process. To evaluate the corresponding total change of entropy it is convenient first to arrive at an expression for the small change of entropy accompanying any differential change of state and then to ascertain the total change of entropy by integration.

The energy equation for any such differential, reversible state change is (equation 8*a*, Art. 36),

$$J d'Q - P dV = J dE, \quad \text{or} \quad d'Q = dE + \frac{P dV}{J}.$$

But $d'Q = T dS$ (Art. 55), $dE = c_v dT$ (Art. 76) and $P = RT/V$ (Art. 74), whence

$$T dS = c_v dT + \frac{RT}{J} \frac{dV}{V}, \quad \text{or}$$

$$dS = c_v \frac{dT}{T} + \frac{R}{J} \frac{dV}{V}.$$

Integrating, the change of entropy in passing from state 1 to state 2 by any reversible process and also, since entropy is a state property, by any process whatsoever, is

$$S_2 - S_1 = \int_{T_1}^{T_2} c_v \frac{dT}{T} + \frac{R}{J} \log_e \frac{V_2}{V_1}, \quad (5)$$

$$= c_v \log_e \frac{T_2}{T_1} + \frac{R}{J} \log_e \frac{V_2}{V_1}, \quad \text{if } c_v \text{ were constant.} \quad (5a)$$

This relation evaluates the entropy change in terms of temperatures and specific volumes. To express it in terms of the initial and final temperatures and pressures write for $P dV$ its equivalent (Art. 74) $R dT - V dP$, whence

$$T dS = c_v dT + \frac{R}{J} dT - \frac{V dP}{J}.$$

But $(c_v + R/J) = c_p$ (Art. 78) and $V = RT/P$ (Art. 74), whence

$$T dS = c_p dT - \frac{RT}{J} \frac{dP}{P},$$

$$dS = c_p \frac{dT}{T} - \frac{R}{J} \frac{dP}{P} \quad \text{and}$$

$$S_2 - S_1 = \int_{T_1}^{T_2} c_p \frac{dT}{T} - \frac{R}{J} \log_e \frac{P_2}{P_1} \quad (6)$$

$$= c_p \log_e \frac{T_2}{T_1} - \frac{R}{J} \log_e \frac{P_2}{P_1}, \text{ if } c_p \text{ were constant.} \quad (6a)$$

Finally, to express the entropy change in terms of the initial and final pressures and specific volumes note that if $P dV + V dP = R dT$ (Art. 74), then

$$\frac{P dV}{RT} + \frac{V dP}{RT} = \frac{R dT}{RT}, \quad \text{or} \quad \frac{dV}{V} + \frac{dP}{P} = \frac{dT}{T}.$$

Substituting this value of dT/T in the expression directly before equation (5),

$$dS = c_v \frac{dV}{V} + c_v \frac{dP}{P} + \frac{R}{J} \frac{dV}{V} = c_p \frac{dV}{V} + c_v \frac{dP}{P} \quad \text{and}$$

$$S_2 - S_1 = \int_{V_1}^{V_2} c_p \frac{dV}{V} + \int_{P_1}^{P_2} c_v \frac{dP}{P} \quad (7)$$

$$= c_v \log_e \frac{P_2}{P_1} + c_p \log_e \frac{V_2}{V_1}, \text{ if } c_p \text{ and } c_v \text{ were constant.} \quad (7a)$$

Again note that these relations, equations 5 to 7a, express the entropy change for any character of gas state change whatsoever.

Example 7. Compute the change of E , H and S for 7 lb. of air in expanding from state 1 at $p_1 = 100$ lb. per sq. in. abs. and $V_1 = 2.44$ cu. ft. per lb. to state 2 in which $p_2 = 14.7$ and $t_2 = 0^\circ \text{ F}$. Take $R = 53.3$, $c_p = 0.241$ and $c_v = 0.173$. Do this by finding T_1 , V_2 and k and then computing each of the required property changes by all of the available, foregoing relations.

81. The Gas Constant, R . — It will have been noticed that no specific value of the gas constant, R , has been indicated; nor did the perfect gas characteristic equation put any limitation of its value, only requiring that it shall be constant and showing that its value might be found experimentally for any particular gas by ascertaining the specific volume at any selected pressure and temperature and then employing the relation $R = PV/T$.

As has been stated, the actual gases and also the very low pressure vapors approach, but only approach, the perfect gas criterion. One point of departure lies in the feature that even any particular gas does not exhibit a strictly constant value of R for different states. However, for many practical purposes it may be sufficiently accurate to neglect this variation and it is therefore customary to assign to the

various gases values of R which correspond to some standard state. In Table VII are presented, for the gases of more usual interest, the values of the gas constant as determined by physical measurement of the specific volume (V_0) at the standard atmospheric pressure (14.7 lb. per sq. in. abs.) and 32° F. There are also tabulated the molecular weights of the fluids (m , referred to oxygen as 32), the product of the molecular weight and the gas constant ($= mR$), and the product mV_0 .

TABLE VII
CHARACTERISTIC PROPERTIES OF GASES

Gas	V_0 , at 14.7 lb. and 32° F.	R	m	mR	mV_0
Sulfur dioxide (SO ₂).....	5.47	23.6	64.0	1512	350
Carbon dioxide (CO ₂).....	8.10	34.9	44.0	1536	356
Oxygen (O ₂).....	11.2	48.3	32.0	1546	358
Atmospheric air.....	12.4	53.3	(29.0)	(1545)	(358)
Nitrogen (N ₂) (atmos.) ³	12.74	54.9	(28.1)	(1545)	(358)
Nitrogen (N ₂) (chem.).....	12.8	55.1	28.0	1543	358
Carbon monoxide (CO).....	12.8	55.1	28.0	1545	358
Ammonia (NH ₃).....	20.8	89.5	17.0	1516	354
Helium (He).....	89.7	386	4.0	1544	359
Hydrogen (H ₂).....	178	767	2.0	1546	358.5

Representative value of $mR = 1545$; of $mV_0 = 358$.

It is significant that in this table the values of mR and of mV_0 are found to show relative uniformity, particularly in the case of the permanent gases of less than three atoms per molecule. For convenience it is customary to select as representative the values of 1545 and 358 for these respective quantities, designating them as the "universal gas constant" (mR) and the "standard mol volume" (mV_0).⁴ It follows from this approximate constancy of the product mV_0 that the specific volumes of the permanent gases at a given temperature and pressure are inversely proportional to their molecular weights, or that the densities are directly proportional.⁵

³ By "atmospheric nitrogen" is meant the residue of atmospheric air after abstraction of the oxygen but retaining the argon, CO₂ et cetera which exists in traces in the atmosphere.

⁴ Since a mass of m pounds of a fluid is commonly known as a "mol" the volume of m pounds is known as the "mol volume."

⁵ This relationship is one of the bases for Avogadro's "Law," which states that for all gases at a given pressure and temperature the number of molecules per unit volume is the same. If that were strictly the case the densities of various gases would necessarily be proportional to their molecular weights and their specific volumes would be inversely proportional. The above relative uniformity of mV_0 is an approximate verification of the hypothesis.

Example 8. From the universal gas constant compute the gas constant of water vapor and its specific volume at 1 lb. per sq. in. abs. and 200° F. Check the latter with the specific volume as found at that state in the Steam Tables. (m for water, H_2O , = 18.)

82. Specific Heat of Actual Gases. — In Arts. 76 and 77 it was indicated that for the perfect gas c_v and c_p must either be constants or functions only of the temperature. Also in Art. 78 it developed that whether they are constants or are variables their difference must be constant ($c_p - c_v = R/J$), again for the perfect gas.

In all of the real gases these specific heats are found actually to vary rather materially with the temperature and to a slight degree with the pressure,⁶ and also there is evidence that their difference is not exactly constant nor does it exactly equal R/J , where R is the gas constant as ascertained from specific volume data (Table VII). However, a degree of accuracy which is sufficient for most practical purposes as well as a great gain in convenience is secured if these specific heats are regarded as effectively constant when the temperature range is moderate and as a function only of the temperature for greater ranges and also if their differences may be regarded as virtually constant.

In Table VIII are presented, for certain gases of more usual engineering interest, expressions for the relations between the specific heat at constant volume (c_v) and the absolute temperature (T) and also corresponding values of the mean ($c_p - c_v$) ($= R/J$), and finally of the mean values of c_p and c_v and of the mean value of the ratio $c_p/c_v (= k)$ for the temperature range between 32° and 400° F. For subsequent work, unless materially greater temperature ranges are involved, these mean values may be employed as generally representative figures. In the table two values are presented for certain of the gases, those designated by the bracketed " a " being ones proposed by Partington and Shilling⁷ as the result of exhaustive analyses of all available data and those designated by the bracketed " b " being ones proposed by Goodenough and Felbeck⁸ as the result of like analyses. The two are observed to check closely except for H_2 and CO_2 . The differences in these at least reflect the difficulties in the way of accurate experimental determinations.

⁶ For air at 140° F. the specific heat has been shown to vary with the pressure according to the relation $c_p = 0.2414 - 0.042014 p - 0.092478 p^2 - 0.0123795 p^3$. For ranges less than 1500 lb. per sq. in. the relation $c_p = 0.2414 - 0.042 p$ is quite adequately accurate. See Ford, Compressor Theory and Practice.

⁷ Partington and Shilling, "The Specific Heat of Gases," Benn Ltd., London, 1924.

⁸ Goodenough and Felbeck, Univ. of Illinois Bulletin 139, 1923.

TABLE VIII
SPECIFIC HEATS OF GASES
(at standard atmospheric pressure)
[according to (a) Partington and Shilling,
(b) Goodenough and Felbeck]

Expressions for c_v at the absolute temperature T (deg. R)	Mean values, 32° to 400° F.			
	$c_p - c_v$ ($= R/J$)	c_v	c_p	k
Carbon dioxide (CO ₂) —				
(a) $0.1260 + 0.0_557 T - 0.0_872 T^2$	.0457	0.162	0.208	1.28
(b) $0.1037 + 0.0_8115 T - 0.0_7284 T^2$ + $0.0_{11}247 T^3$	.0451	0.171	0.216	1.26
Oxygen (O ₂) —				
(a) $0.1539 + 0.0_530 T + 0.0_830 T^2$	.0622	0.156	0.218	1.40
(b) $0.1545 + 0.0_8375 T^2$	.0621	0.156	0.218	1.40
Air —				
(a) $0.1699 + 0.0_533 T + 0.0_833 T^2$	.0687	0.173	0.241	1.40
Nitrogen, atmospheric				
(a) $0.1751 + 0.0_545 T + 0.0_834 T^2$	.0708	0.177	0.248	1.40
Nitrogen (chem.) & Carbon monoxide				
(a) $0.1760 + 0.0_534 T + 0.0_834 T^2$	.0711	0.178	0.249	1.40
(b) $0.1766 + 0.0_8428 T^2$	.0710	0.179	0.250	1.40
Hydrogen (H ₂) —				
(a) $2.314 + 0.0_5193 T$	.985	2.44	3.43	1.40
(b) $1.990 + 0.0_8331 T$	.986	2.21	3.20	1.44

(Note that, for example, $0.0_83 T$ is the symbol for $0.000003 T$ or $0.3 T \times 10^{-6}$.)

All of the relations are observed to be of the form

$$c = a + bT + fT^2 + \dots \quad (\text{see Art. 24}), \quad (8)$$

where a , b and f are numerical coefficients which are established from the experimental evidence. In this expression the coefficient a for c_v is that appearing in Table VIII whereas for c_p it is the tabulated coefficient plus R/J ; that is, $c_p - c_v = a_p - a_v = R/J$ and $a_p = a_v + R/J$. The mean value of the specific heat across any temperature range from T_1 to T_2 becomes

$$\begin{aligned}
 c_{\text{mean}} &= \frac{Q}{T_2 - T_1} = \frac{\int_1^2 c \, dT}{T_2 - T_1} = \frac{1}{T_2 - T_1} \\
 &\times \left[a(T_2 - T_1) + \frac{b}{2}(T_2^2 - T_1^2) + \frac{f}{3}(T_2^3 - T_1^3) + \dots \right] \\
 &= a + b \frac{T_2 + T_1}{2} + f \frac{T_2^2 + T_2 T_1 + T_1^2}{3} + \dots
 \end{aligned}$$

Example 9. Compute the instantaneous values of c_p and of c_p for air at (a) 70° F. , (b) at 2000° F. , and (c) the mean values for the range between 70° and 2000° F.

83. Perfect Gas State Changes. — In the succeeding articles several particular state changes for a perfect gas will be considered, with the understanding that the state changes of the actual "permanent" gases such as air, O_2 , N_2 , and H_2 may with adequate accuracy be analyzed on the perfect gas basis. In the analyses it will again be taken (as in the analyses of vapor processes, Art. 65) (a) that the initial state is adequately designated by the specification of at least two properties, (b) that the process is characterized either by the constancy of some one property or by some definite relationship existing between the properties during the process, and (c) that sufficient additional data are known for determination of the final state.

The typical processes may be either non-flow or steady-flow, for which the energy equations will be recalled to be (Arts. 31 and 34)

$$\text{Non-flow; } \frac{W}{J} + Q = E_2 - E_1.$$

$$\begin{aligned} \text{Steady-flow; } \frac{1}{J}[W + P_1V_1 - P_2V_2 + (U_1^2 - U_2^2)/64.34] \\ + Q = E_2 - E_1. \end{aligned}$$

In ideal circumstances these processes would be mechanically reversible, in which circumstances the above energy equations may be written in the unified form of Art. 36;

$$Q - \frac{1}{J} \int_1^2 P dV = E_2 - E_1;$$

where $-\int_1^2 P dV$ measures the Mechanical Effects. In Art. 36 it was noted that for a non-flow process the sole Mechanical Effect is the shaft-work (W) transferred to or from the system via a shaft or piston rod, whereas for a steady-flow process the Mechanical Effects are the summation of such shaft-work, the difference between the entry and the exit flow-work and any change of kinetic energy. Also recall that if any mechanical irreversibility exists the Mechanical Effects are no longer measured by $-\int_1^2 P dV$.

For the analysis of the several characteristic state changes these energy relations will be written in such specialized forms as fit the particular conditions. It may be remarked that the relations which are developed will usually be ones in terms of the change of the properties E or H , this by reason of the ease of the development and utilization of

equations so expressed. By the use of equations 2, 2a, 3, or 3a, the change of E or H may always be computed from data on the specific heats (c_v or c_p) and on the initial and final values of T , P or V . Of these three properties the temperature and pressure, being the properties for the direct measurement of which engineering instruments are available, may more frequently enter into practical computations.

84. Constant Volume State Change. — Since, by specification, $V_2 = V_1$ (or $dV = 0$), it follows from equation 1 that

$$\frac{P_2}{T_2} = \frac{P_1}{T_1} = \frac{P}{T} = \frac{R}{V} = \text{a constant.} \quad (9)$$

This relation thus provides for the computation of P_2 or T_2 if either is known.

The change of E , H and S for the constant-volume process becomes

$$\Delta E_V = \int_1^2 c_v dT \text{ (Eq. 2), or } = \frac{1}{J(k-1)} (P_2 - P_1) V \text{ (Eq. 2a).}$$

$$\Delta H_V = \int_1^2 c_p dT \text{ (Eq. 3), or } = \frac{k}{J(k-1)} (P_2 - P_1) V \text{ (Eq. 3a).}$$

$$\Delta S_V = \int_1^2 c_v \frac{dT}{T} \text{ (Eq. 5), or } = c_v \log_e \frac{P_2}{P_1} \text{ (Eq. 7a).}$$

The only constant-volume processes of usual interest are non-flow ones which also are mechanically reversible, such as exemplified by the warming or cooling of a gas in a closed container, for which condition,

$$W_V = \text{Mechanical Effects} = - \int_1^2 P dV = \text{zero, since } dV = 0;$$

$$Q_V = E_2 - E_1.$$

Example 10. Given a pound of air initially at 14.7 lb. per sq. in. abs. and 62° F.

(a) What is the specific volume at that state? (b) Compute the change in the properties per pound for a constant-volume temperature rise to 400° F. and the energy required to accomplish the process.

$$(V = 13.17; p_2 = 24.2; \Delta E = 58.5; \Delta H = 81.9; \Delta S = 0.086; Q = + 58.5.)$$

85. Constant Pressure State Change. — With $P_2 = P_1$ (or $dP = 0$), by equation 1,

$$\frac{V_2}{T_2} = \frac{V_1}{T_1} = \frac{V}{T} = \frac{R}{P} = \text{a constant,} \quad (10)$$

whereby V_2 or T_2 may be computed if either is known.

The change of E , H and S for a constant-pressure process becomes

$$\Delta E_P = \int_1^2 c_v dT \text{ (Eq. 2), or } = \frac{1}{J(k-1)} (V_2 - V_1) P \text{ (Eq. 2a).}$$

$$\Delta H_P = \int_1^2 c_p dT \text{ (Eq. 3), or } = \frac{k}{J(k-1)} (V_2 - V_1)P \text{ (Eq. 3a).}$$

$$\Delta S_P = \int_1^2 c_p \frac{dT}{T} \text{ (Eq. 6), or } = c_p \log_e \frac{V_2}{V_1} \text{ (Eq. 7a).}$$

The constant pressure state changes of usual concern are exemplified by a non-flow process in a cylinder with piston or a steady-flow process occurring during the passage of a gas through a heat-interchanger such as an air heater or cooler. For the further and common condition of negligible mechanical irreversibility (at least as regards internal irreversibility, Art. 20)

$$\text{Mechanical Effects} = - \int_1^2 P dV = P(V_1 - V_2);$$

$$Q_P = E_2 - E_1 + \frac{1}{J} \int_1^2 P dV = E_2 - E_1 + \frac{1}{J} (PV_2 - PV_1) = H_2 - H_1.$$

Example 11. Given a pound of air initially at 14.7 lb. per sq. in. abs. and 62° F. ($V_1 = 13.17$, see Ex. 10).

(a) For a constant-pressure temperature rise to 400° F. find the final specific volume and the change of the remaining properties. (b) Compute the heat energy required to accomplish the process and the amount and direction of the energy transition as work accompanying the process if non-flow and reversible.

($V_2 = 21.7$; $\Delta E = 58.5$; $\Delta H = 81.9$; $\Delta S = 0.120$; $Q = + 81.9$; $W = - 18,050$.)

86. Isothermal State Change. — With $T_2 = T_1$, by equation 1,

$$P_2 V_2 = P_1 V_1 = PV = RT = \text{a constant} \quad (11)$$

Also $\Delta E_T = \text{zero}$ (Eq. 2); $\Delta H_T = \text{zero}$ (Eq. 3)

$$\Delta S_T = \frac{R}{J} \log_e \frac{V_2}{V_1} \text{ (Eq. 5), or } = \frac{R}{J} \log_e \frac{P_1}{P_2} \text{ (Eq. 6).}$$

The typical isothermal state change may be a non-flow one in a cylinder with piston and effected by compression accompanied by heat emission (or by expansion accompanied by heat energy entry — see Arts. 35*d* and 42) or it may be an ideal steady-flow one through an operating compressor. For the further ideal circumstance of negligible mechanical irreversibility,

$$\begin{aligned} \text{Mechanical Effects} &= - \int_1^2 P dV = - RT \int_1^2 \frac{dV}{V} \text{ (by Eq. 1)} \\ &= - RT \log_e \frac{V_2}{V_1}, \text{ or } = - RT \log_e \frac{P_1}{P_2} \end{aligned} \quad (12)$$

$$\begin{aligned} Q_T &= + \frac{1}{J} \int_1^2 P dV = \frac{RT}{J} \log_e \frac{V_2}{V_1}, \text{ or } = \frac{RT}{J} \log_e \frac{P_1}{P_2} \\ \text{or also} \quad &= T(S_2 - S_1). \end{aligned} \quad (12a)$$

Example 12. Given a pound of air initially at 14.7 lb. per sq. in. abs. and 62° F. ($V_1 = 13.17$, see Ex. 10).

(a) For an isothermal compression to a pressure of 100 lb. per sq. in. gage find the final specific volume and the change of the remaining properties. (b) Compute the shaft-work required to accomplish the compression if reversible and non-flow or reversible and steady-flow but without sensible difference between the entering and exit kinetic energy, also the requisite heat energy transition.

$$(V_2 = 1.69; \Delta S = -0.141; W = +57,200; Q = -73.5.)$$

87. Reversible Adiabatic State Change. — For this process the basic characteristics are the absence of heat transfer and the constancy of the entropy ($Q = 0$ and $S_2 = S_1$). The property relations may be developed upon this background by the use of equations 5, 6 or 7 if variation of specific heat with temperature needs to be accounted or by equations 5a, 6a or 7a if such variation may be neglected (if c_p , c_v and k may be taken as effectively constant). The latter condition is considered first.

From equation 7a.

$$\begin{aligned} S_2 - S_1 &= c_v \log_e (P_2/P_1) + c_p \log_e (V_2/V_1) \\ &= c_v [\log_e (P_2/P_1) + k \log_e (V_2/V_1)] \\ &= c_v \log_e [(P_2/P_1)(V_2/V_1)^k] = \text{zero.} \end{aligned}$$

Since c_v may not equal zero, $\log_e \left[\left(\frac{P_2}{P_1} \right) \left(\frac{V_2}{V_1} \right)^k \right] = \text{zero.}$

Therefore, taking the antilogarithm,

$$\frac{P_2 V_2^k}{P_1 V_1^k} = 1, \quad \text{or} \quad P_2 V_2^k = P_1 V_1^k = P V^k = \text{a constant.} \quad (13)$$

This relation for the reversible adiabatic is probably the one most frequently encountered. However, by a parallel analysis of equation 6a, or from equations 13 and 1, a very useful P - T relation for the reversible adiabatic may be developed. Thus, by equation 1,

$$P_2 V_2^k = P_2 \frac{R^k T_2^k}{P_2^k} = R^k \frac{T_2^k}{P_2^{(k-1)}}.$$

Similarly,

$$P_1 V_1^k = R^k \frac{T_1^k}{P_1^{(k-1)}},$$

whence, from equation 13,

$$\frac{T_2^k}{P_2^{(k-1)}} = \frac{T_1^k}{P_1^{(k-1)}}, \quad \text{or} \quad \frac{T_2}{P_2^{(k-1)/k}} = \frac{T_1}{P_1^{(k-1)/k}} = \frac{T}{P^{(k-1)/k}} = \text{a constant}^9 \quad (14)$$

⁹ In Physics literature this T - P relation for the reversible adiabatic with a gas is frequently referred to as *Poisson's Equation*.

Again it may be developed from equation 5a, or from equations 13 and 1, that

$$T_2 V_2^{k-1} = T_1 V_1^{k-1} = T V^{k-1} = \text{a constant.} \quad (15)$$

Example 13. Develop equation 14 from equation 6a; develop equation 15 either from equation 5a or from equations 13 and 1.

For temperature ranges such as necessitate consideration of the variation of the specific heats, from equations 5 and 8,

$$S_2 - S_1 = \int_1^2 c_v \frac{dT}{T} + \frac{R}{J} \log_e \frac{V_2}{V_1} = \text{zero, whence} \\ a_v \log_e \frac{T_2}{T_1} + b(T_2 - T_1) + \frac{f}{2}(T_2^2 - T_1^2) = \frac{R}{J} \log_e \frac{V_2}{V_1}. \quad (15a)$$

Similarly from equations 6 and 8 (et sequi),

$$S_2 - S_1 = \int_1^2 c_p \frac{dT}{T} - \frac{R}{J} \log_e \frac{P_2}{P_1} = \text{zero, whence} \\ \left(a_v + \frac{R}{J}\right) \log_e \frac{T_2}{T_1} + b(T_2 - T_1) + \frac{f}{2}(T_2^2 - T_1^2) = \frac{R}{J} \log_e \frac{P_2}{P_1} \quad (14a)$$

The latter expressions obviously are not easily amenable to algebraic solution¹⁰ but they may be solved by trial or graphically without excessive effort. Example 14 suggests the method of procedure in such solution.

Example 14. Given a pound of air initially at 14.7 lb. per sq. in. abs. and 62° F. ($V_1 = 13.17$, by example 10).

(a) For a reversible adiabatic compression to a pressure of 100 lb. per sq. in. gage compute the final temperature and specific volume, presuming that the variation of the specific heat may be neglected (that is, employing the mean value of k of Table VIII). (b) Compute the final temperature and specific volume, giving attention to the variation of the specific heat.

(Hint. Compute initially the value of $R/J \log_e (P_2/P_1)$, which is likewise the required value of the left side of equation 14a; then for values of T_2 at 5 deg. and at 10 deg. below the T_2 computed in (a) evaluate the left side of equation 14a. Finally the correct value of T_2 may be closely estimated by plotting these computed functions of T_2 as ordinates vs. T_2 as abscissae and picking from the curve the value of T_2 which will give the required value for the function. Compute V_2 by equation 1.)

(a; $t_2 = 479^\circ \text{F.}$, $V_2 = 3.03$. b; $t_2 = 473$.)

The change of E and H accompanying a reversible adiabatic may in general be computed to best advantage by the use of the general relations

$$\Delta E = \int_1^2 c_v dT \quad (\text{Eq. 2}) \quad \text{and} \quad \Delta H = \int_1^2 c_p dT \quad (\text{Eq. 3}).$$

¹⁰ They may be solved with close approximation. See E. C. Wadlow, *Phil. Mag.*, 1927, pages 917-922; also W. J. Walker, *Phil. Mag.*, 1922, pages 589-593.

However, the specifications of this state change are so frequently given in terms of the original state and the final pressure (P_2) that there may be some advantage in the evaluation of ΔE and ΔH in terms of such data, although such an evaluation must be limited to the presumption of constant specific heats. To so evaluate them, by equations 2 or 3, 4c and 14,

$$\Delta E_S = c_v(T_2 - T_1)_S = \frac{R}{J(k-1)} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right]; \quad (16)$$

$$\Delta H_S = c_p(T_2 - T_1)_S = \frac{k}{k-1} \frac{R}{J} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right]. \quad (16a)$$

The reversible adiabatic processes which are of common concern may be either non-flow, as in the cylinder of an ideal engine, or steady-flow, as in a nozzle. In either type, by definition

$$Q_S = \text{zero.}$$

It follows that, for a non-flow process,

$$\frac{W_S}{J} = E_2 - E_1;$$

and for a steady-flow process,

$$\begin{aligned} \frac{W}{J} + \frac{1}{J} (U_1^2 - U_2^2)/64.34 &= (E_2 + P_2 V_2/J) - (E_1 + P_1 V_1/J) \\ &= H_2 - H_1. \end{aligned}$$

Example 15. Compute ΔE and ΔH for the process of example 14, (a) presuming that the variation of the specific heats may be neglected, (b) accounting for the variation of the specific heats.

$$(\Delta E = (a) 71.7, (b) 71.4.)$$

$$(\Delta H = (a) 100.5, (b) 99.4.)$$

Example 16. Compute the work ideally required for the adiabatic *compression* of 1000 cu. ft. of air at an initial pressure and temperature of 14.7 lb. abs. and 62° F. to a pressure of 100 lb. gage, the compression taking place in a cylinder with piston; compute the work ideally required for the *induction, compression and delivery* of 1000 cu. ft. from and to the above conditions, the compressor being a steadily operating compressor and the difference between the kinetic energies at entrance and exit being taken as negligible.

(Neglecting specific heat variation: 4,235,000 ft.-lb.; 5,935,000 ft.-lb.)

88. Irreversible Adiabatic State Change.—In many actual flow processes, such as those occurring during the expansion of a gas through a nozzle or the compression through a centrifugal compressor, there is a negligible energy transfer as heat and thus the processes are virtually adiabatic but, by reason of the fluid friction and turbulence accompanying the high velocities, they are internally irreversible. It is found in practice that frequently such processes are characterized (at least

approximately) by exponential relationships between the P , T and V properties which are similar to those of the reversible adiabatic (see equations 13, 14 and 15) except that the exponents are no longer functions of k but ones of some factor n . Thus the following analogous relations are found empirically to fit the actual processes:

$$PV^n = \text{a constant} \quad (17)$$

$$\frac{T}{P^{(n-1)/n}} = \text{a constant} \quad (18)$$

$$TV^{n-1} = \text{a constant} \quad (19)$$

Quite as the k in the reversible relations need not be, nor is a constant but for many purposes may be taken to be effectively such, so there is no theoretical reason that the n in the irreversible relations should be constant but for convenience it is often determined or used as if effectively so.

The changes of E and H for an irreversible adiabatic of the above character may readily be formulated (at least for the condition of effectively constant c_v , c_p and k) by the usual methods. Thus, by equations 2 or 3, 4c and 18,

$$\Delta E_n = c_v(T_2 - T_1) = \frac{R}{J(k-1)} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]; \quad (20)$$

$$\Delta H_n = c_p(T_2 - T_1) = \frac{k}{k-1} \frac{R}{J} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]. \quad (20a)$$

The entropy change during an irreversible adiabatic is necessarily an entropy *increase* (see Art. 55). To evaluate the entropy change the general relations of equations 5a, 6a or 7a may probably be used to best advantage. However again, because of the frequent specification of the state change in terms of the original state and the final pressure, there may be some advantage in having an expression for the entropy change in terms of such data. To so evaluate it, from equations 6a, 18 and 4c:

$$\begin{aligned} \Delta S_n &= c_p \log_e (T_2/T_1) - \frac{R}{J} \log_e (P_2/P_1) \\ &= c_p \log_e (P_2/P_1)^{(n-1)/n} - c_p \frac{k-1}{k} \log_e (P_2/P_1) \\ &= c_p \left(\frac{n-1}{n} - \frac{k-1}{k} \right) \log_e (P_2/P_1) \\ &= c_p \frac{n-k}{nk} \log_e (P_2/P_1) = c_v \frac{n-k}{n} \log_e (P_2/P_1). \end{aligned}$$

In this connection it is interesting to note the requisite magnitude of n relative to that of k , that is, whether n may exceed or shall be less than k . This consideration becomes immediately evident from inspection of the last expression above:

$$\Delta S_n = c_v \frac{n - k}{n} \log_e (P_2/P_1).$$

Since the change of entropy must be an increase and the expression is therefore positive, and since c_v and n must likewise be positive,

$$\begin{aligned} (n - k) \text{ is positive, } & \text{ or } n > k, \text{ if } P_2 > P_1, \text{ as in compression;} \\ (n - k) \text{ is negative, } & \text{ or } n < k, \text{ if } P_2 < P_1, \text{ as in expansion.} \end{aligned}$$

For a compressor delivering air a value of n in excess of 1.75 to 1.85 would represent quite poor performance; for the expansion of air through a device such as a nozzle a value of n less than about 1.3 would likewise represent quite poor performance, although for a pure throttling process (see Art. 89) it would have a value of unity (1.0).

As the usual irreversible adiabatic processes are steady-flow in character only the steady-flow energy equation will be written, or

$$\begin{aligned} \frac{W}{J} + \frac{1}{J} (U_1^2 - U_2^2)/64.34 &= (E_2 + P_2 V_2/J) - (E_1 + P_1 V_1/J)^{11} \\ &= H_2 - H_1. \end{aligned}$$

where $H_2 - H_1$ may be evaluated by the foregoing relations either from data on the initial and final temperatures or on the initial state, the final pressure and n .

¹¹ It is of interest in this connection to recall and to validate the statement of Art. 36 that $-\int_1^2 P dV$ does *not* measure the Mechanical Effects for a mechanically irreversible process. Thus, from the energy equation, for an adiabatic

$$\text{Mechanical Effects} = J(E_2 - E_1)$$

$$= \frac{R}{k-1} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right] \text{ if irreversible;}$$

but for an irreversible adiabatic in which $PV^n = \text{a constant}$ ($= P_1 V_1^n = P_2 V_2^n$),

$$\begin{aligned} -\int_1^2 P dV &= -\int_1^2 \frac{P_1 V_1^n}{V^n} dV = -\frac{P_1 V_1^n}{1-n} \left[V_2^{1-n} - V_1^{1-n} \right] \\ &= \frac{P_1 V_1}{n-1} \left[\left(\frac{V_1}{V_2} \right)^{n-1} - 1 \right] = \frac{R}{n-1} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]. \end{aligned}$$

It thus appears that

$$\frac{-\int_1^2 P dV}{\text{Mech. Effects}} = \frac{k-1}{n-1},$$

and that this ratio is greater or less than unity respectively as k exceeds n , as in expansion, and as n exceeds k , as in compression.

Example 17. Find the change in the properties for an irreversible adiabatic expansion of air from a tank at 100 lb. per sq. in. gage and 300° F. through a nozzle to a discharge pressure of 14.7 lb. per sq. in. abs., the expansion to be representable by the relation $PV^{1.3} = \text{a constant}$. Presuming a negligible entering velocity, compute the exit kinetic energy and velocity and compare with the kinetic energy and velocity ideally attainable with reversible expansion.

$$(t_2 = 13^\circ \text{ F.}; \Delta E = -49.4; \Delta H = -69.2; \Delta S = +0.027; U_2^2/64.34 = 53,800, \text{ vs. } 63,200; U_2 = 1860, \text{ vs. } 2010.)$$

Example 18. In the test of a non-jacketed centrifugal air compressor data were obtained as follows:

Intake

Pressure, atmospheric; temperature, 70° F.; velocity, negligible.

Discharge

Pressure, 5 lb. gage; temperature, 135° F.; velocity, 100 ft. per sec.

Rate of flow

5000 cu. ft. per min. at the intake pressure and temperature.

Assuming that although the air compression was itself adiabatic 5 per cent of the shaft-work was radiated from the bearings and casing, what power would be required to drive the compressor? (*Hint.* Write the energy equation, employing for Q the value $0.05 W/J$.) What value of n would correspond to these data?

$$(W = 13,000 \text{ ft.-lb. per lb.}; \text{Hp.} = 147; n = 1.65.)$$

89. Throttling Processes. — Recalling (Art. 70) that by a throttling process is meant an adiabatic but thoroughly irreversible flow and pressure drop through an obstruction in a line, with no energy delivery as shaft-work and so arranged that the change in kinetic energy is negligible, it developed that for such a process.

$$H_2 = H_1. \quad (21)$$

For such circumstances, with a perfect gas, by equations 3, 2 and 1

$$T_2 = T_1^{12}; E_2 = E_1; P_2 V_2 = P_1 V_1.$$

Also, by equation 6,

$$\Delta S = \frac{R}{J} \log_e (P_1/P_2).$$

It thus appears that in a throttling process with a perfect gas the gas would depart from the region at its initial temperature but we recognize that the process is not in any sense a reversible isothermal. Instead,

¹² In foregoing articles there have been noted various particular features in which the real gases depart from the Perfect Gas. Probably such departure may be indicated qualitatively and determined quantitatively to best advantage by measurements of the temperature change accompanying throttling processes with the actual gases, or in particular by determination of the ratio dT/dP . This ratio is known as the Joule-Thomson coefficient. Its significance and valuable utility is considered further in Part V, Art. 199 et sequi.

in respect to the energy relations,

Mechanical Effects = zero,

(although $-\int_1^2 P dV$ does not equal zero);

$Q = \text{zero},$

(although $\int_1^2 T dS$ does not equal zero).

90. Polytropic State Changes. — In Art. 88 there were noted certain characteristics of the flow processes which occur in *centrifugal machines* and which are virtually adiabatic but quite irreversible by reason of fluid friction and turbulence. Rather opposite characteristics are encountered in the actual expansions and compressions occurring within the cylinders of *reciprocating engines or compressors*. These usually are virtually reversible in so far as concerns fluid friction and turbulence and they are not adiabatic, being invariably accompanied by more or less heat interchange between the gas and the cylinder walls during the process. Interestingly enough, however, it is found in practice that these two distinctly contraventional sorts of processes are parallel in that they both may frequently be characterized (at least approximately) by the same general type of exponential relationships between the P , T and V properties, or (as in Art. 88):

$$PV^n = \text{a constant} \quad (17)$$

$$\frac{T}{P^{(n-1)/n}} = \text{a constant} \quad (18)$$

$$TV^{n-1} = \text{a constant} \quad (19)$$

Probably the outstanding evidence toward validating such a characterization of the processes occurring in reciprocating machines lies in the evidence of the **indicator diagram**. It is recognized that essentially such a diagram portrays the manner of change of the pressure within the cylinder during the change of volume which accompanies the movement of the piston. Referring to the typical indicator diagram of an air compressor in Fig. 28, if on such a diagram one draws a zero pressure line at a scalar distance below the atmospheric line which equals the atmospheric pressure, and if beyond the high pressure end of the card

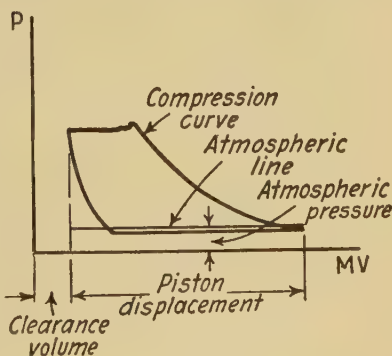


FIG. 28

and at a scalar distance which equals the clearance volume one draws a zero volume line, then scalar distances from these lines to any point on the compression or expansion curves represent the simultaneous values of the absolute pressure (P) and the total volume of the mass of gas which is contained in the cylinder (MV). By investigations of actual indicator diagrams as so prepared it is established that the state changes taking place during compression or expansion are representable with more or less exactness by the " $PV^n = \text{a constant}$ " relationship.¹³ The additional property relations of equations 18 and 19 must follow from the general characteristic equation (equation 1).

An internally reversible but non-adiabatic state change of this character is known as a **polytropic** state change. For such a process the change of the properties E , H and S may be readily formulated (at least for the condition of effectively constant c_v , c_p and k) by the usual methods. Thus, by equations 2 or 3, 4c, and 18,

$$\Delta E_n = c_v(T_2 - T_1) = \frac{R}{J(k-1)} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]; \quad (20)$$

$$\Delta H_n = c_p(T_2 - T_1) = \frac{k}{k-1} \frac{R}{J} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]. \quad (20a)$$

Also, by equations 6a, 18, and 4c (see Art. 88 for detailed steps),

$$\begin{aligned} \Delta S_n &= c_p \log_e (T_2/T_1) - \frac{R}{J} \log_e (P_2/P_1) \\ &= c_v \frac{n-k}{n} \log_e (P_2/P_1), \end{aligned}$$

or further, by introducing equation 18 in this last relation,

$$\Delta S_n = \left(c_v \frac{n-k}{n} \right) \left[\frac{n}{n-1} \log_e (T_2/T_1) \right] = c_v \frac{n-k}{n-1} \log_e (T_2/T_1).$$

¹³ Note that if $P_2 V_2^n = P_1 V_1^n$, or $P_2/P_1 = (V_1/V_2)^n$, then $\log P_2 - \log P_1 = n (\log V_1 - \log V_2)$, or

$$n = \frac{\log P_2 - \log P_1}{\log V_1 - \log V_2}.$$

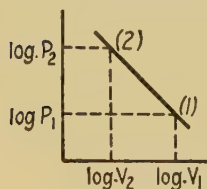


FIG. 29

Referring to Fig. 29, in which simultaneous values of $\log P_1$ and $\log V_1$ and also of $\log P_2$ and $\log V_2$ are plotted, it is evident that n is the slope of the straight line joining the plotted points. It follows that if one plots to logarithmic scales the values of the absolute pressure and the total volume as taken from an indicator diagram and if the plotted points fall in a straight line, then one may

properly conclude that the equation of the original curve is an exponential one, the value of the exponent being equal to the slope of the line as it appears on the logarithmic coördinates.

By comparison of this final expression for the entropy change in terms of the temperature with the general defining relation for the entropy change in any reversible processes (Art. 53, equation 3b):

$$\Delta S = \int_1^2 d'Q/T = \int_1^2 c dT/T = c \log_e (T_2/T_1),$$

it becomes evident that the reversible polytropic is in effect a state change with a *constant* specific heat (c_n) such that

$$c_n = c_v \frac{n - k^{14}}{n - 1}.$$

The energy relations for the reversible polytropic may be developed to best advantage by first evaluating the Mechanical Effects through the introduction of the P - V relation of equation 17 into the general expression $-\int_1^2 P dV$. Thus, if $PV^n = \text{a constant}$ ($= P_1 V_1^n = P_2 V_2^n$),

$$\begin{aligned} -\int_1^2 P dV &= -\int_1^2 \frac{P_1 V_1^n}{V^n} dV = -\frac{P_1 V_1^n}{1-n} (V_2^{1-n} - V_1^{1-n}) \\ &= \frac{P_1 V_1}{n-1} \left[\left(\frac{V_1}{V_2} \right)^{n-1} - 1 \right] \end{aligned}$$

$$\text{or} \quad = \frac{R}{n-1} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right], \text{ (by Eqs. 1 and 17)}$$

$$\text{or} \quad = \frac{R}{n-1} (T_2 - T_1), \text{ (by Eq. 18)}^{15}. \quad (22)$$

These several relations express directly the magnitude of the shaft-work (the sole mechanical effect) in a non-flow polytropic process.

For a steady-flow polytropic

$$\begin{aligned} W + (U_1^2 - U_2^2)/64.34 + P_1 V_1 - P_2 V_2 \\ = -\int_1^2 P dV = \frac{R}{n-1} (T_2 - T_1), \text{ whence} \end{aligned}$$

$$W = \frac{R}{n-1} (T_2 - T_1) + R(T_2 - T_1) + (U_2^2 - U_1^2)/64.34$$

¹⁴ In this connection it is interesting to note that the reversible constant volume, constant pressure, isothermal, and adiabatic state changes may all be regarded as special forms of the polytropic. Thus, it may be shown that

- (a) for constant volume, $c_n = c_v$ and $n = \infty$,
- (b) for constant pressure, $c_n = c_p$ and $n = 0$,
- (c) for isothermal, $c_n = \infty$ (positive or negative) and $n = 1$,
- (d) for reversible adiabatic, $c_n = 0$ and $n = k$.

¹⁵ See also footnote, Art. 88.

$$= \frac{n}{n-1} R(T_2 - T_1) + (U_2^2 - U_1^2)/64.34$$

$$\text{or} \quad = \frac{n}{n-1} RT_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right] + (U_2^2 - U_1^2)/64.34, \text{ (by Eq. 18).}$$

To evaluate the energy transition as heat which accompanies a polytropic, by the general energy equation and the above,

$$\begin{aligned} Q &= E_2 - E_1 + \frac{1}{J} \int_1^2 P dV \\ &= \frac{R}{J(k-1)} (T_2 - T_1) - \frac{R}{J(n-1)} (T_2 - T_1) \\ &= \frac{R}{J} \frac{n-k}{(k-1)(n-1)} (T_2 - T_1) \\ \text{or} \quad &= \frac{R}{J} \frac{n-k}{(k-1)(n-1)} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right], \text{ (by Eq. 18).} \end{aligned} \quad (23)$$

It is of interest to analyze this last relation with the view of ascertaining the requisite magnitude of n with reference to k . Thus,

- (a) for expansion with heat energy emission, as in a water-jacketed internal-combustion engine, since $P_2/P_1 < 1.0$ and Q is negative, then $(n-k)$ must be positive and $n > k$;
- (b) for compression with heat energy emission, as in a water-jacketed compressor, since $P_2/P_1 > 1.0$ and Q is negative, then $(n-k)$ must be negative and $n < k$ (but not less than unity).

Comparison with the corresponding conclusions with regard to the requisite relative magnitude of n in the irreversible adiabatic (Art. 88) shows that in this feature the two processes are again contraventional.

Example 19. Given a pound of air initially at 14.7 lb. per sq. in. abs. and 62° F.
($V_1 = 13.17$, by example 10.)

(a) For a reversible polytropic compression to a pressure of 100 lb. per sq. in. gage with n equal 1.2 compute the final temperature and specific volume and the change of E , H and S .

$$(t_2 = 277^\circ \text{ F.}; V_2 = 2.38; \Delta E = +37.0; \Delta H = +51.9; \Delta S = -0.059.)$$

(b) Compute the work required simply for the compression, the heat energy emission during the compression and the work required for the induction, compression and delivery of the air.

$$(+57,300 \text{ ft-lb.}; -36.7 \text{ B.t.u.}; +68,800 \text{ ft-lb.})$$

Example 20. Data secured by measurements of the indicator card from a water-jacketed air compressor show the following:

	Abs. pressure, lb. per sq. in.	Total cylinder volume, cu. ft.
Beginning of compression.....	14.0	1.05
Middle of compression.....	27.2	0.62
End of compression.....	111.3	0.20

- (a) Find if the P - V curve is a polytropic and, if so, the exponent of the exponential equation which represents the curve. ($n = 1.25$.)
- (b) Find the effective specific heat for this compression process. ($- 0.103$.)

91. Graphical Representation of Gas State Changes. — As in the case of the vapors the property relations and state changes of gases may with advantage be portrayed graphically by curves drawn to coördinate axes of selected properties. Probably the properties which are so employed with most frequency are the absolute pressure P as ordinate and specific volume V as abscissa. To these coördinates may be drawn contour lines of constant volume, constant pressure, or constant entropy (reversible adiabatic) state changes as well as any other sort of state changes. The typical form of curves representing the state changes considered in Arts. 84-90, as they would appear on P - V coördinates, are shown in Fig. 30. One evident feature of the family of curves is their increasing “steepness” with increase in the magnitude of n .

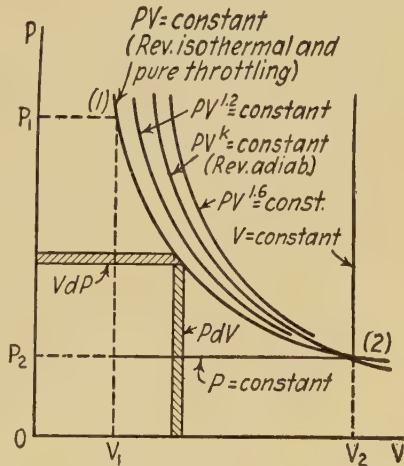


FIG. 30. P - V curves for a gas

Aside from the general utility of the P - V curves they offer the usual particular utility that $-\int_1^2 P dV$ is represented by the area “below” the state change curve and that this area is thus a graphic measure of the Mechanical Effects in a mechanically reversible process, but *only* for a reversible one. In this connection note similarly that the $\int_1^2 V dP$ is the area “back of” the state change curve and that, referring to the figure,

$$P_1V_1 + \int_1^2 P dV - P_2V_2 = - \int_1^2 V dP.$$

However, for a reversible steady-flow process (Art. 36),

$$W + (U_1^2 - U_2^2)/64.34 + P_1V_1 - P_2V_2 = - \int_1^2 P dV,$$

whence,

$$W + (U_1^2 - U_2^2)/64.34 = - P_1V_1 - \int_1^2 P dV + P_2V_2 = \int_1^2 V dP.$$

This is to say that for a steady-flow reversible process the summation of the shaft-work and the change of kinetic energy, or either individually if the other is negligible or not germane, is measured graphically by the area "back of" the state change curve.

Curves of T - S and H - S , such as are so generally employed in connection with vapor properties and processes (Art. 64), are not commonly provided for the gases, preference usually being accorded in process analyses to mathematical computations based on the foregoing formulations. Nevertheless for some purposes the curves may be used to

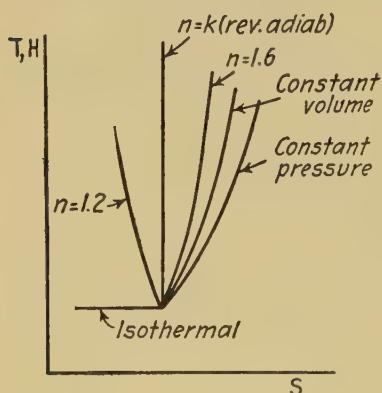


FIG. 31. T - S or H - S curves for a gas

advantage, particularly with T and H plotted jointly against S , each of the former being dependent solely on the other. The appearance of the typical processes when depicted on T , H - S coordinates is shown in Fig. 31.

A scalar T , H - S diagram for air, N_2 , O_2 and CO appears in Fig. 32. For such joint plotting of the properties of these four diatomic gases advantage is taken of the unique phenomenon that for all the specific heat

per mol-mass (per m pounds, where m is the molecular weight) is the same and therefore that their enthalpy and entropy per mol mass are identical.¹⁶ The joint relation for the specific heat per mol-mass, or the "molar" specific heat of these gases is

Partington and Shilling,

$$mc_v = 4.924 + 0.0495 T + 0.0796 T^2; \quad m(c_p - c_v) = 1.99.$$

Goodenough and Felbeck,

$$mc_v = 4.945 + 0.0612 T^2; \quad m(c_p - c_v) = 1.98.$$

¹⁶ This may be verified by reference to the expressions of Table VIII, multiplying the various coefficients in the expressions for air, N_2 , O_2 and CO by the respective molecular weights.

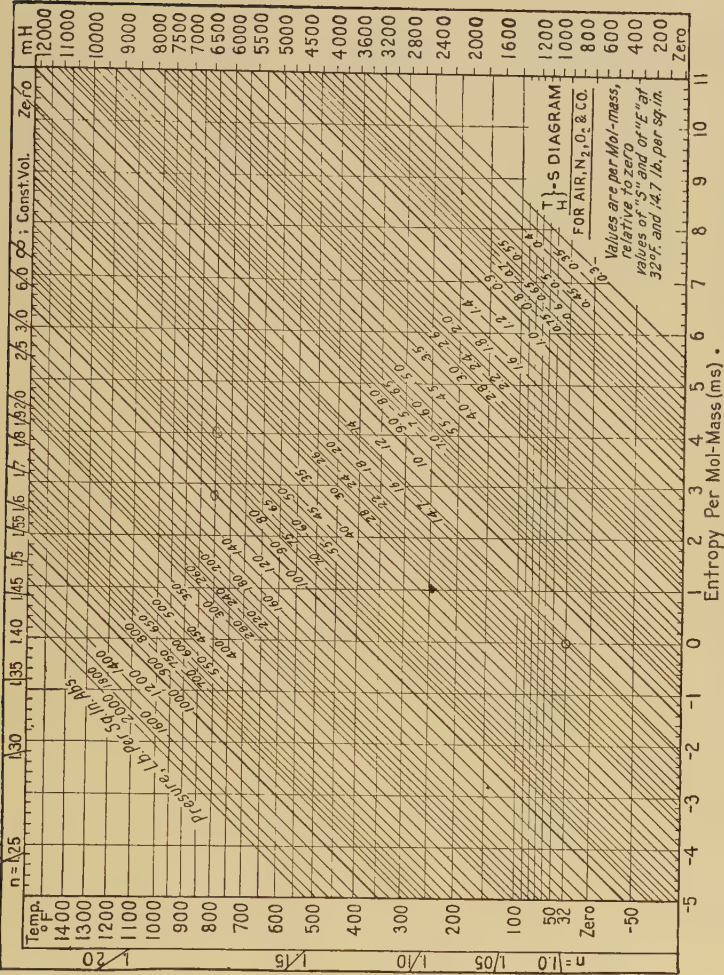


Fig. 32. T - S and H - S diagram for diatomic gases (except hydrogen). Enthalpy and entropy values are per mol-mass (m pounds) and are relative to zero values for the internal energy (E) and entropy at 32° F. and 14.7 lb. per sq. in. Temperature and enthalpy values are to logarithmic scales.

The relation of Partington and Shilling has been used for the plotting of the curves.

The values of mH and mS used in the figure are ones relative to a standard reference state at 32° F. (492° R.) and $14.7\text{ lb. per sq. in.}$ absolute, at which state the entropy and the internal energy are regarded as zero. The values of T and mH have been plotted to logarithmic scales in order to make the pressure contours virtually straight lines (departing from those only by reason of specific heat variation with temperature). On this figure any polytropic or adiabatic state change conforming to the relation " $PV^n = \text{constant}$ " would be representable by a straight line of proper slope. The slope corresponding to various values of n may be found by passing a line through the reference state-point, at 32° and 14.7 lb. , and the proper one of the various indices appearing in left and upper margins. For a state change passing through any other state pass a line of proper slope through the desired state-point.

Example 21. Check various of the results of examples 10 to 19 by use of Fig. 32. (The equivalent molecular weight of air is $\frac{1545}{53.3}$ or 29.)

92. Summary. — The perfect gas was introduced (Art. 48) as the hypothetical substance which, if it should exist, would provide an exact medium for realizing the energy scale of temperature. No substances are found which conform exactly to its requirements but the "permanent" gases (such as air, oxygen, nitrogen, et cetera) and the very low pressure vapors are found to conform sufficiently closely that for many practical engineering purposes they may be regarded as virtually "perfect" and the analyses of processes with such actual fluids may be made upon that basis.

The fundamental requirements of the perfect gas are (a) that for a given gas the product of its specific volume and absolute pressure divided by its absolute temperature shall be an invariable constant for the gas for any and all states in which it may chance to be and (b) that the internal (molecular) energy of the gas depends solely on its temperature.

From these general property relations various other equally general ones may be developed. All that have been developed in the foregoing are collected for convenience in Table IX. At least the first four of the general relations of this table are employed so frequently as to justify memorization.

Associated with the first relation is the phenomenon that for all of the approximately perfect gases the value of R is closely equal to 1545

divided by their molecular weight, which constant (1545) is frequently referred to as the "general" or "universal" gas constant. Table VII provides data on the gas constant for various of the real gases.

TABLE IX
GENERAL PROPERTY RELATIONS; PERFECT GAS

$$\frac{PV}{T} = R, \text{ a constant for any given gas.} \quad (1)$$

$$\begin{aligned} \Delta E &= \int_1^2 c_v dT, \text{ if } c_v \text{ is a function of } T, \text{ or} \\ &= c_v(T_2 - T_1), \text{ if } c_v \text{ is effectively constant, or} \end{aligned} \quad (2)$$

$$= \frac{1}{J(k-1)} (P_2 V_2 - P_1 V_1). \quad (2a)$$

$$\begin{aligned} \Delta H &= \int_1^2 c_p dT, \text{ if } c_p \text{ is a function of } T, \text{ or} \\ &= c_p(T_2 - T_1), \text{ if } c_p \text{ is effectively constant, or} \end{aligned} \quad (3)$$

$$= \frac{k}{J(k-1)} (P_2 V_2 - P_1 V_1). \quad (3a)$$

$$\frac{c_p}{c_v} = k; \quad c_p - c_v = \frac{R}{J}; \quad c_p = \frac{Rk}{J(k-1)} \quad (4)$$

$$\Delta S = \int_1^2 c_v \frac{dT}{T} + \frac{R}{J} \log_e \frac{V_2}{V_1}, \text{ if } c_v \text{ is a function of } T, \quad (5)$$

$$= c_v \log_e \frac{T_2}{T_1} + \frac{R}{J} \log_e \frac{V_2}{V_1}, \text{ if } c_v \text{ is constant;} \quad (5a)$$

$$= \int_1^2 c_p \frac{dT}{T} - \frac{R}{J} \log_e \frac{P_2}{P_1}, \text{ if } c_p \text{ is a function of } T, \quad (6)$$

$$= c_p \log_e \frac{T_2}{T_1} - \frac{R}{J} \log_e \frac{P_2}{P_1}, \text{ if } c_p \text{ is constant;} \quad (6a)$$

$$= c_p \log_e \frac{V_2}{V_1} + c_v \log_e \frac{P_2}{P_1}, \text{ if } c_p \text{ and } c_v \text{ are constant.} \quad (7a)$$

The specific heats of the real gases vary with the temperature to a moderate but negligible degree at moderate temperature ranges but more materially for high temperature ranges. The manner of variation may be ascertained only by experiment. Table VIII quotes the formulations devised by various authorities and fair values for c_p , c_v and their ratio, k , for the range between about atmospheric temperature to 400° F. These relations are of the general form

$$c_v = a + bT + fT^2; \quad c_p = \left(a + \frac{R}{J}\right) + bT + fT^2. \quad (8)$$

The property and energy relations for the various typical state changes of gases may be expressed in numerous forms, some of which have been developed in Arts. 84 to 90. Perhaps those which are directly or in-

directly in terms of the more readily measurable properties, temperature and pressure, offer the greater utility in practical computations. For convenience in reference there are compiled in Table X the relations of that character, together with a few additional ones which may have particular utility.

TABLE X

SPECIAL PROPERTY AND ENERGY RELATIONS FOR PERFECT GAS STATE CHANGES

Constant (specific) volume:

$$\frac{P}{T} = \frac{R}{V} = \text{a constant during the state change.} \quad (9)$$

$$\Delta E = \int_1^2 c_v dT = \frac{1}{J(k-1)} (P_2 - P_1) V;$$

$$\Delta H = \int_1^2 c_p dT = \frac{k}{J(k-1)} (P_2 - P_1) V.$$

$$\Delta S = \int_1^2 c_v \frac{dT}{T}, \quad \text{or} \quad = c_v \log_e \frac{P_2}{P_1} \text{ if } c_v \text{ is constant.}$$

Mechanical Effects, if reversible = zero; $Q = \Delta E$.

Constant pressure:

$$\frac{V}{T} = \frac{R}{P} = \text{a constant during the state change.} \quad (10)$$

$$\Delta E = \int_1^2 c_v dT = \frac{1}{J(k-1)} (V_2 - V_1) P;$$

$$\Delta H = \int_1^2 c_p dT = \frac{k}{J(k-1)} (V_2 - V_1) P.$$

$$\Delta S = \int_1^2 c_p \frac{dT}{T}, \quad \text{or} \quad = c_p \log_e \frac{V_2}{V_1} \text{ if } c_p \text{ is constant.}$$

Mechanical Effects, if reversible = $P(V_1 - V_2)$; $Q = \Delta H$.

Isothermal (constant temperature):

$$PV = RT = \text{a constant during the state change.} \quad (11)$$

$$\Delta E = \text{zero}; \quad \Delta H = \text{zero}; \quad \Delta S = \frac{R}{J} \log_e (P_1/P_2) \quad \text{or} \quad = \frac{R}{J} \log_e (V_2/V_1).$$

$$\begin{aligned} \text{Mechanical Effects, if reversible} &= -RT \log_e (P_1/P_2) \quad \text{or} \\ &- RT \log_e (V_2/V_1). \end{aligned} \quad (12)$$

$$Q = \frac{RT}{J} \log_e (P_1/P_2) \quad \text{or} \quad \frac{RT}{J} \log_e (V_2/V_1). \quad (12a)$$

Reversible adiabatic (isentropic):

$PV^k = \text{a constant during the change}; \quad TV^{k-1} = \text{a constant};$

$$\frac{T}{P^{(k-1)/k}} = \text{a constant.} \quad (13, 14, 15)$$

$$\Delta E = \int_1^2 c_v dT \quad \text{or} \quad = \frac{R}{J(k-1)} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right] \text{ if } k \text{ is constant.} \quad (16)$$

TABLE X — Continued

$$\Delta H = \int_1^2 c_p dT \quad \text{or} \quad = \frac{kR}{J(k-1)} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right] \text{ if } k \text{ is constant.} \quad (16a)$$

$$\Delta S = \text{zero}; Q = \text{zero}; \text{Mechanical Effects} = J \Delta E.$$

Irreversible adiabatics may frequently be characterized approximately by the property relations:

$$PV^n = \text{a constant}; TV^{n-1} = \text{a constant}; \frac{T}{P^{(n-1)/n}} = \text{a constant.} \quad (17, 18, 19)$$

$$\Delta E = \int_1^2 c_v dT \quad \text{or} \quad = \frac{R}{J(k-1)} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]. \quad (20)$$

$$\Delta H = \int_1^2 c_p dT \quad \text{or} \quad = \frac{kR}{J(k-1)} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]. \quad (20a)$$

$$\Delta S = c_v \frac{n-k}{n} \log_e \frac{P_2}{P_1}, \quad \text{where} \quad \begin{cases} n > k \text{ in a compression.} \\ n < k \text{ in an expansion.} \end{cases}$$

$$\text{Mechanical Effects} = J \Delta E \quad \text{but} \quad \neq - \int_1^2 P dV; Q = \text{zero.}$$

Throttling process is primarily characterized by the relation:

$$H_2 = H_1. \quad (21)$$

Polytropics, which virtually are internally reversible but are not adiabatic, may frequently be characterized by the property relations:

$$PV^n = \text{a constant}; TV^{n-1} = \text{a constant};$$

$$\frac{T}{P^{(n-1)/n}} = \text{a constant.} \quad (17, 18, 19)$$

$$\Delta E = \int_1^2 c_v dT = \frac{R}{J(k-1)} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]. \quad (20)$$

$$\Delta H = \int_1^2 c_p dT = \frac{kR}{J(k-1)} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]. \quad (20a)$$

$$S = c_v \frac{n-k}{n} \log_e \frac{P_2}{P_1}, \quad \text{where} \quad \begin{cases} n < k \text{ in a compression with heat emission.} \\ n > k \text{ in an expansion with heat emission.} \end{cases}$$

$$\text{Mechanical Effects} = \frac{R}{n-1} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]. \quad (22)$$

$$Q = \frac{R}{J} \frac{(n-k)}{(k-1)(n-1)} T_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]. \quad (23)$$

Graphic representations of the states and state changes of gases may be made to advantage on P - V , T - S , H - S , or other coördinates. For mechanically reversible state changes the area below the state change curve when plotted to P - V coördinates measures the Mechanical Effects; for mechanically reversible steady-flow processes the area back of the state change curve when plotted to P - V coördinates measures the summation of the shaft-work and the change of kinetic energy or either individually if the other is negligible or not germane. A scalar diagram of T and H vs. S (the two latter being values per mol-mass) is presented for air, N_2 , O_2 and CO in Fig. 32.

93. Review Questions and Topics. —

1. Quote the characteristic P - V - T relations for a (perfect) gas, explain its general significance and describe its utility when the gas constant is known for a particular gas.

2. (a) State "Joule's Law."

(b) Describe and analyze the Joule Experiment, showing in what manner it substantiates Joule's Law. May the Law be substantiated otherwise than by the experiment?

3. State the definite relationship between the change of internal energy and the change of temperature for a (perfect) gas, showing the method of determination of the relationship. In the light of Joule's Law and this relationship what characteristic must be assigned to the specific heat at constant volume for a (perfect) gas?

4. State the definite relationship between the change of enthalpy and the change of temperature for a (perfect) gas, showing the method of determination of the relationship. In the light of Joule's Law and this relationship what characteristic must be assigned to the specific heat at constant pressure for a (perfect) gas?

5. Quote the relationships between the constant pressure and constant volume specific heats of a (perfect) gas, these in terms of their difference and in terms of their ratio. Express the relationship between the change of enthalpy and the change of internal energy for a (perfect) gas in terms of the specific heat ratio.

6. Express the enthalpy change and internal energy change of a (perfect) gas in terms of its specific volume and pressure change and the specific heat ratio.

7. Express the entropy change of a (perfect) gas in terms of (a) its temperature and specific volume change, (b) its temperature and pressure change and (c) its pressure and specific volume change, introducing also its gas constant and specific heats as required.

8. Is the "gas constant" for any actual gas strictly constant? Is it adequately so for purposes of practical computations? What relationship is found to exist (with some degree of approximation) between the molecular weight and the constant for the various actual gases? What is meant by the "universal gas constant" and the "standard mol volume"?

9. Quote the typical form of relationships which are found to exist between the specific heats and the temperature for the actual gases.

10. Quote the P - T relations for a constant volume state change. What property change measures the energy transferred as heat in such a state change?

11. Quote the V - T relation for a constant pressure state change. What property change measures the energy transferred as heat in such a state change?

12. Quote the P - V relation for an isothermal state change. Quote the relations evaluating the Mechanical Effects and the energy transferred as heat in such a state change, in terms of the temperature and the gas constant and the initial and final pressures.

13. Quote the P - V , T - V and P - T relations for a reversible adiabatic state change. What property change measures the energy transition as work in a non-flow reversible adiabatic process and what property change measures the sum of the energy transition as work plus the change of kinetic energy (or either alone if the other is negligible) in a steady-flow reversible adiabatic process?

14. What is the essential characteristic of an irreversible adiabatic state change? What typical P - V , T - V and P - T relations are frequently employed as representing with adequate accuracy the property relations which characterize the irreversible adiabatic? What must be the relative magnitude of n with respect to k in the

typical devices in which such state changes will occur? What property change measures the sum of the energy transition as work plus the change of kinetic energy (or either alone if the other is negligible) in a steady-flow irreversible adiabatic process?

15. Both compare and differentiate between an isothermal and a throttling process with a (perfect) gas.

16. Define a polytropic state change and state the typical devices in which they are taken to occur, comparing such a state change with an irreversible adiabatic. What will be the relative magnitude of n with respect to k in such typical devices?

17. (a) Show on P - V coordinates the typical appearance of a constant pressure process, an isothermal, an irreversible adiabatic expansion or a polytropic compression, a reversible adiabatic, an irreversible adiabatic compression or a polytropic expansion and a constant volume process with a (perfect) gas. Indicate on the diagram for some particular state change the $\int P dV$ and the $\int V dP$ and state the significance of these areas in the case of mechanically reversible processes.

(b) Show on T - S and H - S coordinates the typical appearance of the same processes as designated above.

(c) By virtue of what unique phenomenon may the T - S or H - S diagrams for air, N_2 , O_2 and CO be plotted jointly? What advantage resides in the plotting of the logarithm of the absolute temperature or of the enthalpy in such a diagram, this rather than the temperature or enthalpy directly?

Symbols and Abbreviations, Chapter IX

a = a coefficient in specific heat formulations.

b = a coefficient in specific heat formulations.

c_p = specific heat at constant pressure, B.t.u. per pound.

c_v = specific heat at constant volume, B.t.u. per pound.

Δ = "change of."

E = molecular (internal) energy, B.t.u. per pound.

f = a coefficient in specific heat formulations; also "a function of."

H = enthalpy ($E + PV/J$), B.t.u. per pound.

J = Joule's equivalent (= 778 ft-lb. per B.t.u.).

k = ratio of c_p/c_v (a variable, but approaching constancy).

m = molecular weight.

n = an exponential constant (as in the property relation $PV^n = \text{a constant}$).

P = absolute pressure, pound per square foot.

Q = energy transferred by heat (radiation or conduction), B.t.u. per pound.

R = the gas constant for any given gas.

S = entropy, per pound.

T = absolute temperature, deg. Rankine (= °F. + 460).

V = specific volume, cubic feet per pound.

W = shaft-work, foot-pounds per pound.

CHAPTER X

MIXTURES; PROPERTY RELATIONS AND SIMPLE PROCESSES

Various of the more common engineering fluids are physical mixtures either of several of the more permanent gases or of one or more of those gases with moderately superheated or saturated vapors. Thus dry air is a mixture of oxygen and nitrogen (with traces of argon, CO₂, etc.) and normal atmospheric air is that mixture plus superheated or saturated water vapor or perhaps saturated vapor and liquid (as in clouds, fog or rainfall). The following articles give attention to such mixtures.¹

94. Gas Mixtures. — The analysis of the property relations of gas mixtures would be quite complex indeed if the properties of each constituent should need to be considered individually throughout any required analysis. Fortunately, however, that difficulty may be minimized by reason of the fact that any such mixture may be regarded as a single "equivalent" gas with characteristics which depend on the kind and proportion of each constituent but which are quite as definitely assignable to any specific mixture as they are to any individual constituent.

In the definitive specification of a mixture two procedures are possible and are regularly employed. One, and perhaps the more natural one, is that in which the description is in terms of the *relative mass of each constituent* with respect to the *mass of the whole*, that is, a description in terms of a gravimetric analysis. Thus for a mixture of several constituents which may be designated as gases "x," "y" and "z" the relative mass of these constituents would be, respectively,

$$\frac{M_x}{M_x + M_y + M_z} = \frac{M_x}{M_m}; \quad \frac{M_y}{M_x + M_y + M_z} = \frac{M_y}{M_m}; \quad \frac{M_z}{M_x + M_y + M_z} = \frac{M_z}{M_m};$$

where

M_x , M_y and M_z are the masses of each constituent,
 M_m is the total mass of the mixture.

¹ Study of the material of this chapter may be deferred without disadvantage until the consideration of Combustion and the Internal-Combustion engine. Parts of Art. 100 may perhaps be entirely omitted without disadvantage to the engineering student.

It very frequently happens that in the practical analysis of gas mixtures a second method is followed and measurements are made, not in terms of the masses of the constituents, but in terms of the *volumes* each would occupy when segregated from the others and caused to exist *under the original temperature and pressure of the original mixture*.² Thus, in Fig. 33, the several constituents (x , y , and z) of a mixture are conceived to be separated and to be contained in several compartments x , y , and z where each is at the common temperature T_m and pressure P_m and where the whole volume is the sum of the individual volumes, or $V_x + V_y + V_z = V_m$. The volumetric proportions would be represented by the ratios V_x/V_m , V_y/V_m , and V_z/V_m .

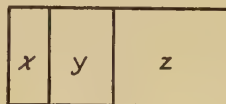


FIG. 33

Frequently a transfer of a volumetric to a gravimetric analysis, or vice versa, is desired. That is readily accomplished if one recalls that, from the characteristic equation for a (perfect) gas,

$$M_x = \frac{P_m}{T_m} \frac{V_x}{R_x} \quad \text{or} \quad V_x = \frac{T_m}{P_m} M_x R_x,$$

where V_x = the total (*not specific*) volume of M_x lb. of constituent x .

R_x = the gas constant for any constituent x .

P_m and T_m = the pressure and temperature of the original mixture or of the constituents when segregated by volumetric analysis.

Employing these relations successively for the several constituents the whole mass of the mixture (M_m) would be

$$M_m = M_x + M_y + M_z = \frac{P_m}{T_m} \left(\frac{V_x}{R_x} + \frac{V_y}{R_y} + \frac{V_z}{R_z} \right),$$

or the whole volume (V_m) would be

$$V_m = V_x + V_y + V_z = \frac{T_m}{P_m} (M_x R_x + M_y R_y + M_z R_z).$$

The relative mass proportion of any constituent would therefore be written, in terms of the volumetric analysis, as

$$\frac{M_x}{M_m} = \frac{V_x/R_x}{V_x/R_x + V_y/R_y + V_z/R_z}. \quad (1)$$

² Such volumetric analysis is typified by the common engineering analysis of furnace gases by means of the Orsat apparatus.

Likewise the relative volume proportion of any constituent would be written, in terms of the mass analysis, as

$$\frac{V_x}{V_m} = \frac{M_x R_x}{M_x R_x + M_y R_y + M_z R_z}. \quad (2)$$

If it is recalled further (Art. 81) that for the gases, with some degree of approximation, the product of the molecular weight (m) and the gas constant (R) equals 1545, then by substituting for each individual value of R in the above equations the quantity $1545/m$ we have (with a like but frequently permissible degree of approximation),

$$\frac{M_x}{M_m} = \frac{V_x m_x}{V_x m_x + V_y m_y + V_z m_z}, \quad (1a)$$

or
$$\frac{V_x}{V_m} = \frac{M_x/m_x}{M_x/m_x + M_y/m_y + M_z/m_z}. \quad (2a)$$

In actual mixtures the constituents will, of course, not be segregated but the molecules of each will be thoroughly dispersed throughout the whole region occupied by the mixture. If one conceives that region to be a closed container and regards the pressure exerted by the fluid on the walls of the container as due to molecular bombardment of those walls (see Art. 8), then one may readily acquire the further concept of the division of the total pressure into various **partial pressures** which are individually attributable to the molecules of the various individual constituents. The partial pressure of or due to any constituent is that which it would exert if it alone should occupy the region at the temperature of the mixture.³ According to the characteristic equation this partial pressure for each constituent would be

$$P_x = M_x R_x \frac{T_m}{V_m}; \quad P_y = M_y R_y \frac{T_m}{V_m}; \quad P_z = M_z R_z \frac{T_m}{V_m}.$$

The proportion between the partial pressure of any constituent and the total pressure of the mixture (P_m) thus becomes, since $P_m = P_x + P_y + P_z$,

$$\frac{P_x}{P_m} = \frac{M_x R_x}{M_x R_x + M_y R_y + M_z R_z} = \frac{M_x/m_x}{M_x/m_x + M_y/m_y + M_z/m_z} \quad (3)$$

$$= \frac{V_x}{V_m} \text{ (by equation 2).} \quad (3a)$$

The latter relation indicates that for a gas mixture the ratio between the partial pressure of any constituent and the total pressure is equal to the volumetric proportion of the constituent.

³ This conclusion is the basis of Dalton's Law for physical mixtures of (perfect) gases.

The relation between the partial pressure of any constituent (as x) and that of any other (as y) would also be

$$\frac{P_x}{P_y} = \frac{M_x R_x}{M_y R_y}, \quad \text{or} \quad = \frac{M_x m_y}{M_y m_x} \text{ (approximately)} \quad (3b)$$

Expressions have thus been developed

- (a) for transferring from a volumetric to a gravimetric analysis of a gas mixture (equations 1 or 1a),
- (b) for transferring from a gravimetric to a volumetric analysis (equations 2 or 2a), and
- (c) for ascertaining the partial pressure relations from either the gravimetric or volumetric analyses (equations 3, 3a or 3b).

With these several relations available one may proceed to develop the expressions which evaluate the characteristics of a mixture when regarded as an equivalent single gas. Those characteristics are developed in the following articles.

95. Equivalent Gas Constant of a Mixture (R_m). — To ascertain the equivalent gas constant of a mixture note again that the total pressure of the mixture is the sum of the several partial pressures, whence we may write, by use of the above expressions for those partial pressures (equation 3 et priori)

$$P_m = P_x + P_y + P_z = \frac{(M_x R_x + M_y R_y + M_z R_z) T_m}{V_m}.$$

The bracketed term in this relation is evidently a constant for a given mixture. Also by comparison with the characteristic P - V - T relation for a gas, that is,

$$P = \frac{MRT}{(MV)},$$

(where MV is the volume of M lb. of the gas) it is further evident that the term is in effect an " $M_m R_m$ " for the mixture, where R_m is an equivalent gas constant for the mixture. It thus develops that

$$M_x R_x + M_y R_y + M_z R_z = M_m R_m, \quad \text{or} \\ R_m = R_x \frac{M_x}{M_m} + R_y \frac{M_y}{M_m} + R_z \frac{M_z}{M_m}. \quad (4)$$

This relation evaluates R_m via the gravimetric analysis and the individual gas constants. To evaluate it via the volumetric analysis one may introduce successively in equation 4 the values of M_x/M_m et cetera as determined by equation 1. Doing so,

$$R_m = \frac{V_x + V_y + V_z}{V_x/R_x + V_y/R_y + V_z/R_z} = \frac{1}{\frac{V_x}{V_m} \frac{1}{R_x} + \frac{V_y}{V_m} \frac{1}{R_y} + \frac{V_z}{V_m} \frac{1}{R_z}} \quad (4a)$$

Frequently it is convenient to employ an "equivalent molecular weight" of a mixture (m_m). If again mR may be taken as approximately constant at 1545 then, from equations 4, with a like degree of approximation,

$$m_m = \frac{1545}{R_m} = \frac{1}{\frac{M_x}{M_m} \frac{1}{m_x} + \frac{M_y}{M_m} \frac{1}{m_y} + \frac{M_z}{M_m} \frac{1}{m_z}}; \quad (5)$$

or from equation 4a,

$$m_m = \frac{V_x}{V_m} m_x + \frac{V_y}{V_m} m_y + \frac{V_z}{V_m} m_z. \quad (5a)$$

Example 1. The volumetric analysis of dry air is about

O ₂	20.9 per cent
N ₂ (atmos.)	79.1 per cent

- Compute (a) the composition by weight. (23.1; 76.9.)
 (b) the equivalent gas constant for the mixture. (53.3.)
 (c) the equivalent molecular weight of the mixture. (29.0.)
 (d) the partial pressures of the O₂ and N₂ at 14.7 lb. per sq. in. total pressure.

Example 2. The products of a combustion show the following volumetric analysis:

CO ₂	10 per cent
O ₂	3 per cent
N ₂	87 per cent

- Compute (a) the composition by weight. (14.8, 3.2, 82.0.)
 (b) the equivalent gas constant of the mixture. (51.9.)
 (c) the equivalent molecular weight of the mixture. (29.8.)
 (d) the density and specific volume of the mixture at 14.7 lb. per sq. in. and 62° F. (0.078.)

96. Specific Heats of Gas Mixtures. — To ascertain the equivalent specific heat of a gas mixture recall that by definition (Art. 24),

$$c = d'Q/dT, \text{ or } Q = \int c dT, \text{ or } MQ = M \int c dT.$$

Consequently the heat transfer accompanying a process or state change of M lb. of mixture made up of M_x , M_y and M_z lb. of the several constituents must equal

$$\begin{aligned} M_m Q &= M_m \int c_m dT \\ &= M_x \int c_x dT + M_y \int c_y dT + M_z \int c_z dT, \end{aligned}$$

where c_m is the equivalent or effective specific heat for the mixture and c_x , c_y and c_z are the individual specific heats of the constituents for the

particular process in question. Solving for c_m ,

$$c_m = \frac{M_x}{M_m} c_x + \frac{M_y}{M_m} c_y + \frac{M_z}{M_m} c_z. \quad (6)$$

If the specific heats may be taken as constant the evaluation of c_m from this relation is simple and self-evident.

If a variation of the specific heat with temperature must be accounted it is necessary to consider the functional relation between the specific heat and temperature, which relation we have seen commonly takes the typical forms (Art. 82),

$$c_v = a + bT + fT^2; \quad c_p = \left(a + \frac{R}{J}\right) + bT + fT^2.$$

In these relations the constants a , b and f (and R) exhibit different values, as evidenced in Table VIII (Art. 82). That may be suitably provided for, however, and equivalent values may be derived for a particular mixture by writing equation 6 in the form

$$\begin{aligned} c_m &= \frac{M_x}{M_m} (a_x + b_x T + f_x T^2) + \frac{M_y}{M_m} (a_y + b_y T + f_y T^2) \\ &\quad + \frac{M_z}{M_m} (a_z + b_z T + f_z T^2) \\ &= \left[\frac{M_x}{M_m} a_x + \frac{M_y}{M_m} a_y + \frac{M_z}{M_m} a_z \right] + \left[\frac{M_x}{M_m} b_x + \frac{M_y}{M_m} b_y + \frac{M_z}{M_m} b_z \right] T \\ &\quad + \left[\frac{M_x}{M_m} f_x + \frac{M_y}{M_m} f_y + \frac{M_z}{M_m} f_z \right] T^2 \\ &= a_m + b_m T + f_m T^2, \end{aligned} \quad (6a)$$

where the equivalent coefficients for the mixture are thus

$$a_m = \frac{M_x}{M_m} a_x + \frac{M_y}{M_m} a_y + \frac{M_z}{M_m} a_z, \text{ etc.}$$

Example 3. Using the mean values of c_v and c_p given for O_2 and N_2 in Table VIII (Art. 82), check the values of c_v and c_p as given for air. See example 1 for the analysis of air.

Example 4. For the mixture of example 2 evaluate the coefficients a_m , b_m and f_m and write the expressions for c_v and c_p for that mixture.

97. State Changes of Gas Mixtures.—In the foregoing articles methods have been devised whereby for any gas mixture one may compute the equivalent gas constant of the mixture (R_m) or its equivalent molecular weight (m_m) and values of the effective specific heats of the mixture or of the coefficients in the functional relations between the specific heat and the temperature. With these constants or coefficients

at hand one may proceed with the analysis of any typical state change of a gas mixture in exactly the same manner as if the fluid were a single gas. The procedures for the single gas have been adequately developed in Art. 84 to 90 of Chapter IX and need not be repeated. In point of fact it may have been observed that in that chapter the practice of regarding a gas mixture as effectively equivalent to a single gas was repeatedly exemplified in the treatment of the properties and energy relations of air jointly with those of the individual gases.

98. Gas and Vapor Mixtures. — Several classes of gas-vapor mixtures are encountered in engineering practice. These may be exemplified by the oxygen, nitrogen and superheated water vapor mixture which constitutes our normal atmosphere; by the air and saturated water vapor which constitutes the atmosphere at the incipency of fog or rainfall; or finally by such air, saturated vapor, and saturated liquid mixtures as the atmosphere during fog or rainfall, the mixture entering the air pump suction of a (steam) condenser or departing from an air-washer or the air and fuel mixture departing from the carburetor of an internal-combustion engine.

Perhaps the distinguishing features by which these several typical mixtures may be classified as well as the distinctive principles by which they each may be analyzed will be developed to best advantage by an illustrative example. Such an example, in which the several classes of mixtures are conceived to be formed successively, is analyzed in example 5.

Example 5. One pound of moisture-free air at standard atmospheric pressure and 76° F. is in a container of $\left(\frac{53.3 \times (460 + 76)}{14.7 \times 144} \right) = 13.5$ cu. ft. capacity. Into this container is introduced 0.010 lb. of water at 76° F. Upon introduction of the water it will evaporate and the water molecules will become thoroughly dispersed throughout the container.⁴ Presuming that the temperature of the air and vapor are caused to remain at 76°, by means of energy reception — see footnote, (1) what will be the state of the water vapor and the total pressure of the mixture *if its volume is retained at 13.5 cu. ft.*; (2) what will be the state of the vapor and the volume of the mixture *if it is permitted to so expand that the pressure shall remain at 14.7 lb. per sq. in.*?

Solution. (1) To ascertain the phase in which the water vapor must exist note that for a volume of 13.5 cu. ft. per 0.01 lb. the specific volume is $(13.5/0.01 =) 1,350$ cu. ft. per lb. From the steam tables the specific volume of saturated water

⁴ This dispersion will occur with a rapidity which depends on the character of the water stream as regards its degree of atomization, et cetera, and on the turbulence of the air. The evaporation would require an energy supply either, primarily, from the air or from some external source. For simplicity it will here be taken that energy is secured from the exterior in sufficient amount for the maintenance of the original 76° F. temperature. The alternative — a cooling of the air in order to supply the energy — is considered in a later article.

vapor at 76° F. is 718, as against the 1,350; hence the vapor must be superheated. The saturation pressure of water at 76° F. is 0.444 lb. per sq. in., whence the partial pressure of the superheated vapor must be less than that figure.

For ascertaining the actual partial pressure of the superheated vapor reference to the superheated steam tables would be indicated but they are found not to extend to the (low) temperature of 76° F. In this predicament recourse must be taken to the fact that very low pressure vapors are found (see pages 137, 140, 155 and 164) to conform approximately to the perfect gas laws.⁵ The mixture may therefore be analyzed as a gas mixture. Consequently, employing equation 3b, Art. 94, and noting that the air, being unchanged in temperature and in specific volume, must have a partial pressure in the mixture which is the same as its initial pressure alone,

$$\begin{aligned} P_{\text{vapor}} &= P_{\text{air}} \times \frac{M_{\text{vapor}}}{M_{\text{air}}} \times \frac{m_{\text{air}}}{m_{\text{vapor}}}, \text{ or} \\ &= 14.7 \times \frac{0.01}{1.00} \times \frac{29.0}{18.0} = 0.236 \text{ lb. per sq. in.} \end{aligned}$$

At 0.236 lb. per sq. in. the saturation temperature of water vapor is 57.7° F., whence the superheat is (76 - 57.7 =) 18.3° F.

The total pressure of the mixture thus becomes (14.7 + 0.236 =) 14.94 lb. per sq. in.

(2) If the mixture is permitted to expand sufficiently to have a total pressure of 14.7 lb. per sq. in. (versus 14.94) at 76° F., the partial pressure of each constituent must be less than 14.7. To ascertain those individual partial pressures note that, by equation 3, Art. 94 (and employing the same approximation as above),

$$\begin{aligned} P_{\text{vapor}} &= P_{\text{mix}} \times \frac{M_{\text{vapor}}/m_{\text{vapor}}}{M_{\text{vapor}}/m_{\text{vapor}} + M_{\text{air}}/m_{\text{air}}} \\ &= 14.7 \times \frac{0.01/18}{0.01/18 + 1.0/29.0} \\ &= 14.7 \times 0.0158 = 0.232 \text{ lb. per sq. in.} \end{aligned}$$

The volume of mixture must be that of either constituent *at its partial pressure* or, for the air,

$$\text{Volume} = \frac{53.3 \times (460 + 76)}{(14.7 - 0.232) \times 144} = 13.7 \text{ cu. ft.}$$

Superheat of vapor = 76 - 57.2 = 18.8° F.

The state of the superheated water vapor in this last condition of a partial pressure of 0.232 lb. per sq. in. (14.7 lb. per sq. in. total pressure) would be represented on *T-S* coördinates by a point such as *a* in Fig. 34.

Example 6. What total mass of water would need to be introduced into and dispersed through the original air of example

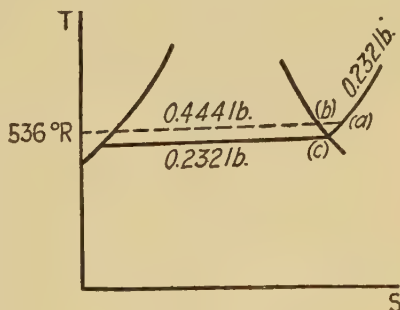


FIG. 34

⁵ Lacking definite *P-V-T* relations for many of the engineering fluids, such as the vapor and gaseous fuels, etc., the engineer is very frequently forced to the use of this approximation. Although unquestionably inaccurate in some instances, its use has enabled many practical developments which otherwise would have been very difficult and probably would not otherwise have been made.

5 in order that it might exist as (dry) saturated vapor at 76° F. (a) if the volume of the mixture were retained at 13.5 cu. ft. and (b) if the mixture were permitted to expand sufficiently to remain at 14.7 lb. per sq. in. total pressure? What would be the partial pressure of the vapor in each instance, the total pressure in the first instance, and the total volume in the second instance?

Solution. (a) From the steam tables the specific volume of saturated water vapor at 76° F. is 718 cu. ft. per lb., whence the specified 13.5 cu. ft. would need to contain exactly $(13.5/718 =) 0.0188$ lb. of vapor. From the steam tables this saturated vapor at 76° F. would be at a (partial) pressure of 0.444 lb. per sq. in., whence the total pressure of the mixture would be $(14.7 + 0.444 =) 15.14$ lb. per sq. in. The state of the vapor in this saturated condition at 0.444 lb. and 76° F. would be represented in Fig. 34 by the point *b*.

(b) At a total pressure of 14.7 lb. per sq. in. and a partial pressure of 0.444 for the vapor the partial pressure of the air would be $(14.7 - 0.444 =) 14.26$ lb. per sq. in.

The volume of the mixture would be measured by that of either constituent *at its partial pressure*. For the 1 lb. of air at 76° F. and 14.26 lb. per sq. in. this would be

$$\text{Volume} = \frac{53.3 \times (460 + 76)}{14.26 \times 144} = 13.9 \text{ cu. ft.}$$

The mass of saturated vapor which would occupy this volume is $(13.9/718 =) 0.0194$ lb. (against 0.0188 lb. as in the above).

The state of the saturated vapor in the mixture would again be represented in Fig. 34 by point *b*

In connection with the foregoing examples it is to be emphasized that for a so-called *saturated mixture*, meaning thereby a mixture of gas and saturated vapor, the partial pressure of the vapor is in every case the saturation pressure corresponding to the temperature of the mixture. This pressure may be ascertained only by reference to the vapor pressure-temperature curve for the vapor (such as Fig. 19 of Art. 60) or, for water vapor, by reference to the steam tables. Such tables also provide the requisite data concerning the density or specific volume of the saturated vapor.

It is obvious that if after saturation additional water were supplied none could vaporize so long as the temperature remained at 76° F. but that instead the water would continue to exist as a liquid in the mixture, perhaps as a fog if atomized on injection but in any event not vaporized. The amount of saturated vapor per cubic foot of mixture would be unchanged by the presence of the additional liquid.

The foregoing illustrative examples serve to typify the two (or three) representative classes of gas-vapor mixtures: (a) that mixture in which the circumstances as regards the temperature and the amount of vapor per unit volume of mixture are such that the vapor is superheated; (b) that mixture in which the temperature and the amount of vapor per unit volume of mixture are such that the whole of the vapor exists in the (dry) saturated state, as at state *b* in Fig. 34; and (c) the mixture

of gas, saturated vapor, and excess saturated liquid. In technical parlance the mixture of gas and saturated vapor would be known as a *saturated mixture*, or in the case of atmospheric mixtures the air would be said to be at 100 *per cent humidity*. Similarly the atmospheric mixture of air and superheated vapor would be designated as having a *relative humidity* which is less than 100 per cent. The quantitative specification and determination of the percentage relative humidity of such mixtures is considered in the following article.

Certain additional features and properties of a saturated mixture are developed in the following example.

Example 7. For the saturated mixture of example 6 find

- the partial pressure of the vapor and of the air — the latter for the total pressures of 15.14 (as in *a*) and 14.7 lb. per sq. in. (as in *b*), all at 76° F.;
- the density of each constituent and of the mixture and also the density of moisture-free air at the specified total pressure;
- the pounds of vapor per pound of air and per pound of mixture and the pounds of mixture per pound of air;
- the internal energy, the enthalpy and the entropy of 1 lb. of *mixture* at a total pressure of 14.7 lb. per sq. in. (taking values of these with respect to 32° F. and 14.7 lb. per sq. in.).

Compare the starred results with the corresponding values as given in the tables of the properties of saturated air as found in the engineering handbooks or in various of the Steam Tables (such as Goodenough). Check all results in second column.

Solution.

	$P_{\text{total}} =$	15.14 ; 14.70
(a) $P_{\text{vapor}} = 0.444$ lb. per sq. in.*		
$P_{\text{air}} = 15.14 - 0.444 =$		14.70 ; 14.26
(b) Vapor density, by tables, =		0.00139 ; 0.00139*
Air density = $\frac{14.7 \times 144}{53.3 \times 536} =$		0.07407* ; 0.07187*
Mixture density =		<u>0.07546</u> ; <u>0.07326</u>
Moisture-free air density = $\frac{15.14 \times 144}{53.3 \times 536} =$		0.07630 ; 0.07407*
(c) Vapor per lb. of air = $\frac{0.00139}{0.07407} =$		0.0188 ; 0.0194*
Vapor per lb. of mixture (M_v) = $\frac{0.00139}{0.07546} =$		0.0185 ; 0.0190
Mixture per lb. of air = $\frac{0.07546}{0.07407} =$		1.0188 ; 1.0194
(d) Internal energy (at 14.7 lb. total pressure) = $M_v E_v + M_a E_a$		
= $0.0190 \times 1034.5 + (1.0 - 0.019) \times 0.173 \times (76 - 32)$		
= $19.66 + 7.46 = 27.12$ B.t.u. per pound of mixture.		
Enthalpy = $M_v H_v + M_a H_a$		
= $0.019 \times 1093.5 + (1.0 - 0.019) \times 0.241 \times (76 - 32)$		
= $20.8 + 10.4 = 31.2$ B.t.u. per pound of mixture.		

$$\begin{aligned}
 \text{Entropy} &= M_v S_v + M_a S_a \\
 &= 0.019 \times 2.0451 + (1.0 - 0.019) \times [0.241 \times \log_e(536/492) \\
 &\quad - (53.3/778) \log_e(14.26/14.7)] \\
 &= 0.0389 + 0.0222 = 0.0611 \text{ units per pound of mixture.}
 \end{aligned}$$

An interesting characteristic of a saturated mixture which will have been observed in example 7 is that the density of the saturated mixture may be less than that of dry gas at the same temperature and (total) pressure, this because of the very low density of the very low pressure vapor constituent. This feature contributes to the barometric depression which usually accompanies or foreruns rainfall.

In carrying out the instructions for comparison of starred items in example 7, reference will have been made to the indicated tables in the steam tables and the handbooks. It is to be remarked that in those tables the values for the *weight of saturated vapor per pound of air*, for the *volume in cu. ft.*, for the *latent heat of vapor* (per pound of air) and for the *heat content (enthalpy) in B.t.u. of 1 lb. of dry air with vapor to saturate it* are strictly applicable only to the condition of a total mixture pressure of 14.7 lb. per sq. in. (standard barometric pressure). For the normal barometric variations at sea level the exact values would not depart sensibly from the values listed but individual computations would be necessary for pressures which differ materially from 14.7.

Example 8. From the proper table in the handbooks or Steam Tables find the pounds of vapor per pound of air for a saturated mixture at 60° F. Compare the result with the result of example 7 and draw conclusions as to the effect of cooling of saturated air.

Example 9. For saturated air at 3.0 in. of mercury (abs.) total pressure and 76° F., as in the air pump suction of a steam condenser, find the same items as called for in example 7 and compare with the results of that example. Also compute the number of pounds of vapor in a cubic foot of the mixture.

$$(M_v/M_a = 0.268; H_m = 240 \text{ B.t.u.; lb. of air per cu. ft.} = 0.00518.)$$

Example 10. Taking ethyl alcohol (C_2H_6O) as the fuel for an internal-combustion engine and taking the suitable air-fuel ratio in the combustible mixture supplied by the carburetor to be 12 to 1, at what minimum temperature would this mixture have to be in order that the fuel should be fully vaporized (saturated) at a total pressure of 29 in. of Hg.? What would be the volume of the mixture per pound of fuel?

Method of Solution. The alcohol may be presumed to approximate the perfect gas characteristics if vaporized (or superheated), in which circumstance equation 3 may be applied to the vapor-air mixture. If that equation is employed, using 1 as the relative mass of vapor and 12 as the relative mass of air, the resulting partial pressure is the vapor pressure which may and must exist if *all* of the alcohol is saturated (or superheated) vapor. The saturation temperature corresponding to this pressure is the *minimum* temperature which would permit the complete vaporization. See the engineering handbooks or other suitable physical tables for vapor pressure-temperature data on alcohol.

Perhaps the simplest method of computing the volume of the mixture is to com-

pute the volume of the air (or of the vapor) at its partial pressure, which volume is also the volume of the mixture. (60° F., 171 cu. ft.)

Example 11. If a fair grade gasoline vapor has the characteristics of a gas with a molecular weight (m) of about 114 and has a saturation pressure of 0.5 in. Hg. at 65° F., and if the intake manifold pressure of a gasoline engine is 28.5 in. Hg., estimate the maximum fuel-air ratio at which the gasoline in the fuel mixture in the manifold could exist wholly vaporized and saturated. (1/14.25.)

99. Humidity and Hygrometry. — In mixtures of a gas, such as air, and a superheated vapor, such as the water vapor mixed with atmospheric air, it frequently is essential that the amount of vapor carried with the air may be designated concretely. To do so several methods or designating measures are employed. These measures and their definitions are

- (a) **Specific humidity**, which is defined as *the mass of vapor in the mixture per unit mass of mixture* [$M_v/M_m = M_v/(M_v + M_g)$].
- (b) **Absolute humidity**, which is defined as *the mass of vapor in the mixture per unit volume of mixture*, that is, the density of the vapor in the mixture.
- (c) **Relative humidity**, which is defined as *the ratio between the density of the (superheated) vapor in the mixture and the density of saturated vapor at the same temperature*. To designate the relative humidity we shall employ the symbol ϕ (phi).

This last measure is the one most used by the engineer. Its application may be illustrated by reference to example 5. Referring to that example, for the first condition (a temperature of 76° F. and a volume of 13.5 cu. ft. for the air and the 0.01 lb. of water vapor) the density of the vapor equals 0.01/13.5 or 0.000741 lb. per cu. ft. The density of saturated vapor at 76° F. is 0.00139 lb. per cu. ft. (from the steam tables), whence the relative humidity of the mixture would be said to be (0.000741/0.00139 =) 0.532 or 53.2 per cent. Similarly for the second condition of example 5 the density of the vapor is (0.01/13.7 =) 0.000729 lb. per cu. ft. and the relative humidity is therefore (0.000729/0.00139 =) 0.523 or 52.3 per cent.⁶

A useful relation may be developed which expresses the relative humidity in terms of the partial pressure of the vapor as it actually

⁶ In practice it is sometimes taken as a convenient approximation that the relative humidity equals the ratio between the mass of vapor *per pound of dry gas* in the actual mixture and the mass of saturated vapor *per pound of dry gas* in a saturated mixture at the same temperature. According to this approximation the relative humidity in each of the conditions of example 5 would be 53.2 per cent. However, the definition given above is the accepted one and employment of the approximation may introduce quite sensible errors and is not commended.

exists in the mixture and its saturation pressure at the mixture temperature. As a very low pressure saturated or superheated vapor may be taken to conform sufficiently exactly to the perfect gas equation of state, $PV = RT$ or $P = RDT$ (where D = density), we may write that

$$\frac{P_{\text{sup. vapor in mixture, at } T_m}}{P_{\text{sat. vapor at } T_m}} = \frac{R_v T_m D_{\text{sup. vapor}}}{R_v T_m D_{\text{sat. vapor}}} = \frac{D_{\text{sup. vapor}}}{D_{\text{sat. vapor}}} \\ = \text{Relative humidity} \quad (7)$$

Illustrating this relation by reference again to the second condition of example 5 and also to Fig. 34,

$$\phi = \frac{P_{\text{sup. vapor}}}{P_{\text{sat. vapor}}} = \frac{0.232}{0.444} = 0.523, \quad \text{and} \quad = \frac{P_a}{P_b} \text{ in Fig. 34.}$$

A mixture may be caused to become saturated by providing additional vapor (as in example 6), by cooling the mixture at constant pressure, by expanding it, et cetera. The temperature at which it becomes saturated *by cooling at constant pressure* is known as the **dewpoint**. Because continued cooling below the dewpoint produces a progressive condensation and because many engineering processes with vapor mixtures proceed at approximately constant pressure some importance attaches to a ready means for predetermining the dewpoint of a mixture of known humidity. This may be developed by observing that, in conformity with equation 3 (Art. 94), the partial pressure of either the vapor or the gas constituent of the mixture is unchanged by cooling at a constant mixture pressure P_m so long as they both act as gases and their relative masses are unchanged. This will be the case at any temperature which is equal to or exceeds the saturation temperature corresponding to the partial pressure of the vapor. At the latter temperature the vapor becomes saturated, whence that temperature is the dewpoint temperature, or

$$\text{Dewpoint temperature of a gas-vapor mixture} = \text{the saturation temperature corresponding to the partial pressure of the (superheated) vapor constituent.} \quad (8)$$

Referring again to the data for the second condition of example 5, and to Fig. 34, the dewpoint of the mixture would be at the saturation temperature corresponding to a vapor pressure of 0.232 lb. per sq. in., or 57.2° F. (from the steam tables). The state of the saturated vapor at the dewpoint would be represented by point *c* in the figure. The state change of the vapor during the cooling from the original temperature to the dewpoint would be represented by the line *ac*.

The determination of the humidity of gas-vapor mixtures, notably that of the atmosphere, is frequently quite essential.⁷ The devices employed for such determination are known as **hygrometers** or **psychrometers**. Several methods for the determination are available. The most direct method but one which is less conveniently and less frequently used employs a polished metal surface which is immersed in the mixture and is so arranged that it may be chilled below the mixture temperature. Upon so chilling, the dewpoint may be closely detected by the incipency of fog precipitation on the surface, whereby from equation 8 the partial pressure of the vapor is ascertained and then its relative humidity from equation 7.

The method most used by the engineer employs two thermometers. The bulb of the one is *dry* and indicates the true temperature of the mixture. The bulb of the other is *wet*, being covered by a wick which is kept thoroughly moistened with the same liquid as exists in the vapor phase in the mixture (as the water in atmospheric air), and an active current of the mixture is caused to flow about this bulb (Fig. 35). The action at the wet bulb wick is a continuous saturation of such mixture as comes in intimate contact with the wick. The vapor required for this saturation is supplied by a continuous evaporation of liquid from the wick and the energy required for the evaporation is supplied primarily by a cooling of the mixture while undergoing saturation at the surface of the wick, with some energy coming to the bulb by radiation from its warmer environs. The wet bulb thermometer registers this lower temperature. The difference between the dry and the wet bulb thermometer readings is known as the wet bulb depression.

A useful energy equation may be developed for this process.⁸ Let it be noted that to the bulb there are flowing steadily the air with the superheated vapor in the air and that water (via the wick) which evaporates in saturating the mixture. From the bulb are flowing the air and the saturated vapor. For writing the energy equation it is convenient to evaluate the energy in B.t.u. per pound of air (not mix-

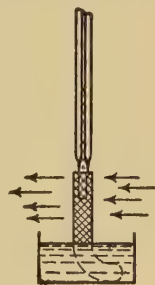


FIG. 35. Wet bulb.

⁷ Instances which may be noted are those of aerological observations, of air conditioning for the benefit of human comfort and efficiency, or of air conditioning in very many manufacturing processes.

⁸ Various empirical or semi-empirical relations for the analysis of psychrometer data have been devised. However, the energy analysis offers the usual advantages. The principles of the energy analysis were originally developed by W. H. Carrier, Trans., A.S.M.E., vol. 33.

ture) which becomes "saturated" at the bulb. Let the following symbols be employed:

t_d = dry bulb (actual mixture) temperature, °F.

t_w = temperature registered by wet bulb, °F.

$M_{v,1}$ = mass of superheated vapor in the original mixture at t_d , pound per pound of air (not mixture).

$M_{v,2}$ = mass of saturated vapor in the final mixture at t_w , pound per pound of air (not mixture).

$(M_{v,2} - M_{v,1})$ = mass of liquid vaporized, pound per pound of air (not mixture.)

$H_{w,2}$ = enthalpy of saturated water at t_w , B.t.u. per pound of water, or $H_{f,2}$ (column 6, Table IV, Art. 61).

$H_{v,1}$ = enthalpy of superheated vapor at t_d , B.t.u. per pound of vapor. (As the low-pressure vapor may again be taken to conform closely to the gas laws and its enthalpy may thus be taken to be the same at any state so long as the temperature is unchanged, $H_{v,1}$ may be determined from the *saturated* vapor tables at the *dry bulb temperature*, or $H_{v,1} = H_{g,1}$.)

$H_{v,2}$ = enthalpy of saturated vapor at t_w , B.t.u. per pound of vapor, or $H_{g,2}$.

$H_{a,1}$ = enthalpy of air at t_d , B.t.u. per pound of air.

$H_{a,2}$ = enthalpy of air at t_w , B.t.u. per pound of air.

c_p = specific heat of air at constant pressure = 0.241.

Q_r = energy to wet bulb as heat by radiation from warmer environs, B.t.u. per pound of air.

Writing the steady-flow energy equation for the wet bulb by equating the sum of the energy coming to the bulb via the water which evaporates from the wick ($= M_{v,2} - M_{v,1}$), via the air and its superheated vapor and by radiation to the sum of the energy leaving via the air and the saturated vapor, in B.t.u. per pound of air (not mixture),

$$[(M_{v,2} - M_{v,1}) \times H_{w,2}] + [1.0 \times H_{a,1} + M_{v,1} \times H_{v,1}] + Q_r \\ = 1.0 \times H_{a,2} + M_{v,2} \times H_{v,2};$$

or, solving for $M_{v,1}$,

$$M_{v,1}(H_{v,1} - H_{w,2}) = M_{v,2}(H_{v,2} - H_{w,2}) - (H_{a,1} - H_{a,2}) - Q_r \\ = M_{v,2}(H_{v,2} - H_{w,2}) - 0.241(t_d - t_w) - Q_r; \\ M_{v,1} = \frac{M_{v,2}H_{f,g,2} - 0.241(t_d - t_w) - Q_r}{H_{g,1} - H_{f,2}}. \quad (9)$$

Measurement of Q_r is obviously rather impractical in normal use but it has been found that with mercury thermometers this radiation to the wet bulb may be accounted with adequate accuracy by reducing the observed wet bulb temperature by about 1.6 per cent of the difference between the dry bulb and wet bulb thermometer readings.

Ingenious psychrometric charts based upon equation 9 (but strictly applicable only to air-water mixtures at a total pressure of or close to the standard barometric pressure of 29.92 in. Hg., that is, 14.7 lb. per sq. in.) have been devised and appear in the various handbooks.

The application of equation 9 and the consequent computation of the relative humidity may be shown to best advantage by a representative example.

Example 12. Observed psychrometric data taken on a wet and dry bulb hygrometer in air at a barometric pressure of 14.7 lb. per sq. in. abs. were

$$t_d = 70^\circ \text{ F.}, \quad t_w = 61^\circ \text{ F.}$$

Compute

- the vapor per pound of dry air ($M_{v,1}$). Compare with chart in engineering handbooks.
- the partial pressure of the vapor (compare with chart) and the dewpoint of the mixture.
- the relative humidity of the mixture. Compare with chart.
- the density of the mixture. Compare with chart.

Solution.

- Corrected wet bulb temperature = $61 - 0.016 (70 - 61) = 60.86^\circ \text{ F.}$
 $M_{v,2}$, at 60.86° F. , = 0.0114 (by saturated air tables in handbooks or steam tables or by computations such as those of example 7c for the circumstance of total pressures other than about 14.7 lb. per sq. in.)

$$H_{v,2} = 1086.6; \quad H_{v,1} = 1090.8; \quad H_{w,2} = 28.9$$

$$M_{v,1} = \frac{0.0114 \times 1057.7 - 0.241(70 - 60.86)}{(1090.8 - 28.9)}$$

$$= (12.06 - 2.20)/1061.9 = 0.00928 \text{ lb. per pound of air.}$$

$$= (0.00928 \times 7000 =) 65.0 \text{ grains. (By chart, 65 grains.)}$$

- Partial pressure of superheated vapor in original mixture ($P_{v,1} = P_a M_{v,1} / (m_a/m_v)$) (Eq. 3b)
 $= (14.7 - P_{v,1}) \times 0.00928 \times (29.0/18).$

$$P_{v,1} = 0.217 \text{ lb. per sq. in.} = 0.441 \text{ in. Hg. (By chart, 0.43 in. Hg.)}$$

$$\text{Dewpoint} = t_{\text{sat. at } 0.441 \text{ in. Hg.}} = 55.4^\circ \text{ F.}$$

- Relative humidity (ϕ) = $P_{v,1}/P_{\text{sat. at } 70^\circ \text{ F.}}$ (Eq. 7)
 $= 0.441/0.7386 = 0.597. \text{ (By chart, 60 per cent.)}$

- Density of vapor = $\phi \times \text{Density, sat. at } 70^\circ \text{ F.}$ (Eq. 7 et priori.)
 $= 0.597/869 = 0.000684.$

$$\text{Density of air} = P_a/RT = (14.7 - 0.217) \times 144/(53.3 \times 530) = 0.07378.$$

$$\text{Density of mixture} = 0.00068 + 0.07378 = 0.07446. \text{ (By chart, } 1/13.5 = 0.074)$$

Example 13. Compute the items as called for in example 12 for the same wet and dry bulb temperatures but for a barometric pressure of 26 in. Hg. (which pressure corresponds to an elevation of about 4000 ft. above sea level).

$$(M_{v,1} = 0.011; \phi = 0.612.)$$

100. State Changes of Gas and Vapor Mixtures. — In analyzing processes with mixtures of gases and vapors general property relations may not be set down directly as they have been for gases and gas mixtures but each problem must be analyzed more or less individually. Several representative ones are considered in more or less detail here. They are arranged in the conventional grouping of (a) constant volume process, (b) constant (total) pressure processes, (c) isothermal process, and (d) reversible adiabatic process.

(a) *Constant Volume Process.* — The controlling feature in the analysis of this type of process is that not only the volume and specific volume of the mixture remains constant but also that of each individual constituent. The details of an analysis are more advantageously indicated by an example.

Example 14. A vessel of 10 cu. ft. volume and open to the atmosphere contains 1 lb. of water. Atmospheric pressure and temperature are 14.7 lb. and 70° F. If the container is closed and heat energy supplied, at what temperature will the water become completely vaporized and what will be the pressure in the container at that temperature? What will be the pressure if the temperature is raised to 500° F.? How much heat was required for each step of the temperature rise?

Solution.

At the initial condition:

$$P = 14.7; P_v \text{ at } 70^\circ \text{ F.} = 0.36; P_a = 14.7 - 0.36 = 14.34.$$

$$\text{Sp. vol. of vapor} = 10.$$

$$\text{Quality} = 10/V_g \text{ at } 70^\circ \text{ F.} = 10/869 = 0.0115.$$

$$\text{Sp. vol. of air} = RT/P = (53.3 \times 530)/(14.34 \times 144) = 13.7.$$

$$\text{Mass of air in container} = 10/13.7 = 0.73 \text{ lb.}$$

At complete vaporization of water:

$$\text{From saturated steam tables at } V_g = 10, P_v = 42.1 \text{ lb. and } t = 270.4^\circ \text{ F.}$$

$$\text{At } V \text{ for air of } 13.7 \text{ and temperature of } 270.4^\circ \text{ F., } P_a = RT/V$$

$$= (53.3 \times 730)/13.7 = 2840 \text{ lb. per sq. ft.} = 19.7 \text{ lb. per sq. in.}$$

$$\text{Total pressure} = 42.1 + 19.7 = 61.8 \text{ lb. per sq. in. abs.}$$

At 500° F.:

$$\text{From superheat tables at } 500^\circ \text{ F. and } V = 10, P_v = 56.4 \text{ lb.}$$

$$\text{For air at specific vol. of } 13.7 \text{ and temperature of } 500^\circ \text{ F.,}$$

$$P_a = (53.3 \times 960)/13.7 = 3730 \text{ lb. per sq. ft.} = 25.9 \text{ lb. per sq. in. abs.}$$

$$\text{Total pressure} = 56.4 + 25.9 = 82.3 \text{ lb. per sq. in. abs.}$$

Heat supplied, 70 to 270.4° F., $= M_w \times \Delta E_w + M_a \times \Delta E_a:$

$$\text{For water} = 1.0(1092.4 - 49.5) = 1042.9 \text{ B.t.u.}$$

$$\text{For air} = 0.73 \times 0.173 \times (270.4 - 70) = 25.3 \text{ B.t.u.}$$

$$\text{Total} = 1042.9 + 25.3 = 1068.2 \text{ B.t.u.}$$

Heat supplied, 270.4 to 500° F., = ΔE :

For steam = $1.0(1177.8 - 1092.4) = 85.4$ B.t.u.

For air = $0.73 \times 0.173 \times (500 - 270.4) = 29.2$ B.t.u.

Total = $85.4 + 29.2 = 114.6$ B.t.u.

(b) *Constant Pressure Processes.* — Illustrative of the processes which are encountered in practice and which operate at approximately constant total pressure are (1) the conditioning of air for ventilating, heating, cooling or humidity control; (2) the cooling of water by an air stream as in the cooling of warm circulating water in the cooling tower or pond; (3) the cooling of the air and vapor mixture in a steam condenser prior to the removal of the air by the air pump; (4) the forming of fuel mixtures in the gasoline engine carburetor et cetera; and (5) the retention of ballast by water recovery from the engine exhaust of airships, et cetera. Being constant pressure processes, and presuming negligible kinetic energy changes during the process, the energy transfer as heat accompanying the process equals the change of enthalpy of the fluid. (See Arts. 67 or 85.)

The following problems will illustrate the methods of analysis.

Example 15. An air water-vapor mixture at 85° F. and 29.92 in. Hg. is known to contain 0.0182 lb. of vapor per pound of air (not mixture). Find (a) the relative humidity and dewpoint of the mixture; (b) the amount of vapor which will have condensed upon cooling the mixture at constant (total) pressure to 70° F., in pound per pound of air and per 1000 cu. ft. of original mixture; (c) the final volume of the mixture per 1000 cu. ft. of original mixture; (d) the energy which must have departed as heat, in B.t.u. per pound of mixture and per 1000 cu. ft. of original mixture.

Solution.

(a) From equation 3 (Art. 94),

$$P_{v,1} = P_m \frac{M_{v,1}/m_v}{M_{v,1}/m_v + M_a/m_a} = 14.7 \times \frac{0.0182/18}{0.0182/18 + 1/29.0} = 0.417 \text{ lb.}$$

Relative humidity = $P_{v,1}/P_{\text{sat., } 85^\circ \text{ F.}} = 0.417/0.596 = 0.70$.

Dewpoint = $t_{\text{sat., } 0.417 \text{ lb.}} = 74.1^\circ \text{ F.}$

(b) Vapor condensed at 70° F. — At this second state, for that portion of the moisture which is still saturated vapor ($M_{v,2}$), equation 3b will continue to hold. Solving the equation for $M_{v,2}$,

$$M_{v,2} = M_a \times \frac{m_v}{m_a} \times \frac{P_{v,2}}{P_m - P_{v,2}},$$

or also, by an analogous solution for $M_{v,1}$ and by comparison,

$$\frac{M_{v,2}}{M_{v,1}} = \frac{P_{v,2}}{P_m - P_{v,2}} \times \frac{P_m - P_{v,1}}{P_{v,1}},$$

where $P_{v,2}$ = pressure of saturated vapor at the second temperature.

Thus, $M_{v,2} = 0.0182 \times \frac{0.363}{14.337} \times \frac{14.283}{0.417} = 0.0158 \text{ lb.}$

Condensate = $0.0182 - 0.0158 = 0.0024$ lb. per pound of air.

Condensate per 1000 cu. ft. of original mixture = condensate per pound of air $\times 1000 \times$ density of air at its original partial pressure.

$$= 0.0024 \times 1000 \times \frac{(14.7 - .417) \times 144}{53.3 \times (460 + 85)} = 0.0024 \times 70.76 = .177 \text{ lb.}$$

$$(c) \text{ Final volume} = 1000 \times \frac{P_{a,1}}{P_{a,2}} \times \frac{T_2}{T_1} = 1000 \times \frac{14.283}{14.337} \times \frac{530}{545} = 968 \text{ cu. ft.}$$

$$(d) \text{ Heat emission} = \Delta H_{\text{moisture}} + \Delta H_{\text{air}} \\ = (0.0182 \times 1097.6 - 0.0024 \times 38 - 0.0158 \times 1090.8) \\ + 1.0 \times 0.241 \times (85 - 70) = 2.7 + 3.6 = 6.3 \text{ B.t.u.} \\ \text{per } (0.0182 + 1.0 =) 1.0182 \text{ lb. of mixture.}$$

Heat emission per pound of mixture = $6.3/1.0182 = 6.18$ B.t.u.

Heat emission per 1000 cu. ft. of original mixture = total emission per pound of air in mixture $\times$ lb. of air per cubic foot of original mixture (see paragraph *b* above) $\times 1000 = 6.3 \times 0.0707 \times 1000 = 445$ B.t.u.

Example 16. For a certain manufacturing process air must be supplied at 70°F. and 75 per cent relative humidity. This may be done under control by first washing the atmospheric air by passing it through a water screen of controlled temperature, thereby enabling the delivery of a saturated mixture at a specified temperature, and then warming this mixture to 70°F. by passing it across heating coils. Assuming a barometer of 29.0 in. Hg., at what temperature must the saturated air leave the washer and enter the heater in order that it may have the designated 75 per cent humidity after warming to 70°F. ? How much heat must be supplied in the heater per 1000 cu. ft. entering the heater? (*Hint.* Find the partial pressure of the vapor constituent in mixture at 70° and 75 per cent (state *a*, Fig. 34), by equation 7, and then the corresponding dewpoint (state *c*, Fig. 34) by equation 8. Find the density of vapor and air constituents as saturated at dewpoint and thus the mass of vapor and of air per 1000 cu. ft., and compute the heat required.)

(61.7°F. , 148 B.t.u.)

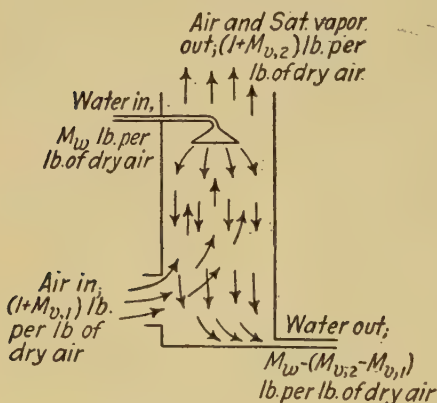


FIG. 36. Cooling tower.

Example 17. In a "cooling tower" warm water (as from a condenser) enters at 135°F. and the atmospheric air supply is at 70°F. and 50 per cent humidity. If the air leaves the tower at 125°F. and saturated (through vaporization of a part of the water) and the (remaining) water leaves at

80°F. , what fraction of the water supply evaporates? Assuming a barometer of 29.92 in. Hg., what volume of air must be furnished per minute if 3000 lb. of water are to be delivered to and cooled in the tower per minute? (*Hint.* Write the steady-flow energy equation, in B.t.u. per pound of dry air, for the tower. Referring to Fig. 36 for interpretation of the symbols,

$$M_w H_{w,1} + (M_{v,1} H_{v,1} + 1.0 H_{a,1}) \\ = [M_w - (M_{v,2} - M_{v,1})] H_{w,2} \\ + (M_{v,2} H_{v,2} + 1.0 H_{a,2}).$$

In this equation M_w is the sole unknown and thus may be computed. Compute the volume of the dry air required per pound of water supplied and thereby the total volume for the specified rate of water supply. Results are: $M_w = 1.95$ lb. of water supplied per pound of dry air; 4.5 per cent of the water supplied is evaporated; 20,900 cu. ft. of air must be supplied per minute.)

Example 18. In a well designed steam condenser the action is first to condense virtually the whole of the vapor and then to segregate and further cool such air as has entered the condenser with the steam, the air finally being withdrawn by the air pump. The objective of the air cooling is to decrease the volume of air to be handled by the pump, thereby increasing the effectiveness and efficiency of the pump in producing a maximum vacuum, and to do this without material subcooling of the condensed steam.

In the following data column *A* represents conditions at the entrance to the condenser; *B*, at the end of the steam condensing zone; *C* and *C'*, at the exit of the air cooling zone (at the pump entrance), the two sets of data being for two final temperatures. The small pressure drop accompanying the flow through the condenser is neglected. The mass of air entering per pound of dry steam is arbitrarily taken as 0.0002 lb.

Check the results which are underscored and supply those which are not given. Plot the required pump capacity against the mixture temperature, and discuss the practical significance of the curve. Why is it desirable to confine the cooling to the air, rather than to cool both the air and condensate?

	<i>A</i>	<i>B</i>	<i>C</i>	<i>C'</i>
Condenser press., inches Hg. abs.	←———— 1.00 —————→			
Air per pound of dry steam entering, pound	←———— 0.0002 —————→			
Air and vapor temperature, deg. F.	79.05	78.7	64.7	55.6
Vapor pressure, inches Hg. abs.	0.9999	0.9884	0.6154	
Vapor density, pound per cubic foot	0.00153	0.00151	0.00097	
Air pressure, inches Hg. abs.	0.000125	0.0116		
Air density, pound per cubic foot	0.0631	0.0429		
Vapor per pound of air, pound	5000	53		
Vapor per pound of steam entering, pound	1.00	0.0106	0.0002	0.0001
Steam condensed, per pound entering	0.00	.9894		
Air volume per pound of steam entering, cubic feet		7.0	0.21	0.14

Example 19. In the process of example 10, if the pressure entering the carburetor does not differ materially from that leaving, at what temperature would (dry) air and the fuel need to be supplied to the carburetor in order that the mixture leaving should be saturated at 60° F.? The specific heat of liquid alcohol may be taken to be 0.6 and its increase of enthalpy (latent heat of vaporization), 425 B.t.u. per pound. (*Hint.* Write an energy equation for the carburetor, in a manner parallel to the procedure of example 17.) (182° F.)

(c) *Constant Temperature (Isothermal) Processes.* — In the analysis of these processes it is necessary to keep in mind the universal feature that the volume of any constituent of a mixture is also the volume of each other constituent and of the mixture.

As the isothermal process is frequently set up as a standard one for the compression phase of an air compressor cycle the following example will analyze such a state change as it would occur for a compressor taking, not dry air but (atmospheric) air containing water vapor.

Example 20. Air at a total pressure of 14.7 lb. per sq. in. abs., 85° F., and 70 per cent relative humidity enters a compressor and is compressed isothermally to a (total) pressure of 100 lb. per sq. in. abs. At what pressure does the mixture become saturated and what are the initial and final densities of the mixture? Has the mixture followed a " $PV = \text{a constant}$ " relation during the compression? Compute the change of internal energy and of enthalpy, the energy departure as heat and the work required for the process, all per pound of dry air.

Solution. The initial state of the mixture is the same as that in example 15. Referring to that example,

$$P_v = 0.417 \text{ lb. per sq. in.}; P_a = 14.28; M_v/M_a = 0.0182;$$

$$\text{Density, vapor} = 0.00131; \text{air} = 0.07076; \text{mixture} = \underline{0.07207}.$$

At 85° F. the saturation pressure of the vapor is 0.596 lb., whence the vapor will become saturated when compressed to that pressure. Also, as the mass ratio during compression is unchanged and as the low pressure superheated vapor acts as a gas up to saturation the ratio between the pressure of the air and that of the vapor is likewise unchanged (from equation 3b of Art. 94). Thus the pressure of the air when saturation is reached $= 0.596 \frac{14.28}{.417} = 20.4$ lb. per sq. in. and the pressure of the mixture at saturation $= 20.4 + 0.596 = 21.0$ lb. per sq. in. abs.

After saturation, as the temperature is constant the vapor pressure must also remain constant at 0.596, whence at $P_{\text{mix, final}} = 100.0$ the partial pressure of the air $= 99.4$. At this state the density of the air $= (144 \times 99.4)/(53.3 \times 545) = 0.492$. The density of the total water content must thus be $(M_v/M_a) \times \text{density of air}$ or $(0.01819 \times 0.492 =) 0.00895$ lb. per cu. ft., or its specific volume $= 111.7$ cu. ft. per lb. The density of the mixture is $(0.492 + 0.00895 =) 0.501$ lb. per cu. ft.

For the mixture $P_2 V_2 = 144 \times 100/0.501 = 28,750$ and $P_1 V_1 = 144 \times 14.7/0.0721 = 29,390$, whence the mixture has not followed a " $PV = \text{constant}$ " relation.

The specific volume of (dry) saturated vapor at 85° F. is 543.8 cu. ft. per lb., whence the final quality of the vapor in the mixture at 100 lb. pressure is $(111.7/543.8 =) 0.205$. Thus 79.5 per cent of the vapor will have condensed during the compression.

The change of internal energy and of enthalpy for the mixture is that for the vapor alone as those properties of the air (as a perfect gas) are unchanged in an isothermal process (Art. 86). Also for the same reason those properties of the original superheated vapor may be taken from the *saturated* steam table at the temperature of the vapor (85° F.).

The change of internal energy (E) for the mixture, but per pound of dry air, equals $M_v/M_a \times (E_2 - E_1)_v = 0.0182 \times (255.1 - 1037.4) = -14.23$ B.t.u.

The change of enthalpy of the mixture, per pound of dry air, equals $M_v/M_a \times (H_2 - H_1)_v = 0.0182 \times (267.4 - 1097.6) = -15.10$ B.t.u.

The change of entropy of the mixture, per pound of dry air, equals $M_a \times A R \log_e (V_2/V_1)_a + M_v/M_a \times (S_2 - S_1)_v = 1.0 \times (53.3/778) \times (-1.94) - 0.0182 \times 1.562 = -0.1604$.

Energy emission as heat (Q) = $T \times \Delta S = -545 \times 0.1604 = -87.4$ B.t.u.

Work required for the compression (not including the intake and delivery) = $J(-Q + E_2 - E_1) = 778 \times (87.4 - 14.2) = \underline{56,900 \text{ ft-lb.}}^9$

(d) *Reversible Adiabatic Processes*.—As may be anticipated, the basic characteristic of such processes is the constancy of the entropy of the mixture. For actual analyses of such processes every advantage must be taken of the general characteristics of gas vapor mixtures, such as those that (1) the volume of the mixture is also the volume of each constituent, (2) the total pressure equals the sum of the partial pressures, (3) for superheated mixtures the partial pressure proportions are computable from the mass proportions (equations 3 and 3b) and are constant within the superheat phase, and (4) for saturated or wet mixtures the vapor pressure is fixed by the temperature of the mixture.

For the isentropic *compression* of a gas-superheated vapor mixture the vapor becomes progressively more superheated, whence the mixture acts throughout the compression as a gas mixture and the process is analyzed by the methods of Arts. 97 and 87. For an *expansion* of such a mixture the same procedure in analysis would obviously apply so long as the vapor remained in the superheat phase. However, the expansion would be accompanied by a decrease in the superheat and a consequent approach toward saturation, whence due care needs to be taken in the analysis to ascertain that saturation has not been reached and passed. Example 21 indicates the methods of ascertaining the point of occurrence of saturation.

The same example analyzes an isentropic expansion of a gas-superheated vapor mixture through a pressure drop sufficient that the vapor does become saturated and then wet. An exemplification of this ideal process is the approximately adiabatic and approximately reversible expansion accompanying the acceleration of atmospheric air in one vicinity by reason of a barometric depression in a neighboring vicinity, or again the pressure and temperature change which would ideally exist at varying elevations in atmospheric air which is drifting vertically without acceleration and adiabatically or is at rest in adiabatic equilibrium.

Certain of the phenomena which the example will show to occur during the expansion of a saturated or a wet mixture are also pertinent and important in connection with the ideally adiabatic *compression* of a wet mixture during the compression stroke of a gasoline engine.

Example. 21. Atmospheric air at 29.0 in. Hg. total pressure, 85° F. and 90 per cent relative humidity is expanded adiabatically and reversibly to a total pressure of

⁹ This quantity exceeds by about 2.3 per cent the work required for the isothermal compression of 1 lb. of dry air at 85° F. between the same pressures. For intake, compression and delivery the excess would be about 1.1 per cent.

26.0 in. Hg. At what total pressure and temperature will the vapor become saturated? What will be the final temperature after the expansion, and what proportion of the vapor will have condensed out?

Solution. Down to saturation of the vapor, by equation 14 of Art. 87,

$$\frac{T_2}{P_2^{(k-1)/k}} = \frac{T_1}{P_1^{(k-1)/k}} = \frac{T_x}{P_x^{(k-1)/k}};$$

where the subscripts (1) and (2) refer respectively to the initial state of the mixture and that at saturation, k is the specific heat ratio for the mixture, and the pressures (P_1 and P_2) are either the total pressures of the mixture (P_m) or the partial pressures of the vapor (P_v). The equation would apply both for the total pressures or the vapor pressures because within the superheat phase the ratio of the two is constant (since the mass proportion is constant — see equation 3b of Art. 94).

For determination of the pressure and temperature at which the mixture will become saturated trial solutions must be made for simultaneous values of T_2 and $P_{v,2}$ which will conform both to the above relation and to the T - P relation of saturated water vapor. If a simple mathematical expression were available which gave the relation between T and P for the saturated vapor line for water vapor, that relation and the above might be solved for the simultaneous values of T and P . Lacking such a simple expression the solution may be made without excessive difficulty by trial solutions and a graphic interpolation such as outlined in the following:

For the original air-vapor mixture at 29.0 in. Hg., 85° F. and 90 per cent relative humidity,

$$P_{v,1} = 0.90 \times 1.212 = 1.091 \text{ in. Hg.}$$

$$\text{Vapor density} = 0.90 \times 0.00184 = 0.00166.$$

$$\text{Air density} = P/RT = (29.0 - 1.091) \times 0.491 \times 144 / (53.3 \times 545) = 0.0679.$$

$$M_v/M_a = 0.00166/0.0679 = 0.0244.$$

$$M_v/M_m = 0.0244/1.0244 = 0.0238.$$

$$\begin{aligned} k_{\text{mix.}} &= \frac{c_{p, \text{mix.}}}{c_{v, \text{mix.}}} = \frac{0.0238 \times 0.461 + 0.9762 \times 0.241}{0.0238 \times 0.351 + 0.9762 \times 0.173} \text{ (Eq. 6 of Art. 96)}^{10} \\ &= \frac{0.246}{0.177} = 1.389. \end{aligned}$$

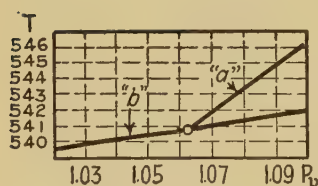


FIG. 37

$(k - 1)/k$, for mixture = 0.280, whence within the superheat region and down to saturation the mixture follows the state change equation $T/P^{0.280} = \text{constant}$.

As down to saturation the vapor pressure will bear a constant proportion to the mixture pressure (by virtue of Eq. 3 of Art. 94) the same T - P relation holds for the vapor alone, whence

$$T/P_v^{0.280} = T_1/P_{v,1}^{0.280} = 545/1.091^{0.280} = 531.9.$$

The T - P_v curve which satisfies this relation appears as curve a of Fig. 37. The T - P curve for saturated water vapor is curve b of the same figure. Curve a meets

¹⁰ c_p for water vapor = $0.462 - 0.0415T + 0.07235T^2$, by Goodenough and Felback. $c_p - c_v = 0.110$ by G. and F. The values used above are those at the temperature of 85° F. In meteorological work c_p for water vapor is frequently taken as constant at 0.47.

the saturated vapor curve at a temperature of 540.8°R. , or 80.8°F. , and a vapor pressure of 1.062 in. Hg., which are therefore the temperature (T_2) and the vapor pressure ($P_{v,2}$) at which the mixture becomes saturated. Again (Eq. 3) the mixture pressure at this vapor pressure must be

$$P_{m,2} = \frac{1.062}{1.091} \times 29.00 = 28.22 \text{ in. Hg.}$$

Further expansion below the pressure and temperature at which the vapor has become saturated will effect a condensation of a portion of the vapor. For the analysis of the expansion it is necessary to write an expression for the entropy change of each constituent of the mixture, to equate their sum to zero and then to scrutinize the resulting expression for the discovery of means whereby it may be solved. That procedure is followed in the following. In the expressions as written the subscripts 2 and 3 will denote respectively the saturation and the final states. Also the subscripts $v,2$ and $v,3$ will denote respectively the (total) mass of vapor at the saturation temperature T_2 and the remaining mass of vapor in the mixture after the further expansion to T_3 .

The difference of entropy of the total mass of water in the mixture, per pound of air, between that of the original saturated vapor at T_2 and the final wet mixture at T_3 equals

$$\begin{aligned} & \left[\frac{M_{v,2}}{M_a} (S_{f,2} + S_{fg,2}) \right] - \left[\frac{M_{v,2}}{M_a} S_{f,3} + \frac{M_{v,3}}{M_a} S_{fg,3} \right] \\ &= \frac{M_{v,2}}{M_a} (S_{f,2} - S_{f,3}) + \frac{M_{v,2}}{M_a} S_{fg,2} - \frac{M_{v,3}}{M_a} S_{fg,3} \\ &= \frac{M_{v,2}}{M_a} \times 1.0 \times \log_e \frac{T_2}{T_3} + \frac{M_{v,2}}{M_a} \frac{H_{fg,2}}{T_2} - \frac{M_{v,3}}{M_a} \frac{H_{fg,3}}{T_3}, \end{aligned}$$

if the specific heat of water is taken as unity (Eqs. 3a and 3c, Art. 53).

The simultaneous change of entropy of the air, per pound of air, will be (Eq. 6 of Art. 80).

$$\begin{aligned} & c_p \log_e \frac{T_2}{T_3} - \frac{R_a}{J} \log_e \frac{P_{a,2}}{P_{a,3}} \\ &= c_p \log_e \frac{T_2}{T_3} - \frac{R_a}{J} \log_e \frac{P_{m,2} - P_{v,2}}{P_{m,3} - P_{v,3}}. \end{aligned}$$

Adding these individual entropy changes and equating their sum to zero,

$$\begin{aligned} & \left[\frac{M_{v,2}}{M_a} \log_e \frac{T_2}{T_3} + \frac{M_{v,2}}{M_a} \frac{H_{fg,2}}{T_2} - \frac{M_{v,3}}{M_a} \frac{H_{fg,3}}{T_3} \right] \\ &+ \left[c_p \log_e \frac{T_2}{T_3} - \frac{R_a}{J} \log_e \frac{P_{m,2} - P_{v,2}}{P_{m,3} - P_{v,3}} \right] = \Delta S_m = 0, \end{aligned}$$

or

$$\frac{J}{R_a} \left[\left(c_p + \frac{M_{v,2}}{M_a} \right) \log_e \frac{T_2}{T_3} + \frac{M_{v,2}}{M_a} \frac{H_{fg,2}}{T_2} - \frac{M_{v,3}}{M_a} \frac{H_{fg,3}}{T_3} \right] - \log_e \frac{P_{m,2} - P_{v,2}}{P_{m,3} - P_{v,3}} = 0.$$

To permit solution of this equation M_v/M_a must be expressed in terms of properties. To do so observe that

$$\begin{aligned} \frac{M_v}{M_a} &= \frac{\text{density of vapor}}{\text{density of air}} = \frac{1/V_g}{(P_m - P_v)/R_a T} \text{ for any saturated vapor, and} \\ &= \frac{P_v/R_v}{(P_m - P_v)/R_a} \text{ for a low pressure vapor (or see Eq. 3b, Art. 94).} \end{aligned}$$

Introducing this relation in the foregoing,

$$\frac{J}{R_a} \left[\left(c_p + \frac{M_{v,2}}{M_a} \right) \log_e \frac{T_2}{T_3} + \frac{R_a}{R_v} \left(\frac{P_{v,2} H_{f_{g,2}}}{(P_{m,2} - P_{v,2}) T_2} - \frac{P_{v,3} H_{f_{g,3}}}{(P_{m,3} - P_{v,3}) T_3} \right) \right] - \log_e \frac{P_{m,2} - P_{v,2}}{P_{m,3} - P_{v,3}} = 0. \quad (10)$$

Scrutinizing this equation it appears that if the final temperature T_3 should be specified the only unknown is $P_{m,3}$, this by reason of the facts that

- (a) $P_{v,3}$ and $H_{f_{g,3}}$ are fixed by T_3 ;
- (b) $R_a, R_a/R_v, J$ and c_p are constants and equal 53.3, 0.62, 778 and 0.241;
- (c) $P_{m,2}$ and $P_{v,2}$ and $M_{v,2}/M_a$ are data which have already been computed above for the given mixture.

In a parallel manner if the final total pressure $P_{m,3}$ should be specified, although $P_{v,3}$, $H_{f_{g,3}}$ and T_3 are all unknown they constitute in effect a single unknown as they are fixed by T_3 .

The equation may perhaps be solved to best advantage by graphic means, that is, by introducing the specified value of T_3 (or of $P_{m,3}$) and then evaluating the expression at three or four values of $P_{m,3}$ (or of T_3) selected at random, plotting the results against the selected values and picking from the resulting curve that value of $P_{m,3}$ (or of T_3) which will give zero magnitude to the expression.

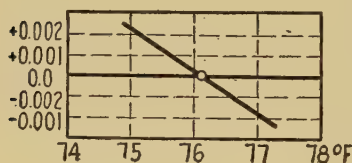


FIG. 38

Proceeding with the solution of example 21, taking $P_{m,3}$ as 26.0 in., substituting the previously determined values of $P_{v,2}$, $P_{m,2}$ and the value of $H_{f_{g,2}}$, taking several random values of T_3 and plotting in Fig. 38 the resulting values of the expression, the temperature of 76.1° F.

is found to give a zero value to the entropy change and thus to satisfy the equation. At that temperature $P_{v,3} = 0.907$ in., whence $P_{a,3} = 26.0 - 0.907 = 25.093$ in. Hg. Also

$$\frac{M_v}{M_a} = \frac{0.907}{25.093} \frac{18}{29.0} = 0.0225.$$

Thus of the original 0.0244 lb. of vapor per pound of air ($0.0244 - 0.0225 =$) 0.0019 lb. or 7.7 per cent will have condensed out upon expanding and cooling to 26.0 in. Hg. total pressure and 76.1° F. temperature.

In connection with the example it is of interest to note that although the entropy of the mixture has remained constant the entropy of the water (vapor plus liquid) constituent reduces rapidly as the mixture expands to pressures and temperatures below the saturation point. Thus in the example the entropy per pound of saturated vapor at 80.8° F. is 2.0322, whereas the vapor entropy in the mixture of $(100.0 - 7.7 =)$ 92.3 per cent quality at 76.1° F. is 1.8944. The appearance of the entire state change of the water is shown on T - S coördinates in Fig. 39.

Summarizing the results of the example; the original mixture at 29.0 in. Hg. total pressure, 85° F. and 90 per cent relative humidity will

become saturated upon adiabatic expansion to 28.22 in. Hg. and 80.8° F. and after further expansion to 26.0 in. Hg. and a temperature of 76.1° F. 7.7 per cent of the vapor will have condensed out, *presuming the condensed vapor to remain as a fog with the mixture and in temperature equilibrium with the remaining saturated vapor and air.*¹¹

The rapid decrease of entropy of the water which accompanies the expansion below the saturation temperature is evidently associated with a rapid rate of condensation as the expansion proceeds. This phenomenon (or its converse) is significant in connection with atmospheric state changes. Also the conversely rapid drying accompanying adiabatic compression of a wet vapor-air mixture is an important and fortunate feature of the mixture compression in the gasoline engine.

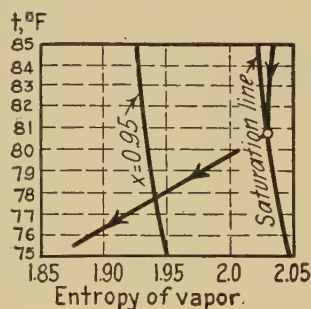


FIG. 39

Example 22. Check the results of example 21 for the vapor pressure and the temperature at saturation and for the vapor pressure, temperature, and the proportion of the vapor which will have condensed out after the complete expansion (check to the extent of testing to ascertain if the results which are quoted conform to the required relations).

101. Summary. — Various of the fluids which are of common engineering concern are physical mixtures (a) of several gases, as the air; (b) of a gas or gas mixture with small quantities of moderately superheated vapor, as humid atmospheric air; or (c) of a gas or gas mixture with saturated vapor and, perhaps, saturated liquid, as the atmospheric air at the incipency of fog and during fog or rainfall.

Gas mixtures are analyzed and described both on the mass proportion basis and on the volumetric proportion basis, the latter being an expression of the proportion of the whole volume which each constituent would occupy if its molecules were segregated and it were caused to exist under the original temperature and total pressure of the mixture. Knowing the gas constant (or the molecular weight) of each constituent of the mixture, the relative proportion of any, as expressed

¹¹ This presumption will not be wholly realized in actual processes because the condensate will not exactly follow the temperature of the vapor and air and also most of it will precipitate out, thus effecting an energy departure from the system. A process in which such an energy departure occurs via the condensed liquid is known as a *pseudo-adiabatic* process. It may be shown that if equation 10 is modified by the elimination of the term $M_{v,2}/M_a$ the resulting expression will provide the relation for ascertaining the final state of the vapor-air mixture in a pseudo-adiabatic.

on either the mass or the volumetric basis, may be transferred to the other basis by the use of the proper one of equations 1 or 1a, or 2 or 2a, Art. 94.

The total pressure of a gas mixture, regarded as a molecular bombardment phenomenon, is the summation of the individual or *partial* pressures attributable to the several individual constituents. Any one of these partial pressures would (in a mixture of perfect gases) be exactly that which it would exert if it alone should occupy the region at the temperature of the mixture. This condition is approached with adequate engineering accuracy in mixtures of actual gases, whence relations may be developed for the evaluation of the partial pressure of any constituent, given the gas constants (or molecular weights) of all of the constituents and either the mass or the volumetric analysis of the mixture. Those relations are presented in equations 3, 3a or 3b of Art. 94.

If, in the practical analysis of processes with gas mixtures, it were necessary to consider throughout the analysis the properties of each constituent such analyses would become quite complex indeed. That complication may fortunately be avoided as it is possible to ascribe constants and coefficients to the mixture quite as they are assignable to the constituents. The equivalent gas constant of a gas mixture (R_m) and also its equivalent molecular weight (m_m) may be evaluated by, respectively, equations 4 or 4a and 5 and 5a, these being designed for use according as the mass analysis or the volumetric analysis of the mixture may be more readily available. Similarly the specific heats of the mixture (c_m) or the coefficients in the typical specific heat-temperature relations may be computed by equations 6 or 6a, given the mass analysis of the mixture and the coefficients for the individual constituents.

With such equivalent constants, coefficients, et cetera thus made available for use in connection with gas-mixture processes and state changes, those state changes may be analyzed quite as they were in Chapter IX with the individual gases.

For those gas-vapor mixtures in which the mass proportion of the vapor constituent is small, whereby the vapor constituent is at a low partial pressure and superheated, the mixture may with adequate accuracy be regarded as a gas mixture and the analysis made as above — by reason of the close approach of low-pressure, superheated vapors to conformity with the perfect gas criteria.

Given the mass proportion of the vapor constituent of a mixture, the existence or non-existence of superheat may be ascertained by computation of the partial pressure of the vapor (Eq. 3) and comparison of that

computed pressure with the saturated vapor pressure corresponding to the temperature of the mixture. Superheat is indicated if the computed partial pressure is less than the saturation pressure.

If the mass proportion of the vapor constituent of a mixture is not directly known the existence or non-existence of superheat may still be ascertained by the use of a hygrometer, such as the wet and dry bulb thermometer device, superheat being indicated by a wet bulb depression. Also by an energy analysis of that device the mass proportion of the vapor may be determined (Eq. 9). For a mass proportion such that the vapor is superheated the ratio between the density of the superheated vapor and the density of saturated vapor at the mixture temperature is known as the relative humidity of the mixture. It develops (Eq. 7) that the relative humidity likewise measures the ratio between the partial pressure of the superheated vapor and the saturation pressure of the vapor at the temperature of the mixture. It also develops that if the gas and superheated vapor mixture were cooled at constant total pressure the vapor would become saturated at a "dew-point" temperature which is the saturation temperature corresponding to the partial pressure of the superheated vapor in the mixture prior to cooling (Eq. 8).

If the mass proportion of the vapor is not directly known but the relative humidity, temperature and total pressure is specified, whereby the partial pressure of the vapor constituent and thus of the gas constituent may be computed, then the density of the gas constituent may be computed from its partial pressure, temperature and gas constant. Having so arrived at the densities of the several constituents *which are occupying jointly the same volume* their mass proportion is equal to their density ratio. Also the density of the mixture is the sum of the several individual densities.

For a gas and saturated-vapor mixture the foregoing procedure is simplified somewhat by reason of the invariable pressure-temperature relation for a vapor at saturation.

Various of the major property relations et cetera for air-water vapor mixtures at standard atmospheric pressure have been presented in convenient tables and charts in the handbooks. However if the total pressure of the mixture is materially above or below the standard barometric pressure certain of these charted and tabulated values will not apply and the routine computations will need be made.

A summarization of the detailed methods for analysis of state changes with gas-vapor mixtures can not well be made, by reason of the variety and complexity of the methods employed. The reader is instead referred to Art. 100.

102. Review Questions and Topics. —

1. (a) Describe the two common bases of expressing the proportional make-up of a mixture of gases.

(b) Quote and explain the several relations whereby a mixture analysis expressed on either basis may be transferred to the other basis.

(c) What is meant by the partial pressure of any individual constituent of a mixture?

(d) Quote and explain the several relations whereby the partial pressure of any individual constituent of a mixture may be evaluated, given the total pressure and the mixture analysis.

2. Quote and explain the several relations whereby the equivalent gas constant and the equivalent molecular weight of a mixture of gases may be computed, given the mixture analysis.

3. Quote and explain the relation whereby the effective specific heat of a gas mixture may be computed (given the mixture analysis and the individual specific heats) and also the equivalent coefficients in the characteristic specific heat-temperature relations.

4. Discuss comparative the methods of analysis of state changes with gases and with gas mixtures.

5. (a) Illustrate the several typical classes of gas-vapor mixtures.

(b) What fundamental basis is usually employed as a necessary but acceptable approximation in the analysis of processes with gas and superheated vapor mixtures?

(c) Outline the procedure for computing the partial pressures of the several constituents, the superheat and the density for a gas-superheated vapor mixture, given the mass analysis, the total pressure and the temperature of the mixture. Depict on T - S coordinates the state of the vapor in such a mixture.

(d) Outline a procedure for computing the mass analysis and the density of a gas-saturated vapor mixture, given the total pressure and the temperature of the mixture. Outline the procedure for computing the relative internal energy, enthalpy, and entropy for such a mixture.

(e) Indicate any limitations to the utility of the usual tables of properties of the mixture of air and saturated water vapor.

6. (a) Define relative humidity and state the relation between the relative humidity of a gas-vapor mixture and the partial pressures of the actual vapor and a saturated vapor at the same temperature.

(b) Define the dewpoint temperature and depict on T - S coordinates the dewpoint for a mixture which was originally at a relative humidity less than 100 per cent.

(c) Describe the wet and dry bulb type of psychrometer and explain its action. What characteristic of a gas-vapor mixture is indicated by a wet bulb depression and what characteristic would be indicated by absence of wet bulb depression?

(d) What relationship for a gas-vapor mixture is ascertained by the energy analysis of the action at the wet bulb?

(e) Outline the procedure for computing the relative humidity of a gas-vapor mixture from the psychrometer data.

Symbols and Abbreviations, Chapter X

a = subscript designating the air constituent of a mixture.

f = subscript designating the saturated liquid state.

g = subscript designating the (dry) saturated vapor state.

- fg = subscript designating the change of a property during change of a fluid from the saturated liquid to the saturated vapor state.
- l = subscript designating a liquid constituent of a mixture.
- m = subscript designating a mixture.
- P = subscript designating constancy of pressure.
- V = subscript designating constancy of specific volume.
- v = subscript designating a vapor constituent of a mixture.
- w = subscript designating the water (vapor, liquid or both) constituent in a mixture.
- x, y, z = subscripts designating the several constituents x, y , and z of a (gas) mixture.
- ϕ = relative humidity.
- a = first coefficient in the c - T relation for a constituent of a mixture (or for the mixture).
- b = second coefficient in the c - T relation.
- c = specific heat, of a constituent or a mixture.
- E = molecular energy, B.t.u. per pound of a constituent or of a mixture.
- H = enthalpy, B.t.u. per pound of a constituent or of a mixture.
- k = ratio of specific heats (c_p/c_v).
- m = molecular weight of a constituent (or of mixture).
- M = mass of a constituent (or of mixture).
- P = partial pressure of a constituent (or total pressure of mixture)
- Q = energy transferred by radiation or conduction (as heat).
- R = gas constant of a constituent (or of a mixture).
- S = entropy per pound of a constituent (or of mixture).
- t = temperature, degree Fahrenheit.
- T = temperature, degree Rankine (degree Fahrenheit absolute).
- $V_{x, y \text{ or } z}$ = volume (not specific volume) of a constituent x, y or z in a mixture if the constituent is segregated and put under the temperature and total pressure of the mixture, as in volumetric analysis.
- V_g = specific volume of (dry) saturated vapor.
- x = quality (saturated-vapor fraction) of a mixture of saturated liquid and vapor.

PART IV — APPLICATIONS

103. Foreword. — The objective of Part I was (a) the acquisition of a broad understanding of the manners in which energy may be stored in various systems and fluids and may be in transit in the various processes of engineering thermodynamics and (b) the development of the relations and equations by which an intelligible accounting might be made of the energy quantities concerned in those processes, these relations being in effect simply formulations of the First Law of Thermodynamics. As an accompaniment of Part I there were acquired adequate concepts of the fluid properties, pressure, specific volume (and density), internal energy, and enthalpy.

Recognizing that, although energy relations based solely upon the First Law are highly necessary and valuable, they alone are wholly inadequate for ascertaining the inherent limitations of our available methods of power production from the energy reserves of nature, it became the function of Part II to inquire into those limitations. That inquiry, based upon the concept of the reversible temperature engine and built about the Carnot Principle and the Second Law of Thermodynamics, developed the important fact that the reversible engine is the most efficient one which may be conceived, developed further a fundamental concept of the temperature property and evolved the entropy concept. Entropy was found to be, basically, an indirect measure of the inherent unavailability of energy which is existing and deliverable by reason of temperature, and also, practically, the fluid property which remains constant in the reversible adiabatic state change. We shall discover subsequently that it is by a far-reaching use, both direct and indirect, of this unique characteristic of the entropy property that the engineer is enabled in practice to judge critically the performance of his power-generating machinery, to design and arrange such equipment intelligently and to predict its operating performance with remarkable accuracy.

The purpose of Part III was the securing of some measure of familiarity with the characteristics of the various sorts of fluids with which the engineer is more commonly concerned and with the methods of ascertaining the relative magnitude of their properties at any specific state or the change of those properties during various conventional sorts of state changes.

Having thus acquired a working knowledge of the basic principles of engineering thermodynamics and an ability to use intelligently the "tools" of that science, we are now in a position to proceed with more detailed consideration of particular processes, machines, or cycles. The considerations will include those of motive power machinery, such as the steam engine or turbine, the steam power plant as a whole and the internal-combustion engine, and also certain power-using apparatus such as compressors, blowers, and refrigerating machinery and certain flow-controlling or metering devices such as the nozzle, orifice, Venturi and Pitot tube, etc. These latter are the subject of the following chapter.

CHAPTER XI

FLOW THROUGH THE NOZZLE, ORIFICE, ET CETERA

Many instances arise in engineering practice in which it is required that the flow of compressible fluids be suitably controlled or that their rate of flow be measured. Also, intimately associated with flow control, it is frequently necessary that jets of high velocity be produced, and produced with minimum energy dissipation. The objective of the following is the delineation of the thermodynamic principles and procedures involved in such flow control and metering and in jet generation.

104. The Nozzle and Orifice, General Forms. — There are two general circumstances under which it becomes desirable to produce high velocity jets of a fluid by its expansion from an initial upper pressure and temperature to some lower pressure and temperature. The one is that in which it is required that a well-formed jet of the maximum attainable velocity be secured; the other and rather opposite circumstance is that in which nicety of jet formation is unnecessary and any velocity increase is only incidental, although it may be highly necessary that the jet velocity be closely calculable. The device in which the pressure drop is carried out under careful control is known as a **nozzle** and the typical machines in which it is employed are the steam turbine and the injector or ejector; the device in which simply a knowledge of the flow conditions is required is known as an **orifice** and its use is primarily that of a flow-control or of a flow-meter. In this connection it should be remarked that a nozzle will in general accomplish all of the usual objectives of an orifice, and also that certain types of orifices may serve adequately as nozzles and in fact may frequently be designated as **flow-nozzles**.

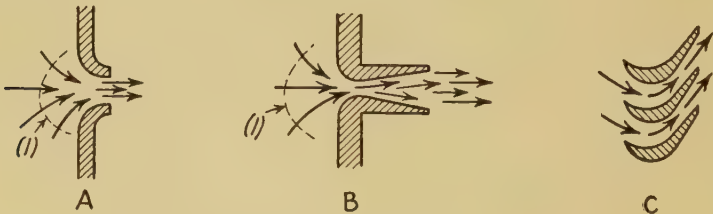


FIG. 40. Nozzle forms.

The general forms which nozzles may take are indicated (in longitudinal section) in Fig. 40. It will be noted in each that the entering

section is smoothly convergent. In nozzles *A* and *C* the convergent section is followed by a short parallel-walled section, giving what is known as a **convergent-parallel** nozzle. Under conditions which will be discussed later it will be found to be necessary that an additional gradually diverging section be provided, as indicated in nozzle *B*, which form would be known as a **convergent-divergent** or simply a **divergent** nozzle. The axis of the nozzle passage may be straight, as in *A* and *B*, or curved as in *C*, which latter condition is quite typical of the steam turbine nozzle block. If it is curved, excessive abruptness of bends is to be avoided. The transverse section of the nozzle passage may be of any convenient form, frequently being rectangular in the steam turbine nozzle.

Regarding further nomenclature, a transverse section across the stream lines of the fluid as it approaches the nozzle will be known as the **entering section** (as the sections marked 1 in nozzles *A* and *B*). The section at the point or region of minimum cross-sectional area will be known as the **throat** of the nozzle. A transverse section perpendicular to the stream lines directly as they leave the nozzle, sometimes known as the nozzle **mouth**, will be known as the **exit section**. Evidently in the convergent-parallel nozzle the throat and exit sections become identical, but differ in the convergent-divergent nozzle. The region into which the jet flows from the nozzle will be known as the **discharge region**.

Since the form of the jet produced by an orifice is in general of no importance the orifice may be of the simplest possible character, such as the two simple orifices shown in Fig. 41, or again it might take diverse other forms. However for use as a flow meter the simpler forms are preferable. The one shown above in the figure is characterized by the smoothly convergent entrance section and would be known as a **rounded-entrance** orifice. Evidently it is effectively identical to a convergent-parallel nozzle. The one shown below would be known as a **thin-plate** or a **sharp-edged** orifice, the first term being applied more particularly to an orifice formed by reaming a circular hole in a thin plate and the latter referring at times to a circular hole in a rather thicker plate but with the downstream edge of the plate chamfered back.

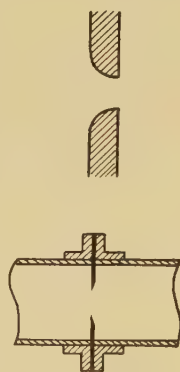


FIG. 41. Orifice forms.

105. The Energy Equation. — Both the nozzle and the orifice are essentially steady-flow devices, whence their energy analysis must be made upon the basis of the steady-flow energy equa-

tion. Repeating for convenience that equation as it appeared in Art. 32 (Eq. 5), in foot-pounds per pound of fluid flowing,

$$\frac{U_1^2}{64.34} + JH_1 + W + JQ = \frac{U_x^2}{64.34} + JH_x,$$

where the subscript 1 refers to the entering section of the nozzle or orifice and the subscript x refers to any subsequent section in the stream between the entrance and the discharge region.

Scrutinizing this general equation to ascertain if any terms are omissible it becomes evident that for the conditions of the nozzle or orifice the *shaft-work* term W disappears as there is obviously no reception or departure of energy from the system via a shaft. As regards the Q term, though quite probably there is some slight energy transition to or from the fluid by radiation and conduction, while enroute from sections 1 to x , its amount per pound of fluid would be quite negligible indeed and thus that term also would disappear. Expressed alternatively, the expansion is essentially an adiabatic one. The general energy equation as particularized for the nozzle or orifice would thus become (see also Art. 33c),

$$\frac{U_1^2}{64.34} + JH_1 = \frac{U_x^2}{64.34} + JH_x, \quad \text{or}$$

$$\frac{U_x^2 - U_1^2}{64.34} = J(H_1 - H_x); \quad U_x = \sqrt{64.34 J(H_1 - H_x) + U_1^2} \quad (1)$$

In the usual circumstances in which an energy analysis is desired a frequent requisite is the determination of the velocity of the fluid (U_x) at some particular section, this determination to be made however without having direct information concerning the entering velocity (U_1) but usually with quantitative data available concerning the cross-sectional area of the entering stream and concerning the initial state of the fluid. Equation 1 may be put in a form which is more convenient for that circumstance by combining the equation with the **Continuity Equation** (Art. 31, Eq. 2), that is, with the relation

$$M' = \frac{U_1 A_1}{V_1} = \frac{U_x A_x}{V_x}, \quad \text{or}$$

$$U_1 = U_x \left(\frac{A_x}{A_1} \right) \left(\frac{V_1}{V_x} \right), \quad (2)$$

where M' = mass-rate of flow, pounds per second.

A = cross-sectional area of stream at any section, square feet.

U = velocity, feet per second.

V = specific volume of fluid at any section, cubic feet per pound.

Introducing this expression for U_1 into equation 1,

$$\frac{U_x^2 - U_x^2 \left(\frac{A_x}{A_1}\right)^2 \left(\frac{V_1}{V_x}\right)^2}{64.34} = J(H_1 - H_x), \quad \text{or}$$

$$U_x = \sqrt{\frac{64.34 J(H_1 - H_x)}{1 - (A_x/A_1)^2 (V_1/V_x)^2}}$$

$$= 223.7 \sqrt{\frac{(H_1 - H_x)}{1 - (A_x/A_1)^2 (V_1/V_x)^2}}. \quad (3)$$

This relation will be of very considerable subsequent utility and should be retained by the reader.

Example 1. Check the constant 223.7 appearing in the above equation.

In many instances encountered in the analysis of practical flow problems the cross-sectional area of the entering section of the stream (A_1) may be of such considerable magnitude in comparison with any other sectional area in question (A_x) that one may regard the entering velocity (U_1) as wholly negligible or, from an alternative aspect, one may regard the denominator of the right-hand term of equation 3 as effectively equal to unity. In that event, either from equation 1 or from equation 3,

$$U_x = 223.7 \sqrt{(H_1 - H_x)}. \quad (3a)$$

Comparing equations 3 and 3a in the light of the preceding it appears that the factor

$$\sqrt{\frac{1}{1 - (A_x/A_1)^2 (V_1/V_x)^2}}$$

is one which enters if the initial velocity of the entering stream is of such relative magnitude as to require accounting and is absent if the entrance velocity is of minor and negligible magnitude. The factor is therefore known as a **correction factor for initial velocity**, or

$$\text{Correction factor for initial velocity} = \sqrt{\frac{1}{1 - (A_x/A_1)^2 (V_1/V_x)^2}}. \quad (4)$$

In numerous subsequent relations its appearance or absence will be based upon exactly the same premises.

Equations 1 and 2 provide a valuable method of energy accounting for a nozzle or orifice and they are wholly general in their applicability. However, they suffer the same inadequacy as was indicated in Art. 39, that is, an inadequacy in the feature that they offer no information concerning the maximum velocity and kinetic energy ideally attainable as the result of the expansion of a fluid through a specified pressure (and

temperature) range; nor do they offer any assistance in predicting the kinetic energies and velocities actually attainable. For such information recourse must be had to the various conclusions based upon the Carnot Principle and the Second Law. These conclusions as interpreted for the case of the flow through a nozzle or orifice are summarizable in the statements:

- (a) that, in general, the ideal state change of the fluid while enroute through the device would be a frictionless or mechanically reversible one, or
- (b) that, more concretely, the maximum kinetic energy and velocity which are ideally attainable as the result of the adiabatic expansion from a pressure P_1 to any pressure P_x would therefore be that securable upon a reversible adiabatic or *isentropic* state change through the pressure range in question.

To reflect these conclusions in the general equations 3 or 3a those equations may be written in the special forms

$$U_x, \text{ ideal} = 223.7 \sqrt{\frac{(H_1 - H_x)_S}{1 - (A_x/A_1)^2 (V_1/V_x)_S^2}}; \quad \text{or} \quad (5)$$

$$= 223.7 \sqrt{(H_1 - H_x)_S} \text{ if } U_1 \text{ is negligible} \quad (5a)$$

where the subscript S indicates that the values of H_x , of $(H_1 - H_x)$, of V_x and of (V_1/V_x) are those for an isentropic expansion of the fluid from the initial to the second pressure.

Scrutinizing these latter relations it appears that, given the initial state of the fluid entering the nozzle or orifice and the several pressures at the entering and any subsequent section, the problem of ascertaining the ideally attainable velocity at the second section would become solely that of determining the enthalpy and specific volume of the fluid after an expansion of the fluid *at constant entropy* to the pressure which it is proposed shall exist at the section. That is true and these ideal relations, together with the more general ones, will be extensively employed throughout the subsequent analyses. However there are certain very definite limitations as regards the allowable manner of their application which must be clearly recognized and which will be developed in the following articles. Because the details of their application will differ somewhat for a gas and for a vapor the flow of those several types of fluids will be considered individually from this point on.

106. Nozzle Form for Reversible Expansion of a Gas (*with negligible initial velocity*). — In order to develop more conveniently and intelligibly certain distinctive characteristics of both the nozzle and the orifice, it

is desirable first to investigate certain outstanding features of the form which a nozzle must take in order that it may provide properly for the flow of a fluid through it. To facilitate the development of those characteristics it is desirable to limit our initial attentions to the quite common circumstance of a *negligible velocity in the fluid as it approaches the nozzle* and to the ideal circumstance of *frictionless flow*. The particular form of the energy equation which fits such circumstances is provided in equation 5a.

To investigate the necessary nozzle form let it first be recalled (a) that in conformity with the specification of steady-flow the *mass-rate* of flow past all successive sections throughout the length of the nozzle must be the same and (b) that associated with the expansion of the fluid there is not only a velocity increase but also a specific volume increase, with no requirement, however, that the two shall increase at the same rate. These items are correlated in the **Continuity Equation**. Writing that equation as solved for the area A_x ,

$$A_x = M' \frac{V_x}{U_x}.$$

From this relation it becomes evident that to properly accommodate the steady-flow and progressive expansion of the fluid the cross-sectional area of the stream, and thus of the nozzle, must be caused to vary directly as the specific volume and inversely as the velocity. It remains to evaluate these concretely at any stage in the progressive expansion, evaluating them at present however for the condition of an ideal frictionless and non-turbulent expansion of a gas, such as air.

The ideal specific volume, $V_{x, \text{ideal}}$, may be expressed to best advantage in terms of data on the initial state of the gas approaching the nozzle and the pressure P_x to which the expansion shall have proceeded. As the ideal expansion is reversible and adiabatic the P - V relation of equation 13, Art. 87, is evidently indicated, or (for the circumstance of a negligible specific heat variation):

$$V_{x, \text{ideal}} = V_1(P_1/P_x)^{1/k}. \quad (6)$$

The velocity ideally attainable, $U_{x, \text{ideal}}$, is expressed in terms of the change of enthalpy of the gas in equation 5a, or

$$U_{x, \text{ideal}} = 223.7 \sqrt{(H_1 - H_x)_S} = 8.02 \sqrt{J(H_1 - H_x)_S}.$$

However, the enthalpy change for a reversible adiabatic expansion of a gas (and for the circumstance of a negligible specific heat variation) may be expressed in terms of data on the initial state and the pressure

P_x by equation 16a of Art. 87. Thus, by use of that equation,

$$U_{x, \text{ideal}} = 8.02 \sqrt{\frac{k}{k-1} R T_1 [1 - (P_x/P_1)^{(k-1)/k}]^2} \quad (7)$$

$$= 109.7 \sqrt{T_1 [1 - (P_x/P_1)^{0.286}]} \text{ for air.}$$

This relation is evidently an exact one and it will be referred to frequently. However it is sometimes regarded as something of a nuisance to evaluate numerically in any circumstances and is actually quite bothersome to evaluate accurately when the ratio P_x/P_1 approaches unity. For that reason it is of value to note that by performing a series expansion by the binomial theorem on the term

$$\frac{k}{k-1} [1 - (P_x/P_1)^{(k-1)/k}]^2$$

and throwing out the resulting minor terms a simpler relation is obtained which is quite adequately accurate for any value of P_x/P_1 which is not less than 0.90. The resulting relation is

$$U_{x, \text{ideal}} = 8.02 \sqrt{R T_1} \sqrt{\frac{P_1 - P_x}{P_1} \left[1 + \frac{1}{2k} \frac{P_1 - P_x}{P_1} \right]} \quad (7a)$$

$$= 58.6 \sqrt{T_1} \sqrt{\frac{P_1 - P_x}{P_1} \left[1 + 0.36 \frac{P_1 - P_x}{P_1} \right]} \text{ for air.}$$

For the circumstance of a value of $(P_1 - P_x)/P_1$ which is not in excess of 0.01 the bracketed term in equation 7a differs negligibly from unity and so for such a circumstance the equation may be used in the further simplified form

$$U_{x, \text{ideal}} = 8.02 \sqrt{\frac{R T_1}{P_1} (P_1 - P_x)} = 8.02 \sqrt{(P_1 - P_x) V_1}. \quad (7b)$$

¹ Useful alternative forms of equation 7 may be developed by the introduction of equation 4c of Art. 78, or by the use of equation 3a of Art. 79, giving the relations

$$U_{x, \text{ideal}} = 223.7 \sqrt{c_p T_1 [1 - (P_x/P_1)^{(k-1)/k}]}, \text{ or}$$

$$= 8.02 \sqrt{\frac{k}{k-1} (P_1 V_1 - P_x V_x) S}.$$

For an analysis accounting for specific heat variation see Wadlow, Philosophical Magazine of Nov. 1927.

² Thus, writing for $\frac{P_x}{P_1}$ its equivalent $\left(1 - \frac{P_1 - P_x}{P_1}\right)$,

$$\begin{aligned} \frac{k}{k-1} \left[1 - \left(\frac{P_x}{P_1} \right)^{(k-1)/k} \right] &= \frac{k}{k-1} \left[1 - \left(1 - \frac{P_1 - P_x}{P_1} \right)^{(k-1)/k} \right] \\ &= \frac{k}{k-1} \left[1 - \left\{ 1 - \frac{k-1}{k} \frac{P_1 - P_x}{P_1} + \frac{k-1}{2k} \left(\frac{k-1}{k} - 1 \right) \left(\frac{P_1 - P_x}{P_1} \right)^2 - \dots \right\} \right] \\ &= \frac{P_1 - P_x}{P_1} + \frac{1}{2k} \left(\frac{P_1 - P_x}{P_1} \right)^2 - \dots = \frac{P_1 - P_x}{P_1} \left(1 + \frac{1}{2k} \frac{P_1 - P_x}{P_1} - \dots \right). \end{aligned}$$

In the two relations of equations 6 and 7 and the above form of the Continuity Equation there are available expressions whereby, given the initial state of the gas and any selected pressure P_x to which the gas shall have expanded at any point in a nozzle, one may compute the specific volume and velocity of the gas at that point and the corresponding requisite cross-sectional area of the nozzle at the point in order that flow at the rate M' shall be properly accommodated. These several relations will subsequently be amalgamated into a useful single one, but prior to that it will be illuminating to employ them individually for computing the necessary progressive sectional areas of a nozzle to provide for progressive expansion, and thereby to discover certain unique characteristics of a proper nozzle form. That is done in the following example.

Example 2. Determine the necessary characteristic form of a nozzle which shall provide for the flow and expansion of 2 lb. of dry air per second from a supply region at 100 lb. per sq. in. abs. pressure and 70° F. to a discharge region at 15 lb. per sq. in. abs. Do this by computing the necessary successive sectional areas of the nozzle after progressive expansion to pressures of 80, 60, 55, 50, 45, 30 and 15 lb. Assume the ideal case of a reversible adiabatic flow and also assume a negligible entrance velocity. Show by diagram the resulting values of the specific volumes, velocities, and areas at the several pressures.

Solution for $P_x = 80$ lb. per sq. in.

$$V_1 = RT_1/P_1 = 53.3 \times (460 + 70)/(100 \times 144) = 1.963 \text{ cu. ft. per lb.}$$

$$V_{80} = 1.963 \times (100/80)^{1/1.4} = 2.303 \text{ cu. ft. per lb.}$$

$$U_{80} = 8.02 \times \sqrt{3.5 \times 53.3 \times 530 \times [1 - (80/100)^{0.286}]} = 626 \text{ ft. per sec.}$$

$$A_{80} = 2 \times 2.303/626 = 0.00736 \text{ sq. ft.} = 1.06 \text{ sq. in.}$$

The complete results of like computations for the specified succession of values of P_x are tabulated herewith. Also in the two diagrams of

P_x lb./sq. in.	V_x cu. ft./lb.	U_x ft./sec.	A_x sq. in.
100	1.963	negl.	large
80	2.303	626	1.059
60	2.824	929	0.876
55	3.01	1000	0.867
50	3.22	1070	0.868
45	3.47	1140	0.878
30	4.64	1360	0.982
15	7.61	1630	1.344

Fig. 42 the results are portrayed graphically. In diagram *a* the several volumes, velocities and areas are plotted against the pressures. Several significant features of the resulting curves will shortly be considered. In the meantime it is to be noted that the area curve in this diagram

evidently portrays the necessary area distribution axially along the nozzle if it were required that the pressure drop should proceed at a rate which is uniform with respect to axial distances traveled. Such an axial distribution of pressures and area would satisfy all thermodynamic requirements but it is not a necessary one. In fact it has been found that certain hydrodynamic considerations require that the rate of divergence of the nozzle beyond the throat must be much less rapid

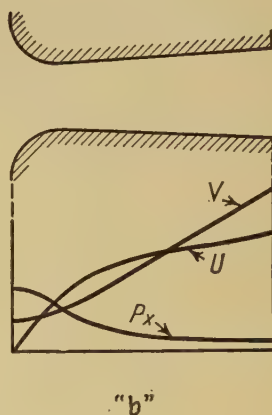
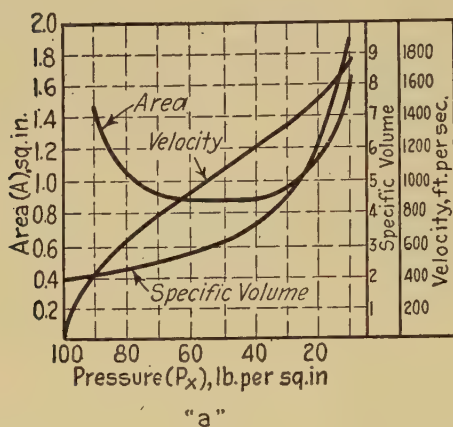


FIG. 42

than shown in order to avoid excessive turbulence in the stream. A nozzle form such as is portrayed in diagram *b* minimizes the latter difficulty and is a typical one. Thermodynamically it simply effects a different axial distribution of the volumes, velocities and pressures along the nozzle. The curves of diagram *b* depict these as so redistributed.

Example 3. Check all of the results for the values of P_x of 55 and of 15 lb. per sq. in. in the foregoing table.

Let us now observe several significant features of the nozzle form as those are indicated by the above tabular and graphical results, and also draw several important conclusions from the observations.

In the first place it appears that during the progress of the expansion there exists the unique circumstance that *initially* a relatively more rapid rate of increase of velocity combines with a relatively less rapid rate of increase of specific volume so as to require a *convergent passage* in the fore part of the nozzle, whereas *subsequently* a reversal in the relative rates of velocity and specific volume increases acts to require

a *divergent passage* in the after part of the nozzle. This circumstance thus acts to produce a minimum area or **throat** in the nozzle at a pressure which was found in the example to lie between 55 and 50 per cent of the supply pressure. We shall find subsequently that this percentage ratio between the throat and initial pressures is an effectively constant and invariable one. Also we shall discover further that this relative pressure to which the gas will have expanded at the end of the convergent channel of the nozzle is a particularly significant pressure, for which reason it is designated in technical parlance as the **critical pressure**.

As a result of this interesting circumstance it appears that if it were desired to select a nozzle which should properly provide for an expansion from the initial pressure to a discharge region pressure which is *greater than or equal to the critical*, then in general any convergent passage would be suitable. (The nozzle should have a short, parallel-walled exit section.) However, the requisite exit area of that passage would be fixed jointly by the initial state, the discharge pressure, and the desired mass-rate of flow. Thus, employing specific data from the problem, for flow to a discharge region at 80 lb. per sq. in. absolute pressure a convergent-parallel nozzle would be suitable and would deliver 1 lb. of air per second per $(1.059/2 =) 0.530$ sq. in. of exit area. Likewise for a 55-lb. discharge region pressure a convergent-parallel nozzle of $(0.867/2 =) 0.434$ sq. in. exit area per pound per second flow would be indicated.

In contrast to the foregoing it appears that if it were desired to select a nozzle which should properly provide for an expansion from the initial pressure to a discharge region pressure which is *less than the critical*, then only a convergent-divergent passage would be suitable. Again the requisite area of the nozzle at the exit would be exactly fixed by the initial state, the discharge pressure, and the desired mass-rate of flow; but it is important to note that the area of the *throat* would be fixed by the initial state, the desired mass-rate of flow, and the *critical pressure*. Again employing data from the problem for purposes of illustration, for flow to a discharge region at 30 lb. a convergent-divergent nozzle would be required with a throat area of $(0.867/2 =) 0.434$ sq. in. per pound per second flow and an exit area of $(0.982/2 =) 0.491$ sq. in. per pound per second. For a 15-lb. discharge region pressure exactly the same throat area would be required per unit mass-rate of flow but the requisite exit area in order to provide properly for the further expansion to 15 lb. per sq. in. would be $(1.344/2 =) 0.672$ sq. in. per unit mass-rate of flow.

The foregoing important observation may be summarized briefly as follows:

- (a) For the ideal expansion of a gas from a given initial state to a discharge region which is at a pressure not less than the critical pressure (a discharge pressure $\geq$ about 50 per cent of the initial pressure) a convergent-parallel nozzle will suitably provide for the formation of a well-formed jet in the discharge region. The proper exit area of the nozzle will be determined jointly by the initial state of the fluid, the desired mass-rate of flow, and the discharge region pressure.
- (b) For the ideal expansion of a gas to a discharge region which is at a pressure less than the critical pressure a convergent-divergent nozzle channel will be required in order to provide properly for the expansion and for the formation of the jet. The proper exit area of the nozzle will still be determined by the initial state of the gas, the desired mass-rate of flow and the discharge region pressure, but the throat area will be determined independently by the initial state, the desired mass-rate and the critical pressure.³

It is important that these several characteristics of the nozzle form be clearly recognized.

In lieu of the foregoing step-by-step procedure in area computation by individual use of equations 6 and 7 and the Continuity Equation, the three equations may be amalgamated to give a single useful relation by which the areas per unit mass-rate of flow or the mass-rate per unit area may be computed directly. It will in general be more convenient to have this unified relation expressed in terms of the mass-rate per unit area, M'/A_x . The relation is developed directly from the three equations as follows:

$$\begin{aligned}
 \frac{M'}{A_x} (\text{ideal}) &= \frac{U_{x, \text{ideal}}}{V_{x, \text{ideal}}} \\
 &= \frac{8.02 \sqrt{\frac{k}{k-1} RT_1 [1 - (P_x/P_1)^{(k-1)/k}]} }{V_1 (P_1/P_x)^{1/k}} \\
 &= 8.02 \sqrt{\frac{k}{k-1}} \frac{P_1}{\sqrt{RT_1}} \sqrt{\left(\frac{P_x}{P_1}\right)^{2/k} - \left(\frac{P_x}{P_1}\right)^{(k+1)/k}} \quad (8) \\
 &\quad (\text{since } V_1 = RT_1/P_1); \\
 &= 2.055 \frac{P_1}{\sqrt{T_1}} \sqrt{\left(\frac{P_x}{P_1}\right)^{1.43} - \left(\frac{P_x}{P_1}\right)^{1.714}}, \text{ for air.}
 \end{aligned}$$

³ This statement is amplified in the following article.

For convenience there are plotted in Fig. 43 values of the term

$$\sqrt{\left(\frac{P_x}{P_1}\right)^{1.43} - \left(\frac{P_x}{P_1}\right)^{1.714}}$$

plotted against the pressure ratio P_x/P_1 .

It may be noted that, as with equation 7, equation 8 is an exact one and one to which frequent reference will be made, but it is also frequently regarded as a nuisance to use and it is actually very bothersome to solve accurately when P_x/P_1 approaches unity. For that reason it is again of value to note that by performing a series expansion by the binomial

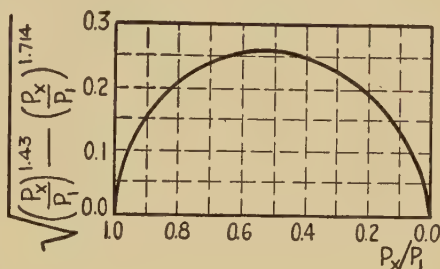


FIG. 43

theorem on the term $\frac{k}{k-1} \left[\left(\frac{P_x}{P_1}\right)^{2/k} - \left(\frac{P_x}{P_1}\right)^{(k+1)/k} \right]^4$ and throwing out

⁴ Thus, writing for $\frac{P_x}{P_1}$ its equivalent $\left(1 - \frac{P_1 - P_x}{P_1}\right)$,

$$\begin{aligned} & \frac{k}{k-1} \left[\left(\frac{P_x}{P_1}\right)^{2/k} - \left(\frac{P_x}{P_1}\right)^{(k+1)/k} \right] \\ &= \frac{k}{k-1} \left[\left(1 - \frac{P_1 - P_x}{P_1}\right)^{2/k} - \left(1 - \frac{P_1 - P_x}{P_1}\right)^{(k+1)/k} \right] \\ &= \frac{k}{k-1} \left[\left\{ 1 - \frac{2}{k} \frac{P_1 - P_x}{P_1} + \frac{1}{k} \left(\frac{2}{k} - 1\right) \left(\frac{P_1 - P_x}{P_1}\right)^2 - \dots \right\} \right. \\ & \quad \left. - \left\{ 1 - \frac{k+1}{k} \frac{P_1 - P_x}{P_1} + \frac{k+1}{2k} \left(\frac{k+1}{k} - 1\right) \left(\frac{P_1 - P_x}{P_1}\right)^2 - \dots \right\} \right] \\ &= \frac{k}{k-1} \left[\left(\frac{-2}{k} + \frac{k+1}{k}\right) \frac{P_1 - P_x}{P_1} + \left(\frac{2-k}{k^2} - \frac{k+1}{2k^2}\right) \left(\frac{P_1 - P_x}{P_1}\right)^2 + \dots \right] \\ &= \left[\left(\frac{P_1 - P_x}{P_1}\right) - \frac{3}{2k} \left(\frac{P_1 - P_x}{P_1}\right)^2 + \dots \right]. \end{aligned}$$

This relation may be put into a preferable form by subtracting and adding $\left(\frac{P_1 - P_x}{P_1}\right)^2$ and further rearranging as follows:

$$\begin{aligned} & \left(\frac{P_1 - P_x}{P_1}\right) - \frac{3}{2k} \left(\frac{P_1 - P_x}{P_1}\right)^2 \\ &= \left(\frac{P_1 - P_x}{P_1}\right) - \left(\frac{P_1 - P_x}{P_1}\right)^2 - \frac{3}{2k} \left(\frac{P_1 - P_x}{P_1}\right)^2 + \left(\frac{P_1 - P_x}{P_1}\right)^2 \\ &= \left(1 - \frac{P_1 - P_x}{P_1}\right) \left(\frac{P_1 - P_x}{P_1}\right) - \frac{3-2k}{2k} \left(\frac{P_1 - P_x}{P_1}\right)^2 \\ &= \frac{P_x}{P_1} \left(\frac{P_1 - P_x}{P_1}\right) - \frac{1.5-k}{k} \left(\frac{P_1 - P_x}{P_1}\right)^2. \end{aligned}$$

the minor terms a more convenient equation is obtained which will be adequately accurate for any value of P_x/P_1 which is not less than 0.65. The resulting relation is:

$$\begin{aligned}\frac{M'}{A_x} \text{ (ideal)} &= 8.02 \frac{P_1}{\sqrt{RT_1}} \sqrt{\frac{P_x}{P_1} \left(\frac{P_1 - P_x}{P_1} \right) - \frac{1.5 - k}{k} \left(\frac{P_1 - P_x}{P_1} \right)^2} \\ &= 8.02 \frac{1}{\sqrt{RT_1}} \sqrt{P_x(P_1 - P_x) - \left(\frac{1.5}{k} - 1 \right) (P_1 - P_x)^2}; \quad (8a) \\ &= \frac{1.1}{\sqrt{T_1}} \sqrt{P_x(P_1 - P_x) - 0.071 (P_1 - P_x)^2} \text{ for air.}\end{aligned}$$

For values of $P_1 - P_x$ which are not in excess of about 10 per cent of P_1 the second term under the radical becomes negligible and the latter equation may be used in the simplified form

$$\begin{aligned}\frac{M'}{A_x} \text{ (ideal)} &= 8.02 \sqrt{\frac{P_x(P_1 - P_x)}{RT_1}}; \quad (8b) \\ &= 1.1 \sqrt{\frac{P_x(P_1 - P_x)}{T_1}} \text{ for air.}^6\end{aligned}$$

For values of $P_1 - P_x$ which are not in excess of about 1 per cent of P_1 it develops that a wholly negligible error is introduced if in equation 8b P_x/RT_1 is written as P_1/RT_1 , or $1/V_1$, whence for that circumstance the equation may be used in the form

$$\frac{M'}{A_x} \text{ (ideal)} = 8.02 \sqrt{\frac{P_1}{RT_1} (P_1 - P_x)} = 8.02 \sqrt{\frac{P_1 - P_x}{V_1}}. \quad (8c)$$

Fig. 43 serves to corroborate the evidence of Fig. 42a in the feature that for any given initial state of the gas the mass-rate of flow per unit area, M'/A_x , is observed to reach a maximum at a **critical pressure** P_x of about one-half of P_1 , or conversely that the area for a given mass-

⁵ This very useful relation, equation 7a, and the one immediately following, 8b, were developed rationally by S. A. Moss, A.S.M.E. Paper on the Measurement of Flow of Air and Gas, December 1927. Other references might also be quoted.

⁶ An equation of this form was devised in about 1870 by Fliegner, wholly from experimental data on air flow, and has had much use as a convenient approximation under the name of the "Fliegner equation." The Moss development serves to indicate a rational basis for the Fliegner equation and to show the limits within which it may be used with adequate accuracy.

⁷ This equation, and also the second arrangement of equation 7b, will be recognized as essentially those which are used in Hydraulics for predicting the ideal flow rates of a liquid through a nozzle or orifice. For that reason they frequently are designated as the "hydraulic equations."

rate of flow attains a minimum value at that critical pressure. Alternatively, this is again to say that for the ideal reversible expansion and accelerating flow of a gas the stream and the nozzle enveloping the stream must be convergent to a section of minimum area or throat at which the critical pressure exists. Beyond this point the channel must be divergent for further reversible expansion.

Example 4. Provide the algebraic steps for developing equation 8 from the expression immediately preceding it.

Example 5. Recompute the requisite channel areas for the nozzle of example 2, employing the direct relations of equations 8b, 8a or 8 as those become applicable or necessary.

107. The Critical Pressure. — In the light of the foregoing the *critical pressure*, $P_{\text{crit.}}$, may now be defined formally as *that minimum pressure to which an expansive fluid will expand in passage through a convergent channel connecting a higher and a lower pressure region*. It was suggested in the foregoing that, for a given fluid when approaching the channel with negligible velocity and expanding and accelerating reversibly therein, the critical pressure will bear an effectively constant proportion to the supply region pressure. This ratio is known as the **critical ratio**, $P_{\text{crit.}}/P_1$.

The fact of the effective constancy of this ratio under the specified conditions has indeed already been established by the evidence of equation 8. In that equation a minimum value of A_x/M' will evidently occur when the variable function $[(P_x/P_1)^{2/k} - (P_x/P_1)^{(k+1)/k}]$ is maximum, and it is further evident that the value of P_x/P_1 at which the function reaches its maximum can depend only on the value of k for the fluid. The magnitude of the ratio which will give the maximum may be readily determined analytically by the common mathematical device of taking the first derivative of the function with respect to the variable P_x , setting the derivative equal to zero and solving for the resulting value of P_x/P_1 . Thus, noting that in the derivation P_1 is a constant and k may be taken as effectively such,

$$\frac{d[(P_x/P_1)^{2/k} - (P_x/P_1)^{(k+1)/k}]}{dP_x} = \frac{2}{k} \left(\frac{1}{P_1}\right)^{2/k} P_x^{(2-k)/k} - \frac{k+1}{k} \left(\frac{1}{P_1}\right)^{(k+1)/k} P_x^{1/k} = 0.$$

Solving, $P_x/P_1 = \left(\frac{2}{k+1}\right)^{k/(k-1)}$ when $[(P_x/P_1)^{2/k} - (P_x/P_1)^{(k+1)/k}]$ is maximum or A_x/M' is minimum. Thus it is established that, as the pressure at the section of minimum area, or throat, of the nozzle is the

critical pressure,

$$\begin{aligned}\frac{P_{\text{crit.}}}{P_1} &= \left(\frac{2}{k+1}\right)^{k/(k-1)}, \quad \text{or} \\ P_{\text{crit.}} &= P_1 \left(\frac{2}{k+1}\right)^{k/(k-1)}; \\ &= 0.528 P_1 \text{ for air.}^8\end{aligned}\tag{9}$$

If this value of the critical ratio is introduced into equation 8 there is obtained a *special* but very useful form of that equation which will serve to evaluate directly the mass-rate of flow at the point at which the critical pressure exists, that is, at the throat of a convergent-divergent nozzle. Thus, performing the substitution,

$$\begin{aligned}\frac{M'}{A_{\text{throat}}} \text{ (ideal)} &= 8.02 \sqrt{\frac{k}{k-1}} \frac{P_1}{\sqrt{RT_1}} \sqrt{\left(\frac{2}{k+1}\right)^{2/(k-1)} - \left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}} \\ &= 8.02 \frac{P_1}{\sqrt{RT_1}} \sqrt{\frac{k}{k-1} \left(\frac{2}{k+1}\right)^{2/(k-1)} \left[1 - \left(\frac{2}{k+1}\right)^{\left(\frac{k+1}{k-1} - \frac{2}{k-1}\right)}\right]} \\ &= 8.02 \frac{P_1}{\sqrt{RT_1}} \sqrt{\frac{k}{k-1} \left(\frac{2}{k+1}\right)^{2/(k-1)} \frac{k-1}{k+1}} \\ &= 8.02 \frac{P_1}{\sqrt{RT_1}} \sqrt{\frac{k}{k+1} \left(\frac{2}{k+1}\right)^{2/(k-1)}}; \\ &= 0.532 P_1 / \sqrt{T_1} \text{ for air.}\end{aligned}\tag{8x}$$

Similarly, substituting the value of the critical ratio in the general velocity and specific volume relations of equations 7 and 6 respectively, special but direct relations are evolved for the velocity and volume at the throat, or

$$\begin{aligned}U_{\text{throat, ideal}} &= 8.02 \sqrt{RT_1} \sqrt{\frac{k}{k-1} \left[1 - \left(\frac{2}{k+1}\right)\right]} \\ &= 8.02 \sqrt{RT_1} \sqrt{\frac{k}{k+1}}^9 \\ &= 44.7 \sqrt{T_1} \text{ for air,}\end{aligned}\tag{7x}$$

⁸ Referring to Fig. 43 it is evident that for air M'/A_x changes relatively gradually with changes of P_x/P_1 in the vicinity of 0.5. The same phenomenon would appear with virtually all of the gases, and the vapors as well. For that reason a high degree of exactness in the evaluation of the critical ratio is not essential, whence it is not an unusual practice to take it as effectively equal to 0.5.

⁹ By equation 14 of Art. 87 this relation may be expressed in terms of T_{throat} , giving the relation $U_{\text{throat, ideal}} = \sqrt{32.17 kRT_{\text{throat}}}$. By any general physics text, the velocity of any adiabatic compression waves such as those of sound, in a compressible gas, is $\sqrt{32.17 kRT}$. The two relations are evidently parallel, whence the critical velocity is frequently referred to as the "acoustic velocity."

and

$$V_{\text{throat, ideal}} = V_1 \left(\frac{k+1}{2} \right)^{1/(k-1)}; \quad (6x)$$

$$= 1.58 V_1 \text{ for air (and other diatomic gases).}$$

In the preceding article, in connection with the second summarization of observations on the results of example 2, it was remarked that "the throat area will be determined independently by the initial state, the desired mass-rate and *the critical pressure*." In view of the fact that for a given gas the critical pressure depends only on the initial pressure it now develops that a more accurate or illuminating statement would be that *the throat area will be determined wholly by the initial state and the desired mass-rate of flow*. Conversely, *a given throat area and initial state will establish the mass-rate of flow obtainable through the nozzle, irrespective of the discharge region pressure so long as that is equal to or less than the critical*.

Example 6. For the data of example 2 compute by the above relations the pressure, specific volume, velocity and requisite area at the throat and compare with the previous results.

Example 7. The following table gives miscellaneous data for various nozzles. In the table the subscript 2 refers to the exit section of the nozzle (i.e., at entry to the discharge region) and in those instances in which a convergent-divergent nozzle is required the subscript *t* refers to the throat section. Compute all results as indicated by the blank spaces in the table, except as any may not be pertinent. Take dry air as the gas and regard the fluid as approaching the nozzle with negligible velocity and the flow to be frictionless. Use the simplified forms of the several equations when their use is permissible.

	(a)	(b)	(c)	(d)	(e)	(f)	(g)
p_1 , lb. per sq. in. abs.	150	150	150	150	150	150	200
t_1 , deg. Fahr.	100	100	100	100	100	100	100
V_1 , cu. ft. per lb.							
p_2 , lb. per sq. in. abs.	148	115	100	79	50	15	15
V_2 , cu. ft. per lb.							
V_t , " "							
U_2 , ft. per sec.							
U_t , " "							
A_2 , sq. ft.	.01	0.01	.01			.0193	
A_t , " "					.01		
M' , lb. per sec.				4.85			10.0

108. Flow of a Gas through an Orifice (ideal). — In investigating the characteristics of the flow of a gas through an orifice it is to be recalled (Art. 104) that essentially the orifice is simply an aperture with a more or less well-formed entrance section but always without a divergent downstream portion. In this aspect an orifice is virtually equivalent

to any part or the whole of the convergent portion of a nozzle, and thus it would appear that the orifice should be able to do all that the convergent section of the nozzle may do, but could do no more. More concretely, the orifice may provide adequately for the (ideally) *reversible* expansion of the fluid down to any pressure which exceeds or equals the critical pressure but not for further reversible expansion to any lesser pressure.

As a result of these features of the flow through an orifice two characteristic conditions of flow are to be considered. These may be classified as follows.

Condition A. — *Flow to a discharge region which is at a pressure greater than or equal to the critical pressure.* As in this case a complete and ideally a reversible expansion to the discharge region pressure may be accomplished by and “in” the orifice the ideal mass-rate of flow may be computed directly by the preferable or permissible one of the equations 8, 8a, 8b, or 8c (Art. 106), or by equation 8x (Art. 107) *only* if the discharge region pressure should chance to equal the critical pressure. Quite as was the case with the nozzle, for a given size of orifice the ideal mass-rate of flow will depend on both the supply pressure and the discharge region pressure. The ideal exit velocity may be computed by the equations 7, 7a or 7b (Art. 106).

Condition B. — *Flow to a discharge region which is at a pressure less than the critical pressure.* For this condition the orifice follows exactly the same unique law of flow as does the nozzle, that (Art. 107) the ideal mass-rate of flow depends solely on the throat (orifice) area and the initial state of the fluid and is wholly independent of the discharge region pressure so long as that is less than the critical. This unique phenomenon is again due to the fact that the critical pressure is the minimum pressure to which a compressible fluid will expand in a converging stream, whence the pressure at the throat of an orifice persists at the critical pressure irrespective of the reduced pressure beyond the throat.

The fact of the persistence of the critical pressure at the throat of an orifice in spite of a reduced pressure in the discharge region may be readily verified by an exploration of an orifice with a “search-tube,” or by a pressure connection to the throat. In this connection it is to be remarked that obviously a complete expansion of the fluid from the critical to the discharge region pressure must unquestionably occur eventually. In a nozzle it occurs in an orderly and (ideally) reversible manner in the divergent section. With an orifice it occurs in the open zone or region directly after the orifice but, lacking the benefit of the suitably formed divergent channel, the expansion is largely in lateral

directions and takes place in a markedly turbulent, semi-explosive, and thoroughly irreversible manner without serving to develop additional unidirectional velocity and kinetic energy in the stream.

By reason of the critical pressure phenomenon *the ideal mass-rate of flow may be computed directly by equation 8x* (Art. 107). A possible alternative procedure might be the use of equations 8 or 8a but only with the P_x of those equations taken as the critical pressure. However, it will be recalled that equation 8x is in fact only the special form taken by equation 8 when $P_x = P_{\text{crit}}$.

Example 8. Compute the ideal rate of air flow through an orifice of 1-in. diameter from a region at 100 lb. per sq. in. abs. and 70° F. to a discharge region at a pressure of (a) 80, (b) 60, (c) 50, (d) 30 and (e) 15 lb. per sq. in. abs. Draw a graph of the rates of flow versus the discharge region pressure.

Example 9. For the supply conditions and orifice size of example 8 compute the ideal rate of flow if the pressure difference across the orifice were (a) 18 in. of water, (b) 18 in. of mercury. (0.302; 1.07 lb. per sec.)

Example 10. Compute the ideal exit velocity and the ideal mass-rate of discharge to atmosphere for helium (see Table VII, Art. 81) from a tank which is at 70° F. and a pressure of 2 in. of water above atmospheric, discharge taking place through a 2-in. diameter orifice. Also compute the ideal volume-rate of flow. What would be the approximate maximum tank pressure, in inches of water, for which the use of equations 7b and 8c would be permissible?

(254 ft. per sec.; 0.0575 lb. per sec.; 331 c.f.m.; 4 in.)

109. Influence of Approach Velocity. — For reasons of convenience the relations of equations 7 to 9 were developed for the circumstance of a negligible velocity U_1 in the approach channel to a nozzle or orifice. It remains to consider the influence of approach velocity and the means by which it may properly be accounted for.

Two procedures are available. The one to which reference has already been made in Art. 105 requires only the introduction of the "correction factor for initial velocity," equation 4 of that article, when that factor departs sufficiently from unity to require its consideration. More concretely, the procedure is in general that of performing the required computations without attention to the initial velocity by the use of Equations 7 or 8 (including the simplified and special forms of those equations), and then the multiplication of the results by the correction factor. Alternatively, the correction factor may be introduced directly in those equations quite as it was incorporated in equation 3 of Art. 105.

For convenience the correction factor is repeated herewith, and also is presented in several modified forms, to wit:

$$\text{Correction factor for initial velocity} = \sqrt{\frac{1}{1 - (A_x/A_1)^2 (V_1/V_x)^2}}, \quad (4)$$

or in terms of P_1 and P_x , since $P_x V_x^k = P_1 V_1^k$,

$$= \sqrt{\frac{1}{1 - (A_x/A_1)^2 (P_x/P_1)^{2/k}}} \quad (4a)$$

Moss has shown that for values of P_x/P_1 in excess of 0.5 the factor may be modified to the more convenient form

$$\text{Correction factor for initial velocity} = \sqrt{\frac{1}{1 - (A_x/A_1)^2 \left[1 - \frac{2}{k} \left(\frac{P_1 - P_x}{P_1} \right) \right]}}. \quad (4b)$$

Finally if $(P_1 - P_x)/P_1$ does not exceed about 1 per cent the factor may be further simplified to the form

$$\text{Correction factor for initial velocity} = \sqrt{\frac{1}{1 - (A_x/A_1)^2}}. \quad (4c)$$

The real value of the factor will always exceed unity but in many practical cases it will approach so closely to unity as to permit its neglect. No positive specification may well be set down as to the concrete circumstances under which it may be neglected but it may be suggested that in general if the area ratio A_x/A_1 does not exceed about 0.1 neglect is permissible.¹⁰

An alternative method of procedure in accounting for approach velocity depends upon the use of a device which is considered subsequently, the **impact tube** (see Art. 112). Deferring the development of the principles and use of that device, it may be remarked here that the tube is simply an open-ended one which shall be so located in a stream that the plane of the open end is at right angles to the direction of flow of the stream, with the other end connected outside of the stream to a suitable pressure gage. It may be shown that if, instead of measuring the true

¹⁰ For the rather unusual circumstance of flow to a region which is at or below the critical pressure, and through a nozzle or orifice the throat diameter of which is in excess of about one-half of the approach section diameter, the critical ratio will increase sensibly above that given by equation 9 (Art. 107). Also the throat velocity, specific volume and mass-rate of flow will be correspondingly modified. It may be developed that for other than negligible values of A_{throat}/A_1 the critical ratio for isentropic expansion is expressed exactly by the relation

$$\frac{P_{\text{crit.}}}{P_1} = \left(\frac{2}{k+1} \right)^{k/(k-1)} \left[1 + \frac{k-1}{2} (A_{\text{th.}}/A_1)^2 (P_{\text{crit.}}/P_1)^{\frac{k+1}{k}} \right]^{\frac{k}{k-1}}.$$

At values of $A_{\text{th.}}/A_1$ of 0.6, 0.75 and 0.9 the critical ratio exceeds that as given for air by, respectively, 4, 8.5 and 24 per cent. The mass-rate of flow through the throat increases in about the same proportions.

pressure of the fluid in the approach channel (through a port in the channel wall), the impact tube is employed to determine the **impact pressure**, and if the indicated impact pressure is used as P_1 in the numerical computations based on equations 7 and 8 or the modified forms of those equations, then the results obtained will have automatically been compensated for the influence of the approach velocity.

Example 11. Compute the correction factors for examples 8 and 9 if the orifice of those examples were located in a line of 2 in. diameter.

110. Actual Flow through Orifice and Nozzle (for Gas). — By the analyses of the several preceding articles various relations have been developed which will serve for determining the velocities and mass-rates of flow ideally attainable through the nozzle or orifice. The velocities and rates actually attained will depart from the ideal in greater or less degrees and for several reasons. As the degrees and reasons for this departure may differ somewhat in the two devices, as do also the practical schemes which usually are employed by the engineer for expressing the degree of departure, the two will be considered individually in the following.

Nozzle. — Recalling that the objective of the nozzle is the establishment of a well-formed high-velocity jet it may reasonably be anticipated that the measure by which the performance of a nozzle shall be judged should also be a fairly direct measure of the degree with which the velocity and kinetic energy actually attained approaches that ideally attainable. Such a measure is the **nozzle efficiency**. This the engineer defines as the ratio between (a) the actual gain in kinetic energy as a result of the expansion of the fluid from the initial pressure (and temperature) to the discharge region pressure, and (b) the ideal increase of kinetic energy for an expansion through the same pressure range, or

$$\text{Nozzle efficiency} = e_n = \frac{(U_2^2 - U_1^2)_{\text{actual}}}{(U_2^2 - U_1^2)_{\text{ideal}}} . \quad (10)$$

Several methods are available for ascertaining the efficiency of an actual nozzle, such as by simultaneous measurement of the mass-rate of flow of fluid through it and of the reaction or impact force generated by the jet, but no method has as yet been devised for rationally predicting that efficiency. The development of such awaits further advancement in the science of hydromechanics, whence in lieu of that the engineer must depend largely on prior tests and on experience in selecting suitable nozzle efficiencies for use in design. He finds that the maximum attainable values of these efficiencies range from perhaps 0.95 to 0.98, that they are sensibly higher in simple convergent nozzles than in con-

vergent-divergent ones, that in a divergent portion efficiency is decreased both by increased angle of divergence and by increase of axial length (whence an optimum compromise must be made in these two contraventional features), that the efficiency of a divergent nozzle is materially decreased by failure to provide an exit area which corresponds properly to the throat area and the discharge region pressure, et cetera.

All of these influences must be considered by the designing engineer but further attention will not be given to them here. However, presuming that a prior estimate of a fair and reasonable efficiency may be made, it is fitting that the thermodynamic features of the actual nozzle design should be considered.

The attainment of efficiencies less than unity is obviously due to irreversibility occasioned by the existence of fluid friction and turbulence during the adiabatic expansion through the nozzle. This in turn predicates an increase of entropy of the fluid instead of the isentropic state change which is characteristic of the ideal reversible expansion. It is important that the influences of this entropy increase of the gas be recognized.

Referring to equation 6a (Art. 80) it becomes evident that, for a given value of P_2/P_1 , an increase of entropy must be accompanied by a greater magnitude of T_2 than would exist for an isentropic expansion. It follows further that, as for a gas $(H_1 - H_2) = c_p(T_1 - T_2)$, the final enthalpy H_2 must also exceed that which would have resulted from isentropic expansion. Finally, by a parallel analysis of equation 7a (Art. 80), it is again evident that for a given value of P_2/P_1 the final specific volume V_2 must exceed the volume ideally attained.

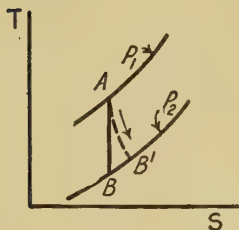


FIG. 44

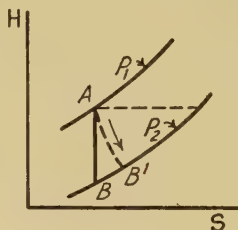


FIG. 45

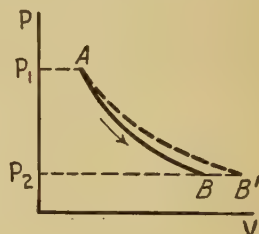


FIG. 46

These significant conclusions are both corroborated and illustrated in Figs. 44, 45, and 46. In Figs. 44 and 45, which are drawn to T - S and H - S coordinates respectively, the typical constant pressure lines for a gas as those appear on such coordinates are represented by lines P_1 and P_2 (see also Fig. 31, Art. 91). An ideal isentropic expansion is represented in those figures by a line such as AB , whereas a state change

of increasing entropy from the same initial state to the same final pressure is depicted by a line such as AB' . The greater final temperature and enthalpy occasioned by the irreversibility are evident. Similarly in Fig. 46 a reversible adiabatic state change is depicted on P - V coördinates by the line AB , and the actual irreversible adiabatic state change by line AB' .

For use in the design of an actual nozzle no composite flow equation parallel to equation 8 of Art. 106 may be readily formulated. Instead, the preferable procedure is that of individual computation of the actual specific volume and velocity of the gas upon progressive expansion through the nozzle and the use of these in the Continuity Equation, quite as in the solution of example 2.

To ascertain the actual specific volume at any section x after expansion to pressure P_x , let $e_{n,x}$ represent the efficiency of the portion of the nozzle preceding the section, and note that

$$\begin{aligned} e_{n,x} &= \frac{(U_x^2 - U_1^2)_{\text{actual}}}{(U_x^2 - U_1^2)_{\text{ideal}}} = \frac{(H_1 - H_x)_{\text{actual}}}{(H_1 - H_x)_{\text{ideal}}} \text{ (by Eq. 1, Art. 105).} \\ &= \frac{(P_1 V_1 - P_x V_x)_{\text{actual}}}{(P_1 V_1 - P_x V_x)_{\text{ideal}}} \text{ (by Eq. 3a, Art. 79)} \\ &= \frac{1 - (P_x/P_1) (V_x/V_1)_{\text{actual}}}{1 - (P_x/P_1) (P_1/P_x)^{1/k}} \text{ (by Eq. 13, Art. 87).} \end{aligned}$$

Solving,

$$\begin{aligned} V_{x,\text{actual}} &= V_1 \frac{-e_n + e_n(P_x/P_1) (P_1/P_x)^{1/k} + 1}{(P_x/P_1)} \\ &= V_1 [(P_1/P_x)^{1/k} e_n + (P_1/P_x) (1 - e_n)] \end{aligned} \quad (11)$$

To express the actual velocity at section x in terms of the initial state, the pressure at x and the nozzle efficiency, from equation 3 (Art. 105), the above enthalpy-efficiency relation and equation 16a of Art. 87,

$$\begin{aligned} U_{x,\text{actual}} &= 223.7 \sqrt{\frac{(H_1 - H_x)_{\text{actual}}}{1 - (A_x/A_1)^2 (V_1/V_x)^2_{\text{actual}}}} \\ &= 223.7 \sqrt{\frac{(H_1 - H_x)_S e_{n,x}}{1 - (A_x/A_1)^2 (V_1/V_x)^2_{\text{actual}}}} \\ &= 8.02 \sqrt{e_n} \sqrt{\frac{k}{k-1} R T_1 \frac{1 - (P_x/P_1)^{(k-1)/k}}{1 - (A_x/A_1)^2 (V_1/V_x)^2_{\text{actual}}}} \end{aligned} \quad (12)$$

This relation is recognized to be parallel to equation 7 of Art. 106 except for the introduction of the efficiency term e_n and for the inclusion

of the correction factor for initial velocity.¹¹ In accordance with the discussions of Art. 109, the inclusion of this latter factor may or may not be necessary. By quite the same treatment which was accorded to equation 7 the numerator of this expression may be modified to a form parallel to equation 7a, in which form however it is again limited in utility to the condition of a value of P_x/P_1 not less than 0.75.

A conclusion which is directly evident from joint attention to equations 12 and 7 is that the ratio between the actual and the ideal velocities virtually equals the square root of the nozzle efficiency. This velocity ratio is known as the **coefficient of velocity**.

$$\text{Coefficient of velocity, } C_v, = \frac{U_{2, \text{actual}}}{U_{2, \text{ideal}}} = \sqrt{e_n} \quad (\text{closely}) \quad (13)$$

We have thus seen that, associated with the greater exit entropy which is occasioned by friction and turbulence, there is a lesser attained velocity and a greater specific volume than would accompany an ideal expansion. Considered in connection with the Continuity Equation ($M'/A = U/V$) the result of these features must be a lesser mass-rate of flow per unit area or, conversely, a greater requisite area for a given mass-rate of flow. Specific evidence of this is obtained by comparison of the results of example 2 of Art. 106 with the accompanying tabulation which presents results of recomputations of the same example, these recomputations having been made upon the basis of a 96 per cent nozzle efficiency.

p_x lb./sq. in.	V_x cu. ft./lb.	U_x ft./sec.	A_x sq. in.
100	1.963	negl.	large
80	2.30 (vs. 2.30)	615 (vs. 626)	1.08 (vs. 1.06)
60	2.84 (vs. 2.82)	910 (vs. 929)	0.90 (vs. 0.88)
55	3.03 (vs. 3.01)	980 (vs. 1000)	0.89 (vs. 0.87)
50	3.25 (vs. 3.22)	1050 (vs. 1070)	0.89 (vs. 0.87)
45	3.51 (vs. 3.47)	1115 (vs. 1140)	0.90 (vs. 0.88)
30	4.72 (vs. 4.64)	1330 (vs. 1360)	1.08 (vs. 0.98)
15	7.83 (vs. 7.61)	1600 (vs. 1630)	1.41 (vs. 1.34)

Scrutiny of the results shows also the definite association of area and pressure still to be evident, as well as the existence of the minimum

¹¹ It may also be taken that the irreversible adiabatic state change of the gas is representable by an exponential relation of the form $PV^n = \text{a constant}$, as was discussed in Art. 88. In that article it was also shown that for an expansion the exponent must be less than k . For such a procedure $V_{x, \text{actual}} = V_1(P_1/P_x)^{1/n}$ and

$$U_{x, \text{actual}} = 8.02 \sqrt{\frac{k}{k-1} RT_1 \frac{1 - (P_x/P_1)^{(n-1)/n}}{1 - (A_x/A_1)^2 (P_x/P_1)^{2/n}}}$$

area or throat at a critical pressure which bears practically the same proportion to the initial pressure as it did in the ideal expansion.

Example 12. Check all of the above results for values of P_x of 55 lb. and of 15 lb. per sq. in. abs. Compute the coefficient of velocity of the nozzle.

Orifice. — Recall again (Art. 104) that the orifice is to be regarded simply as a flow-control or flow-metering device. For that reason in the use of or the analysis of performance of an orifice the interest of the engineer lies not in a kinetic efficiency, as it did with the nozzle, but rather in the degree of approach of the actual mass-rate of flow through the orifice to the ideal rate. The ratio between these rates is known as the **coefficient of discharge** (C_d):

$$\text{Discharge Coefficient } (C_d) = \frac{\text{actual mass-rate of flow}}{\text{ideal mass-rate of flow}}; \quad \text{or} \\ (M'/A_2)_{\text{actual}} = C_d(M'/A_2)_{\text{ideal}}, \quad (14)$$

where the ideal rate is that as computed upon the basis of the actual pressure drop through the orifice and the area of the throat of the orifice.

The magnitude of this ratio or coefficient as attained with the flow of gases will vary quite materially, ranging from about unity downward. The amount of its departure from unity is markedly influenced by the type of orifice employed and also by the discharge-supply pressure ratio which may exist. The following paragraphs will consider broadly the values of the coefficient which are obtained in the use of the two conventional types of orifice.

For an orifice with well-rounded entrance,¹² in which there is thus provided in the orifice itself a suitably shaped convergent passage, the actual mass-rate of flow closely approaches the ideal and discharge coefficients of about 0.96 to 0.99 or more are attained, the lower values corresponding to small orifice diameters and lower rates of flow. Recall in this connection that in the computation of the ideal rate due attention must invariably be given both to the accounting for approach velocity and to the critical pressure feature.

The sharp-edged or thin-plate orifice is particularly simple to make and also easy to install and thus it offers certain advantages. However, this type of orifice unfortunately exhibits a quite perplexing variety of discharge coefficients, the values ranging from a lower limit of about 0.60 up to nearly unity. No attempt will be made here to present the hydrodynamic reasons for this very considerable variation but we

¹² An orifice with a particularly well-formed entrance section is frequently used in flow metering and is very commonly designated as a **flow nozzle**.

may at least indicate certain of the operating conditions which are found to influence the coefficient more outstandingly.

A very convenient and common engineering procedure in flow-metering is the installation of a thin-plate orifice between a pair of flanges in or at the end of a pipe line. When installed in a line a further frequent practice is to measure the approach region pressure and temperature by, respectively, a manometer or gage and a thermometer in the line at about one pipe diameter "ahead of" the orifice plate, and to measure the difference between the approach and discharge region pressures by a manometer connected differentially between the approach region and some point which is "downstream" from the orifice. It is in the selection of the proper point for this downstream pressure tap that a first difficulty appears. Test results indicate that, for the common condition of flow rates such that the downstream pressure is well above the critical pressure, there is a very pronounced variation in the pressure indicated as the tap may be located nearer to or farther from the orifice plate.

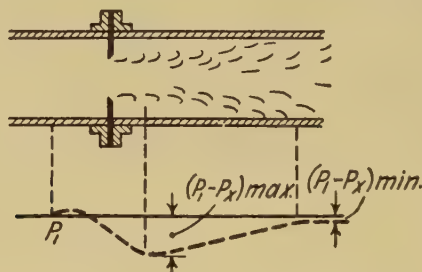


FIG. 47

The typical appearance of the resultant pressure gradient is represented in Fig. 47. In the figure it appears that there is a section of minimum pressure at a short distance from the orifice plate and a subsequent gradual but partial recovery of pressure up to a plane at about five pipe diameters below the plate. This phenomenon is taken as evidence of a convergence and velocity acceleration in the stream lines issuing from the orifice, which convergence continues up to a **vena contracta** at the section of minimum pressure, with subsequent divergence and deceleration as pressure recovery takes place.

The practical results of this condition are that, following the standard procedure of basing the ideal mass-flow computations on the observed pressure and the known orifice area (rather than on the unknown vena contracta area), the computed ideal mass-rate of flow must be definitely affected by the location of the downstream pressure tap. One engineering practice is to attempt to locate that tap at the vena con-

tracta,¹³ for which reason extensive tests have been made for the purpose of ascertaining the proper location and the proper corresponding discharge coefficient for use when the tap is so located. However further difficulties are encountered in this selection of the tap location and also of the coefficient. These difficulties arise by reason of the facts (a) that the vena-contracta location depends materially on the relative diameter of the orifice with respect to the pipe diameter and to some degree on the rate of flow and (b) that the coefficient varies moderately with the relative orifice size and increases very materially with the rate of flow (this in addition to the influence of approach velocity as that enters in the correction factor for initial velocity, Art. 105). To illustrate, when the flow rate is such as to give a pressure difference not exceeding about 10 per cent of the absolute upstream pressure the vena contracta occurs at a distance from the orifice which equals some 0.4 to 0.6 of the pipe diameter and the coefficient will be about 0.61.¹⁴ In distinction to this, however, when the discharge region pressure is equal to or less than the critical (at the maximum flow rate) there is evidence that the vena contracta area approaches that of the orifice and the discharge coefficient becomes about 0.80.

It should be evident that in spite of its apparent simplicity and utility an accurate use of the sharp-edged orifice will require either actual calibration or reliable and fairly extensive information concerning its idiosyncrasies.

Example 13. Selecting reasonable values for the discharge coefficient estimate the actual rates of flow through the orifice of example 10, if (a) a rounded-entrance orifice, (b) a sharp-edged orifice.

Example 14. It is desired to test an air compressor the capacity of which is expected to be about 1000 cu. ft. of "free air" per minute (the volume of air at atmospheric temperature and pressure which enters the compressor per minute). It is proposed to use a sharp-edged orifice in the discharge line, in which the pressure is 80 lb. per sq. in. gage and the temperature about 100° F. Select a suitable size orifice for a specification that the downstream pressure tap shall be located at the point of minimum pressure and that the pressure difference across the orifice shall not exceed 2.65 in. Hg. (Neglect pipe velocity in this preliminary estimate.)

(Diam. = $2\frac{1}{4}$ in.)

¹³ Sometimes the procedure employed is that of placing the downstream pressure tap at the point of maximum pressure recovery. In this event the observed pressure difference measures virtually the energy degradation due to the friction and turbulence accompanying the flow, rather than the kinetic energy gain in the orifice. However, the observed difference might be used in the flow formulas and a pseudo ideal mass-rate of flow computed. The actual flow materially exceeds this computed rate and thus apparent or pseudo discharge coefficients are obtained which definitely exceed unity.

¹⁴ For more complete data, discussions and bibliography see A.S.M.E. Research Reports on Fluid Meters.

111. The Venturi Meter. — A device which is very generally used in the metering of the flow of liquids through a pipe line and has distinct utility in the metering of gases consists essentially of an orifice which is formed with a smoothly convergent entrance section and is followed directly by a gradually tapering **diffuser** section which diverges to the original diameter of the entrance to the orifice and thus to the diameter of the pipe line in which the device is located. Such a device is known as a **Venturi tube or meter**. Its form is represented in Fig. 48a. In that figure there is also indicated the character of the velocity and pressure changes occurring enroute through the tube and the provisions by which the pressure P_1 , temperature T_1 , and the pressure difference ($P_1 - P_2$) between entrance and throat may be measured. These data constitute all of the data necessary for the use of the meter.

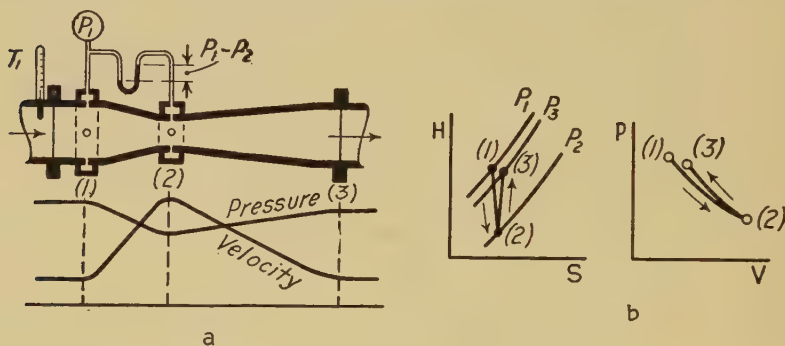


FIG. 48. Venturi meter

The action in the meter is that of an adiabatic (and ideally reversible) expansion and acceleration of the gas while enroute between sections 1 and 2, followed by a deceleration and adiabatic recompression in the diffuser between sections 2 and 3 to nearly the entrance pressure. The over-all pressure drop is only that due to the friction and turbulence encountered through the meter.¹⁵ The several successive state changes of the fluid enroute are typified by curves 1-2 and 2-3 on the H - S and

¹⁵ The meter evidently has the general form of a convergent-divergent nozzle. However, its action and conditions of use are wholly distinctive. Recall that such a nozzle is employed under conditions where by extraneous circumstances the discharge region pressure P_3 is maintained at a pressure less than the critical and the throat pressure is the critical, whereas the Venturi tube operates under conditions of a P_3 which very nearly equals P_1 . It may be remarked in this connection however, that again the critical pressure is the minimum which might be attained in the throat of the tube and that therefore the attainment of that pressure marks the limiting flow capacity of the meter. Recall that, as shown in footnote 10, the critical pressure ratio depends also on the area ratio of the meter.

P - V coordinates of Fig. 48b. Quite as is the case with the orifice the mass-rate of flow through the meter is directly dependent on the pressure difference between entrance and throat or, conversely, the greater the rate of flow the greater the pressure difference.

Recognizing that the initial section of the meter is in all respects simply an orifice with well-formed approach section, differing from an orifice of that character only in the features that special provisions are made for measuring the entrance and throat pressures and that the approach velocity is rarely negligible, it follows that *for computations of the ideal rates of flow through the meter the standard velocity relations of equations 7, 7a, or 7b may be employed or the standard mass-rate relations of equations 8, 8a, 8b, or 8c, except that the correction factor for initial velocity of equations 4, 4a, 4b, or 4c will commonly need to be introduced.* The meter is usually used at such rates of flow that the use of equations 7a or 7b, 8b or 8c, and 4b or 4c are either permissible or may be virtually necessary. The coefficient of discharge of a well-constructed meter will range from about 0.96 for one with an orifice diameter less than about 1 in. up to 0.99 or more for a meter with orifice diameter of 12 in. or greater.

Example 15. A 10-in. $\times$ 6-in. Venturi meter delivering air shows a differential pressure of 9.2 in. of mercury with a pipe pressure of 5.1 lb. per sq. in. gage and a pipe temperature of 85° F. Compute the throat velocity, the mass-rate of flow and the volume rate in the pipe. (700 ft. per sec.; 11.5 lb. per sec.; 7000 c.f.m.)

112. The Impact Tube and Pitot Tube. — A small-bore, open-ended tube (usually with a chamfered or smoothly rounded tube wall at the open end) which is so placed in a moving stream of a fluid that the open

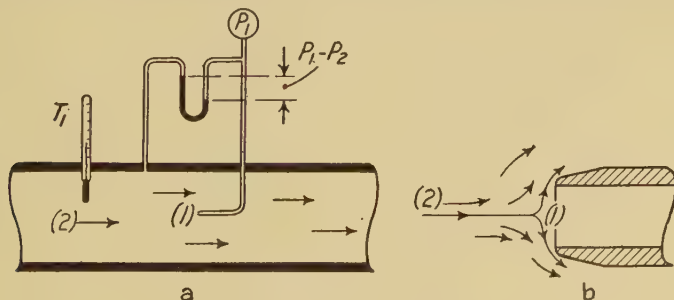


FIG. 49. Impact tube

end faces squarely into the stream is known as an **impact tube**. The form and a customary arrangement for the use of such a tube are indicated in Fig. 49a. It will be observed in the figure that the outer end of the tube is closed by a manometer or other pressure-measuring device so that there may be no flow of the fluid through the tube.

The action of the tube becomes evident if in Fig. 49b one regards the thread of the stream which is approaching along the line of the tube axis. A section in that stream we shall designate as section 2. As such a thread of the fluid approaches closely to the tube end it must necessarily diverge and decelerate, until upon reaching a section 1 directly at the tube mouth it has decelerated to a very negligible velocity. This virtually complete deceleration of the stream thread is accompanied by a practically isentropic compression of the fluid in the diverging stream, in a manner parallel to the deceleration and recompression of the stream in the diffuser section of the Venturi tube. The fact of the compression is well evidenced by the excess pressure which will be indicated by a manometer which is connected on the one side to the outer end of the impact tube and on the other side is so connected that only the pressure of the fluid in the free stream will act.

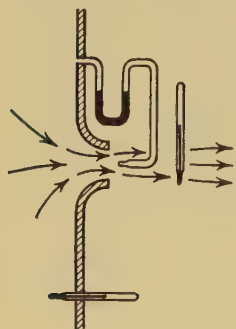


FIG. 50

The action in the stream line approaching the tube mouth would thus appear to be the effective reverse of the pressure decrease and acceleration which occurs in the flow from a region of negligible velocity to and through an orifice. This conclusion is well validated by experimental evidence which is secured by placing an impact tube in a gas stream which has been generated by an expansion through an orifice from a region of

superior pressure but negligible velocity. An arrangement for securing such evidence is indicated in Fig. 50. Two significant phenomena will appear from the general experiment. These are that

- (a) the **impact pressure** which is registered in the tube will be found to equal practically exactly that of the region from which the fluid is flowing, as shown by the absence of pressure difference in the differentially-connected manometer, and
- (b) if thermometers are located with the bulb of one in the original higher pressure region and the bulb of the other in the jet the readings of the two will differ by only a small amount, evidencing that the thermometer in the stream likewise registers an **impact temperature** which is virtually that of the fluid *after a compression against the bulb and an almost complete deceleration*, this instead of the reduced air temperature which must have resulted from the prior expansion in the flow nozzle.¹⁶

¹⁶ The failure of the impact temperature exactly to attain the original temperature is due to radiation from the thermometer bulb to the cooler air in the surrounding

Both of these phenomena will be found to maintain through any pressure range within the critical.

By virtue jointly of the physical action in the stream line approaching the tube and of the further experimental evidence secured with the impact tube and flow nozzle one is justified in applying to the stream-line the usual energy equation for reversible steady-flow:

$$\frac{U_2^2}{64.34} + JH_2 = JH_1, \quad \text{or} \quad U_2 = \sqrt{64.34 J(H_1 - H_2)_s},$$

where Subscript 2 refers to the lower pressure but higher velocity section in the approaching stream, and

Subscript 1 refers to the higher pressure but negligible velocity section at or in the tube mouth.

By further analysis of this energy relation along lines which are wholly identical with those already followed in Art. 106 the velocity relations of equations 7, 7a, or 7b would result. These may properly be applied therefore to the impact tube, interpreting the symbol U_x in those relations as the stream line velocity U_2 . Also in those relations the item T_1 would evidently be the *impact* temperature secured with thermometer bulb exposed to the stream, P_1 would be the impact tube pressure or *impact* pressure and P_x would be the so-called *static* pressure of the region in which a free stream is flowing or that obtained through a flush opening in the wall of a duct through which a confined stream is passing.

It will have become evident that strictly speaking the impact tube measures the velocity only of that small portion of a stream which is approaching directly in line with the tube mouth. This condition gives rise to at least two characteristic conditions which must be recognized in the use of the tube.

One of these conditions is that of a stream in which the velocity is effectively the same at all points across the stream section, as it would be in the discharge from a well-rounded orifice such as the one illustrated in Fig. 50. For such an orifice or such a condition the **mass-rate** of flow from the orifice may be determined by combining the Continuity Equation with the above velocity relations. Exactly such a combination was made in developing equations 8, 8a, 8b, and 8c (Art. 106). It follows therefore that those equations may be employed for computing

stream and to the region of somewhat depressed pressure and temperature which exists directly aft of the bulb. The error is quite negligible. Dr. S. A. Moss of the General Electric Co., who has done much valuable research work in connection with the impact tube and other velocity meters, finds that the temperature deficit depends primarily on the stream velocity and may be closely expressed by the relation $(T_{\text{impact}} - T_1) = 0.0005 U^{1.5}$.

from impact tube and impact thermometer data the mass-rate of flow from a well-rounded orifice, interpreting T_1 , P_1 , and P_x in those equations in the manner noted above and regarding A_x as the orifice area.

A second condition under which an impact tube may be used is that of flow through a duct, within which the velocity may vary materially from point to point across the duct and an *average* velocity must therefore be ascertained in order to determine the volume-rate or mass-rate of flow through the entire duct. For finding the average velocity it obviously is necessary to measure the local velocities at various selected points about the duct. Several schedules are available for doing this but, for a circular duct, probably the most advantageous one is to measure the velocities at selected points across one or more diameters of the duct, the objective in the selection of the points being that of adequately

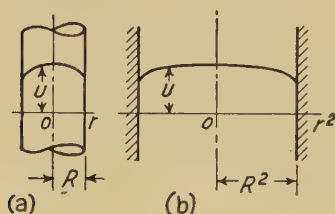


FIG. 51

determining the character of the velocity distribution. The curve of Fig. 51a shows a typical distribution as determined by such a *traverse* of the duct. If the measured velocities are plotted against the *square* of the distance of the impact tube from the center-line of the duct, as in Fig. 51b, the mean ordinate of the resulting curve measures the average velocity.¹⁷ An alternative procedure, but one which is not adequate in the case of an erratic and non-symmetrical velocity distribution, requires velocity determinations at distances of $R\sqrt{1/(2N)}$, $R\sqrt{3/(2N)}$, $R\sqrt{5/(2N)}$, . . . and $R\sqrt{(2N-1)/(2N)}$ from the center line, where R = interior radius of the duct and N = the number of tube locations on each side of the center-line. If the traverse is reasonably symmetrical and a sufficient number of points are taken an arithmetical average of the velocities so determined equals closely the effective average velocity in the duct.

It has been established in the foregoing that the impact pressure and temperature registered in a flowing stream are those which would exist if the stream were brought to rest by a reversible adiabatic compression

¹⁷ The volume-rate of flow through any semi-circular ring of differential width dr and radius r equals $U(\pi r dr)$, which equals $U\frac{\pi}{2}d(r^2)$. The total volume-rate through a circular duct of radius R equals $\int_{-R^2}^{+R^2} U\frac{\pi}{2}d(r^2)$, the area of the duct equals πR^2 and the average velocity thus equals $\int_{-R^2}^{+R^2} U d(r^2)/2R^2$. The numerator of this fraction is graphically the area under a curve of U plotted against (r^2) , the denominator is the total length of the base line and the fraction is thus the mean ordinate of the curve.

or, conversely, they are the pressure and temperature in a *zero* velocity region an expansion from which would give the existing stream velocity. It follows from the latter that if one were to measure the impact pressure and temperature in a stream approaching a nozzle or orifice, instead of the static data, one might thereby make an automatic correction for the approach velocity in the stream. This condition has been mentioned in Art. 109, in which it was indicated that if the impact pressure and temperature were employed as the P_1 and T_1 in the various flow equations of those devices no correction factor for approach velocity would need to be introduced. That contention is thus validated.

An industrial instrument known as the **Pitot tube** which is very frequently employed for velocity determinations in a moving stream consists essentially of a pair of small concentric tubes, the inner one of which forms an impact tube and the outer one of which is closed at the upstream end but is provided with small openings through its walls so located that the static pressure in the stream may be communicated accurately to the interior of the tube. The instrument thus provides for the measurement of both the static pressure and the impact pressure with but a single opening into the duct.

Example 16. A Pitot tube traverse of a 12-in. circular duct through which air at atmospheric pressure and 70° F. is flowing showed the following schedule of values of $(P_1 - P_2)$ at the indicated points across one diameter of the duct.

	Above center					(center line)	Below center				
Distance from center of duct, in.	5.7	5.0	4.25	3.3	1.9	(0.0)	1.9	3.3	4.25	5.0	5.7
$P_1 - P_2$, inches of water	3.7	4.1	4.4	4.6	4.8	(4.8)	4.8	4.8	4.7	4.3	3.9

Compute the average velocity in the duct, the volume-rate of flow through the duct (c.f.m.), and the ratio of mean to center velocity. Draw the traverse to scale and ascertain a point at which the local velocity equals the average.

113. Summary, Flow of Gas Through the Nozzle, Orifice et Cetera. — The state change taking place in a fluid which is enroute through a nozzle is effectively an adiabatic one and is ideally a reversible and isentropic one. The energy equation for such a process thus becomes

$$\frac{U_x^2 - U_1^2}{64.34} = J(H_1 - H_x) \text{ for any character of adiabatic flow, and}$$
$$= J(H_1 - H_x)_S \text{ for an ideal isentropic flow.} \tag{1}$$

For practical use of these equations in connection with the analysis of flow through a nozzle it becomes necessary to eliminate one of the unknowns U_x or U_1 . It is usually more advantageous to eliminate the latter. That may be accomplished by introduction of the Continuity

Equation,

$$M' = \frac{U_1 A_1}{V_1} = \frac{U_x A_x}{V_x}. \quad (2)$$

Thereby the energy equation may be modified to the form

$$U_x = 8.02 \sqrt{\frac{J(H_1 - H_x)}{1 - (A_x/A_1)^2 (V_1/V_x)^2}} \text{ for any adiabatic flow, or } \quad (3)$$

$$= 8.02 \sqrt{\frac{J(H_1 - H_x)_S}{1 - (A_x/A_1)^2 (V_1/V_x)_S^2}} \text{ for ideal flow} \quad (5)$$

In these relations the denominator under the radical is to be regarded as constituting simply a correction term for initial velocity, or

$$\text{Correction factor for initial velocity} = \sqrt{\frac{1}{1 - (A_x/A_1)^2 (V_1/V_x)^2}} \quad (4)$$

Inclusion of this factor may or may not be necessary as the velocity and kinetic energy of the approaching fluid is or is not negligible. Also any necessity for the use of the factor may be obviated if a pseudo initial state for the approaching fluid is ascertained in terms of the impact pressure and impact temperature of the approaching stream.

For the ideal flow and expansion of a gas (through a sufficiently moderate temperature range that the specific heat ratio may be taken as effectively constant) the specific volumes at successive pressures may be computed by the conventional relation

$$V_{x,S} = V_1(P_1/P_x)^{1/k}. \quad (6)$$

Again for ideal flow, and for convenience omitting the correction factor for initial velocity, the velocities obtained by a gas become

$$U_{x, \text{ideal}} = 8.02 \sqrt{\frac{k}{k-1} RT_1 [1 - (P_x/P_1)^{(k-1)/k}]}. \quad (7)$$

This velocity relation may be expressed in several modified forms which are convenient and permissible in certain limited circumstances, namely:

If $(P_1 - P_x)/P_1$ does not exceed about 0.10,

$$U_{x, \text{ideal}} = 8.02 \sqrt{RT_1} \sqrt{\frac{P_1 - P_x}{P_1} \left[1 + \frac{1}{2k} \frac{P_1 - P_x}{P_1} \right]}; \quad (7a)$$

If $(P_1 - P_x)/P_1$ does not exceed about 0.01,

$$U_{x, \text{ideal}} = 8.02 \sqrt{RT_1 \frac{P_1 - P_x}{P_1}} = 8.02 \sqrt{(P_1 - P_x) V_1} \quad (7b)$$

For air $8.02 \sqrt{R} = 8.02 \sqrt{53.3} = 58.6$.

By introducing these volume and velocity relations in the Continuity Equation a direct expression is obtained for the mass-rate of flow,

in the form

$$\frac{M'}{A_x} (\text{ideal}) = 8.02 \sqrt{\frac{k}{k-1}} \frac{P_1}{\sqrt{RT_1}} \sqrt{\left(\frac{P_x}{P_1}\right)^{2/k} - \left(\frac{P_x}{P_1}\right)^{(k+1)/k}} \quad (8)$$

This mass-flow relation may likewise be expressed in several modified forms which are convenient and permissible in certain limited circumstances:

If $(P_1 - P_x)/P_1$ does not exceed 0.35,

$$\frac{M'}{A_x} (\text{ideal}) = 8.02 \frac{1}{\sqrt{RT_1}} \sqrt{P_x(P_1 - P_x) - \frac{1.5-k}{k} (P_1 - P_x)^2}; \quad (8a)$$

If $(P_1 - P_x)/P_1$ does not exceed about 0.1,

$$\frac{M'}{A_x} (\text{ideal}) = 8.02 \sqrt{\frac{P_x(P_1 - P_x)}{RT_1}}; \quad (8b)$$

If $(P_1 - P_x)/P_1$ does not exceed about 0.01,

$$\frac{M'}{A_x} (\text{ideal}) = 8.02 \sqrt{\frac{P_1(P_1 - P_x)}{RT_1}} = 8.02 \sqrt{\frac{P_1 - P_x}{V_1}}. \quad (8c)$$

For air $8.02/\sqrt{R} = 1.10$.

If by the joint use of equations 6 and 7 and the Continuity Equation, or by the use of equation 8, one computes the necessary successive areas required to properly accommodate the flow of a gas during reversible expansion and acceleration through a nozzle it is discovered that

- (a) for any given initial state of a particular fluid a particular value of A_x/M' (or M'/A_x) is associated with each successive pressure;
- (b) a succession of areas of decreasing magnitude is adequate for an expansion to a pressure which equals or is in excess of about one half of the initial pressure, whence a convergent channel will provide suitably for an expansion to such a *critical* pressure, or to any greater pressure;
- (c) for a continued reversible expansion to a pressure which is less than the critical pressure a subsequent succession of areas of increasing magnitude is required, whence a convergent-divergent channel of exactly defined proportions is necessary for a reversible expansion to a pressure less than the critical. The plane of smallest area of the convergent-divergent channel is known as the *throat*. At that plane the mass-rate of flow per unit area is the maximum and the throat area fixes the mass-rate of flow obtainable through the nozzle.

By a mathematical analysis of equation 8 it may be shown that for a given gas the critical pressure bears a unique proportion to the initial pressure, namely:

$$\frac{P_{\text{critical}}}{P_1} = \left(\frac{2}{k+1} \right)^{k/(k-1)} (= 0.528 \text{ for air}). \quad (9)$$

For this particular critical value of the pressure P_x equation 8 takes the special form

$$\begin{aligned} \frac{M'}{A_{\text{throat}}} (\text{ideal}) &= 8.02 \frac{P_1}{\sqrt{RT_1}} \sqrt{\frac{k}{k+1} \left(\frac{2}{k+1} \right)^{2/(k-1)}} \\ &= 0.532 P_1 / \sqrt{T_1}, \text{ for air.} \end{aligned} \quad (8x)$$

Equations 6 and 7 also take the special forms,

$$V_{\text{throat, ideal}} = V_1 \left(\frac{k+1}{2} \right)^{1/(k-1)} \quad (6x)$$

$$U_{\text{throat, ideal}} = 8.02 \sqrt{RT_1} \sqrt{\frac{k}{k+1}} \quad (7x)$$

As an orifice fails to furnish a properly formed divergent channel downstream from its throat it is able to provide for reversible expansion only to pressures equal to or in excess of the critical pressure. As a result of this feature of an orifice it is convenient to consider two characteristic conditions under which the flow of a gas through an orifice may occur. These are

Condition A, in which the discharge region pressure is in excess of or equal to the critical pressure. For this condition a reversible expansion to the discharge region pressure may ideally be accomplished by the orifice, whence for computing the ideal mass-rate of flow through the orifice the preferable or permissible ones of equations 8 to 8c may be employed.

Condition B, in which the discharge region pressure is less than the critical pressure. For this condition a gas will expand in the orifice only down to the critical pressure, any further expansion occurring in a thoroughly irreversible manner in the region downstream from the orifice. As a result the ideal mass-rate of flow may be computed by the special relation of equation 8x.

The actual kinetic energy and mass-rate of flow attained by even the best character of nozzle can not equal the ideal. The performance of an operating nozzle is evaluated in terms of its efficiency, that is, the ratio between the actually attained and the ideally attainable kinetic energy. Aside from the effect of reduced nozzle efficiency on velocity it also acts to increase the specific volume of the fluid, both of which

influences join to decrease the mass-rate of flow through a nozzle or to require greater nozzle area for a given specified rate. A good nozzle will exhibit an efficiency of 0.96 or more.

The performance of an orifice is invariably expressed in terms of a coefficient of discharge, that is, the ratio between the actually attained and the ideally attainable mass-rate of flow through the orifice. A rounded-entrance orifice will give discharge coefficients of 0.96 and greater, whereas with a sharp-edged orifice the coefficient may range from 0.60 to nearly unity, the lower coefficients being occasioned by a stream contraction directly after the orifice which appears when the orifice is used at lower rates of flow.

The Venturi meter is essentially an orifice with smooth entrance, with provision made for pressure measurement in the approach channel and directly in the orifice and also with a diffuser section provided downstream from the orifice for the efficient deceleration and recompression of the departing fluid. Equations 8 to 8c are applicable according to circumstances, usually, however, with a necessity for introducing the correction factor for approach velocity.

The impact tube is essentially a point-velocity meter which operates by virtue of the adiabatic compression of a gas stream which occurs in the thread of the stream which is approaching the tube mouth. As the action is the virtual reverse of the adiabatic expansion through an orifice the conventional velocity relations of equations 7 to 7b are applicable. As the tube provides data only on a point velocity a stream must be adequately traversed in order to ascertain its mean velocity.

114. Nozzle Form for the Ideal Expansion of Vapor. — The general energy equations for a nozzle are invariably applicable whether the fluid flowing is a gas or a vapor, whence we may refer immediately to the basic energy equations 1, 2, 3, and 5 (Art. 105). Those equations are repeated here for convenience:

$$\frac{U_x^2 - U_1^2}{64.34} = J(H_1 - H_x) \quad (1)$$

$$M' = \frac{U_1 A_1}{V_1} = \frac{U_x A_x}{V_x} \text{ (Continuity Equation)} \quad (2)$$

$$U_x = 223.7 \sqrt{\frac{H_1 - H_x}{1 - (A_x/A_1)^2 (V_1/V_x)^2}} \quad (3)$$

$$U_{x, \text{ideal}} = 223.7 \sqrt{\frac{(H_1 - H_x)_S}{1 - (A_x/A_1)^2 (V_1/V_x)_S^2}} \quad (5)$$

Quite as it was with the flow of a gas, given the state of the fluid as supplied, the problem of applying these equations in developing either

the ideal or the actual nozzle form is that of ascertaining the simultaneous values of H_x and V_x as expansion proceeds to successively lower pressures. However, as may be anticipated, the detailed procedure in ascertaining these values may differ in the cases of the gas and of the vapor, particularly if the vapor is one for which tables of properties are available. Such tables being available for the vapors of major engineering importance, it is the practice (at least among engineers of the United States) to utilize them, although it will appear subsequently that in considerations of actual flow certain difficulties are encountered in that procedure.

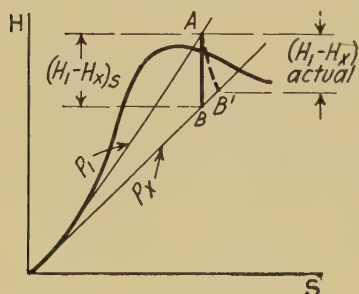


FIG. 52

For the circumstance of the ideal expansion the values of the desired properties are easily obtained by recognizing the isentropic character of the expansion and then following the methods of examples 19 and 20 of Art. 68, using either the vapor tables alone or those jointly with a Mollier (H - S) chart.¹⁸ The appearance of an ideal state change as represented on

H - S coördinates would be that of a

line such as AB in Fig. 52. The essential characteristic of the line is the ideal constancy of the entropy.

The procedure in investigating the nozzle form for ideal expansion is perhaps shown to best advantage by an illustrative example.

Example 17. Determine the characteristic form and dimensions of a steam nozzle which shall provide for the flow and reversible expansion of 2 lb. of steam per second from an initial state of 200 lb. per sq. in. abs. and 420° F. to a discharge region at a pressure of 60 lb. abs. Do this by computing the areas required upon expansion to successive pressures of 160, 120, 115, 109, 85 and 60 lb. Assume the ideal case of reversible adiabatic expansion and also presume negligible entrance velocity.

Solution for P_x of 160 lb. per sq. in.

$$H_1 = 1222.3; V_1 = 2.436; S_1 = 1.5735.$$

$$S_x = S_1 = 1.5735; t_x \text{ (from Superheat Table)} = 376.4^\circ \text{ F.}$$

$$H_x = 1202.7; \Delta H = 19.6; U_x = 223.7 \sqrt{19.6} = 990 \text{ ft. per sec.}$$

$$V_x = 2.894; M'/A_x = 990/2.894 = 342; A_x = 0.00585 \text{ sq. ft.}$$

Solution for P_x of 120 lb. per sq. in.

$$S_x = S_1 = 1.5735; x_x = 1 - \frac{1.5874 - 1.5735}{1.0956} = 1 - 0.0127 = 0.987.$$

$$H_x = 1189.8 - 0.0127 \times 877.4 = 1178.7; \Delta H = 43.6; U_x = 1480.$$

$$V_x = 3.725 - 0.0127 \times 3.707 = 3.678.$$

$$M'/A_x = 1480/3.678 = 401.5; A_x = 0.00498.$$

¹⁸ It is to be remarked that a *Pressure-Entropy* table or chart offers definite advantages in this connection.

The major results at these and the other specified pressures are tabulated below.

Tabular Results, Example 17

P_x (lb. per sq. in.)	t , or x	H_x	ΔH	U_x	V_x	M'/A_x	A_x (sq. ft.)
200	420	1222.3			2.436		
160	376	1202.7	19.6	990	2.894	342	0.00585
120	0.987	1178.7	43.6	1480	3.678	402	0.00498
115	0.984	1175.3	47.0	1530	3.817	402	0.00498
109	0.981	1171.1	51.2	1600	4.001	400	0.00500
85	0.964	1151.1	71.2	1890	4.978	379	0.00528
60	0.943	1124.4	97.9	2210	6.7605	327	0.00612

Example 18. Check the above results for the value of P_x of 115 and compute similar results for a P_x of 14.7 lb. abs. ($U = 3120$, $M'/A = 133$, $A = 0.0150$.)

Scrutiny of the above tabular results indicates the same significant features in the steam nozzle that were apparent and to which attention was directed in connection with the air nozzle of example 2, Art. 106. Restating those features briefly, they are that

(a) For a given initial state a particular enthalpy, specific volume, and area (per unit mass-rate of flow) are again associated with each successive pressure.

(b) The critical pressure phenomenon is again evidenced by the minimum area, or throat, which occurs in the above example upon expansion to about $(115/200 =)$ 57 per cent of the supply pressure.

(c) For expansion to any pressure in excess of the critical or to the critical a convergent passage is proper.

(d) A subsequent divergent passage is required in order to provide properly for a controlled and directed expansion to a pressure which is less than the critical.

(e) For expansion to a discharge region pressure which is greater than the critical the required exit area for a given mass-rate of flow will depend on both the supply state and the discharge region pressure or, conversely, the mass-rate of flow at given supply and discharge region pressures will be proportional to the exit area.

(f) On the contrary, for expansion to a discharge region which is at a pressure equal to or less than the critical the invariable existence of the critical pressure at the throat and a fixed value of the critical ratio will act to cause the required throat area for a given mass-rate of flow to depend only on the state of the fluid supply or, conversely, the mass-rate of flow obtained with a given supply state depends only on the throat area and is independent of the discharge region pressure

(although obviously the proper final exit area for suitable accommodation of the entire expansion must depend also on the discharge region pressure).

One feature in which vapor flow and gas flow will differ slightly is that the critical ratio for a vapor does not appear to be strictly a constant. Thus in the example, in which the steam supply was moderately superheated, the critical ratio appears to have the value of about 0.57. By similar investigation it would be found that the critical ratio for steam would apparently range from about 0.55 for highly superheated steam supply to about 0.58 for saturated steam supply. However this variation does not require much practical concern, due to the very slight change of the cross-sectional area of the nozzle at pressures between about 55 and 60 per cent of the supply pressure.

An important characteristic of this ideal expansion which is clearly indicated in the above tabulation and was evident in Fig. 52, is the progressive condensation of a portion of the vapor with progressive expansion. Quoting specific figures, the reversible adiabatic expansion of the initially superheated supply would require the actual condensation of about 2 per cent of the vapor prior to reaching the throat and about 6 per cent prior to reaching the 60 lb. exit pressure, and example 18 would indicate that 13 per cent should condense if the expansion were continued to atmospheric pressure. In making these quality computations it was inherently presumed that a stable thermal equilibrium was maintained between the liquid and the vapor as the expansion proceeded, whence the expansion would be designated as being of the **equilibrium** type. A further consideration will be given to the possibility of an actual expansion of this type.

115. Actual Expansion of a Vapor. — Any actual expansion through a nozzle is necessarily irreversible by reason of the unescapable friction and turbulence accompanying the high velocity flow. The influence of such irreversibility and the general manner of accounting for it in air nozzle design has been considered in Art. 110. It will similarly be considered for steam nozzle design in the following paragraphs, but before doing so it needs be noted that without question additional causes for irreversibility exist in the case of vapor flow, or more particularly in the case of a vapor which is supplied in a saturated or only slightly superheated state. This additional irreversibility is occasioned by a temporary unstable condition which has been found to exist in the rapidly expanding vapor. The condition is that known as **supersaturation** and is characterized by a retarding or deferring of the partial condensation which we have seen would accompany the equilibrium type of expansion.

In the following there are presented two analyses of the actual expansion of a vapor through a nozzle. The first one disregards the supersaturation phenomenon and is inadequate in that feature but is presented for the reason that it has been for many years the conventional one employed by American engineers, and also because it may be used with success in actual nozzle design if suitable coefficients are employed in connection with it. The second considers briefly the effect of supersaturation. It will also serve to clarify certain perplexing features of the coefficients which are found to apply when the first method is used.

(a) *Actual Expansion, disregarding Supersaturation.* As with the air nozzle, the *efficiency* is also employed as the figure of merit of a vapor nozzle and is again defined (see Art. 110) as

$$\begin{aligned}\text{Nozzle efficiency } (e_n) &= \frac{(U_x^2 - U_1^2)_{\text{actual}}}{(U_x^2 - U_1^2)_{\text{ideal}}} \\ &= \frac{(H_1 - H_x)_{\text{actual}}}{(H_1 - H_x)_S}.\end{aligned}\quad (10)$$

Rearranging,

$$H_{x,\text{actual}} = H_1 - e_n(H_1 - H_x)_S \quad (15)$$

As the $H_{x,\text{actual}}$ of equation 15 is that attained after expansion from a given initial state to a *specified lower pressure* P_x the two properties H_x and P_x are thus determined at the second state and, if the expansion is presumed to take place with the vapor and liquid in thermal equilibrium, the remaining properties in the second state are directly determinable from the steam tables or charts. The more immediately necessary property is the specific volume, $V_{x,\text{actual}}$. That property being determined the velocity and mass-rate of flow per unit cross-sectional area are computed in the usual manner by equation 3 and the Continuity Equation. The second state would be represented on H - S coordinates by a point such as B' , Fig. 52. The actual enthalpy decrease and kinetic energy increase would similarly be measured by the vertical distance of A above B' in the same figure.

The application of the method is illustrated by the following example.

Example 19. Redetermine the throat and exit area for the nozzle of example 17, presuming a 97 per cent efficiency for the convergent portion of the nozzle, an over-all efficiency of 93 per cent for the entire nozzle and the equilibrium type of expansion. (Neglect approach velocity.)

Solution.

At $P_x = \text{critical} = 115$ (approx.): — $(H_1 - H_x)_S = 47.0$ (by Ex. 17);

$(H_1 - H_x)_{\text{actual}} = 47.0 \times 0.97 = 45.6$;

$H_{x,\text{actual}} = 1222.3 - 45.6 = 1176.7$

$U_{x,\text{actual}} = 223.7 \sqrt{45.6} = 1510$ (vs. 1530, ideal).

$$x_x = 1 - \frac{1189.1 - 1176.7}{880.0} = 1 - 0.014 = 0.986 \text{ (vs. 0.984).}$$

$$V_x = 3.878 - 0.014 \times 3.860 = 3.824 \text{ (vs. 3.817).}$$

$$M'/A_x = 1510/3.824 = 395; A_x = 0.00506 \text{ (vs. 0.00498).}$$

$$\text{At } P_x = 60: -(H_1 - H_x)_S = 97.9; \Delta H_{\text{actual}} = 97.9 \times 0.93 = 91.0.$$

$$H_{x, \text{actual}} = 1222.3 - 91.0 = 1131.3.$$

$$U_x = 223.7 \sqrt{91.0} = 2130 \text{ (vs. 2210).}$$

$$x_x = 1 - \frac{1177.0 - 1131.3}{915.0} = 1 - 0.050 = 0.950 \text{ (vs. 0.943).}$$

$$V_x = 7.172 - 0.050 \times 7.155 = 6.82 \text{ (vs. 6.76).}$$

$$M'/A_x = 2130/6.82 = 313; A_x = 0.00639 \text{ (vs. 0.00612).}$$

Example 20. Compute the exit specific volume, velocity and area for the nozzle of example 19 except for an atmospheric discharge region pressure.

The above nozzle would be said to exhibit a **Coefficient of Velocity** (C_v) of $(2130/2210 =) 0.965$. It is to be noted that for the not uncommon circumstance of negligible initial velocity,

$$\text{Coef. of Vel. } (C_v) = \frac{U_{x, \text{actual}}}{U_{x, \text{ideal}}} = \sqrt{\text{Nozzle Eff. } (e_n)}.$$

Nozzle efficiency determinations are made by simultaneous measurements of the mass-rate of flow (as by condensing and timed weighings of the condensate) and of the reaction force exerted by the jet. Tests have indicated that efficiencies of from about 0.90 to 0.97 are obtainable in actual steam turbine nozzles, with the values depending upon the density, viscosity and compressibility of the vapor, the pressure range, the nicety of the nozzle design and freedom from too great and too abrupt curvature in the flow passages et cetera. The designer selects the anticipated efficiency primarily on an empirical basis, with the higher efficiency expectation for pressure ranges well within the critical and the lower expectancy for the divergent portion of a nozzle (as in the example). The efficiency of the divergent portion will also be markedly lower if the so-called **expansion ratio** of the nozzle¹⁹ is either greater than or less than the correct ratio corresponding to any particular throat and discharge region pressures. For such conditions the nozzle will in fact cause the fluid to be respectively **over-expanded** or **under-expanded** in the nozzle. Over-expansion has rather a more deleterious effect than under-expansion.

In accordance with the above example the actual mass-rate of flow per unit area of throat would be $(2/0.00506 =) 395.0$ lb. per sec. per sq. ft., as against an ideal rate of (from example 17, $2/0.00498 =) 402.1$ lb. per sec. per sq. ft. A discharge coefficient of $(395.0/402.1 =)$

¹⁹ The **expansion ratio** of a nozzle is defined as the ratio between the exit area and the throat area.

0.985 would thus be indicated and values of about that magnitude or even somewhat lower might apparently be anticipated for any smoothly convergent nozzle passage or orifice, just as such coefficients were stated to be usual for a well rounded orifice with air (Art. 110). An anomalous and perplexing circumstance is disclosed however by actual test results on vapor flow, to wit that the above discharge coefficients of about 0.98 are found when the supply is at a fairly high superheat, whereas when only slightly superheated or saturated vapor is supplied (as in the example) coefficients of unity or greater are obtained. This condition is regarded as one of the contributory evidences that the expansion of such a vapor is not accompanied by the progressive condensation which is associated with thermal equilibrium but that instead the vapor becomes temporarily supersaturated. We shall see that supersaturation would presage a less specific volume (greater density) of the vapor at the throat and would explain the unique circumstance of a discharge coefficient which is greater than unity.

(b) *Actual Expansion, considering Supersaturation.* It was remarked above that supersaturation is characterized by a retarding of condensation during the (adiabatic) expansion of a slightly superheated or a saturated vapor in a nozzle. Various causes enter to effect this retardation and it is beyond the scope of this book to consider them in detail. However, it may be noted that the passage of a given particle of vapor through an ordinary nozzle occupies only some 1/10,000 to 1/1,000 of a second, that it may be shown that extremely small drops of liquid can not exist in thermal equilibrium with their surrounding vapor²⁰ and also that for condensation to take place nuclei must be present or be formed about which the molecules of liquid may collect. These and other fundamental considerations point to a conclusion that actual condensation could take place only partially or perhaps not at all during the extremely rapid expansion which occurs in a nozzle and that therefore instead of the equilibrium type of expansion the flowing vapor must pass into a momentary metastable phase in which the normal action of condensation is to some degree suspended and the fluid acts much as would a dry gas.

This conclusion is further substantiated by other phenomena. One which has already been mentioned is the unreasonable value of the discharge coefficient which is obtained with a steam nozzle or orifice when the ideal rate of flow is computed upon the equilibrium basis, whereas reasonable values obtain when the ideal rates are computed with the expectation of supersaturation. Another item is the signifi-

²⁰ Analyses show that the smallest drop which could exist in equilibrium would have a diameter of about 10^{-4} in. See Stodola, *Steam Turbine*, 1927.

cant fact that in observing a vapor which is expanding in a transparent nozzle no fogging or clouding of the jet appears until expansion is advanced well beyond the point at which condensation would begin if thermal equilibrium were maintained.

Regarding supersaturation as therefore an established fact, it remains to ascertain more definitely what laws are followed during such expansion. Unfortunately there exists at this writing a considerable uncertainty and lack of agreement among experimenters concerning the exact character of all of these laws. However it seems to be quite well established that in the earlier portion of the expansion there is an entire absence of condensation and that if the vapor is initially slightly superheated or dry saturated it will for a period follow the laws of the adiabatic expansion of a superheated vapor. Experiments indicate that this complete supersaturation and action like a superheated vapor persists until a pressure is reached which is about one half of that at which condensation would normally begin. This is approximately equivalent to stating that for initially dry and saturated steam complete supersaturation persists down to and somewhat beyond the critical pressure.

The reversible adiabatic expansion of superheated steam is found to follow quite closely a P - V relation which is parallel to that of a perfect gas, or specifically

$$PV^{1.3} = \text{constant}, \quad (16)$$

and the evidence indicates that the same relation represents adequately the property relation existing during a frictionless adiabatic expansion of wholly supersaturated steam.²¹ If the exponent 1.3 in this relation is regarded as the equivalent of the specific heat ratio k in the corresponding P - V relation for the expansion of a gas (i.e., $PV^k = \text{constant}$), and if further for superheated and supersaturated steam there is sufficient conformity to the characteristic gas equation $PV = RT$ that P_1V_1 may be employed in the flow relations in lieu of the term RT_1 , then for the flow of superheated and supersaturated steam one may write by analogy a group of flow relations which are parallel to equations 8 to 8x as derived in Arts. 106 and 107. These relations as particularized for superheated and supersaturated steam become (without attention to initial velocity),

$$\frac{M'}{A_x} (\text{ideal}) = 16.7 \sqrt{\frac{P_1}{V_1} \left[\left(\frac{P_x}{P_1} \right)^{1.54} - \left(\frac{P_x}{P_1} \right)^{1.77} \right]}; \quad (17)$$

²¹ This is in distinction to the condition that saturated steam which is expanding isentropically *under conditions of equilibrium* displays the P - V relation, $PV^{1.14}$, approximately = constant. The exponent varies with the initial condition of the steam and changes as the expansion proceeds but 1.14 is a fairly average value.

or, if $(P_1 - P_x)/P_1$ does not exceed about 0.35,

$$\frac{M'}{A_x} (\text{ideal}) = 8.02 \frac{1}{\sqrt{P_1 V_1}} \sqrt{P_x(P_1 - P_x) - 0.15(P_1 - P_x)^2}; \quad (17a)$$

or, if $(P_1 - P_x)/P_1$ does not exceed about 0.1,

$$\frac{M'}{A_x} (\text{ideal}) = 8.02 \sqrt{\frac{P_x(P_1 - P_x)}{P_1 V_1}}; \quad (17b)$$

or, if $(P_1 - P_x)/P_1$ does not exceed about 0.01,

$$\frac{M'}{A_x} (\text{ideal}) = 8.02 \sqrt{\frac{P_1 - P_x}{V_1}}. \quad (17c)$$

Also for the special condition of a value of P_x equal to the critical pressure, in which circumstance the flow rate per unit area is recalled to be the maximum,

$$\frac{M'}{A_{\text{throat}}} (\text{ideal}) = 3.78 \sqrt{P_1/V_1}^{22} \quad (17x)$$

Employing the value for the equivalent value of k in equation 9 of Art. 107 the critical ratio for the flow of superheated and supersaturated steam becomes

$$\frac{P_{\text{critical}}}{P_1} = \left(\frac{2}{1.3 + 1} \right)^{1.3/0.3} = 0.545.$$

It will have been noted that the foregoing relations are for flow which is ideal in the feature of being frictionless but actual with respect to the consideration of the supersaturation phenomenon. In order to account for the influence of friction, as that is reflected in reduced nozzle efficiency, the gas-flow relations of equations 11 and 12 of Art. 110 may be employed if proper attention is given to the suitable value of k , whereby in conjunction with the Continuity Equation sufficient relations are available for the actual design of a nozzle passing superheated or supersaturated steam within the range of expansion through which supersaturation may be taken to persist.

²² An approximate empirical relation known as **Napier's Equation** has for many years been in common use as a ready rule-of-thumb for estimating the maximum rate of flow. The equation is

$$M' = A_{\text{throat}} \frac{P_1}{C},$$

where C is a coefficient originally given as 70 but now recognized as having that approximate value only for dry saturated and moderately superheated steam supply and as increasing to about 80 for fairly high superheat. The expression is not dimensionally consistent nor particularly accurate but its simplicity and convenience often justify its use where a considerable degree of approximation is permissible.

For purposes of illustration and comparison the throat of the nozzle of example 17 is again redesigned in the following example, this time with due attention to supersaturation.

Example 21. Redetermine the throat velocity and area for the nozzle of example 17, presuming a 97 per cent efficiency for the convergent portion of the nozzle and accounting for supersaturation.

Solution.

$$P_{\text{critical}} = 0.545 \times 200 = 109 \text{ lb. per sq. in. abs.}$$

$$V_{\text{throat, ideal}} = 2.436 \times (1/0.545)^{1/1.3} = 3.88$$

$$V_{\text{throat, actual}} = 2.436 [0.97(1/0.545)^{1/1.3} + 0.03(1/0.545)] \\ = 3.90 \text{ (by Eq. 11, Art. 110)}$$

$$U_{\text{throat, ideal}} = 8.02 \sqrt{4.33 \times 144 \times (200 \times 2.436 - 109 \times 3.88)} = 1600$$

$$U_{\text{throat, actual}} = 8.02 \sqrt{0.97} \sqrt{\frac{1.3}{0.3}} \times 144 \times 200 \times 2.436 \times (1 - 0.545^{0.3/1.3}) \\ = 1575 \text{ (by Eq. 12)}$$

$$M'/A_{\text{throat, ideal}} = 1600/3.88 = 412 \text{ lb. per sec. per sq. ft.}$$

$$M'/A_{\text{throat, actual}} = 1575/3.90 = 404 \text{ lb. per sec. per sq. ft.}$$

Referring to the tabular results of example 17, for the ideal nozzle as designed without attention to supersaturation, the ideal mass-rate of flow per square foot at the throat was $(1530/3.82 =)$ 402 lb. per sec., whereas the above computation with consideration of supersaturation indicates an ideal rate of 412 and an actual rate of 404 lb. per sec. per sq. ft. of throat. The usual attainment of mass-rates which equal or slightly exceed the ideal as computed upon the basis of equilibrium may therefore be anticipated and justified upon the basis of supersaturation.²³ In this connection observe that the increased mass-rate of flow is due to a greater density of the supersaturated steam $(1/3.88, \text{ vs. } 1/4.00, \text{ at } P_x = 109 \text{ lb. per sq. in.})$.

Note several unique features of the supersaturated flow which are disclosed by the computations of example 21. Thus observe that if the perfect gas law, $P_1 V_1/T_1 = P_x V_x/T_x = \text{a constant}$, may be regarded as holding for the supersaturated vapor then, at the throat,

$$T_{\text{throat, ideal}} = T_1 \frac{P_x V_{x, \text{ideal}}}{P_1 V_1} \\ = (420 + 460) \frac{109 \times 3.88}{200 \times 2.436} = 765^\circ \text{ R.} = 305^\circ \text{ F}$$

The saturation temperature of steam at a pressure of 109 lb. per sq. in. abs. is 334° F. , wherefore the supersaturated vapor at 305° F. is re-

²³ Mass-rates in excess of the equilibrium ideal also are obtained with initially wet steam. The explanation there is based partially on supersaturation and partially on a relatively slower speed which it may be shown must exist in drops of moisture.

garded as being **subcooled** by 29° F. A considerable proportion of the energy which would normally be freed for purposes of jet acceleration by vapor condensation is evidently obtained instead by an excess temperature drop in the unstable supersaturated vapor. This is accompanied by a corresponding volume decrease, the specific volume of equilibrium steam at the 109 lb. point in the expansion being 4.00 as against the 3.88 of the supersaturated steam. Note also that, from an alternative viewpoint, the *saturation* pressure of steam at 305° F. is 72 lb. per sq. in. abs., wherefore the supersaturated vapor would be said to have a $(109/72 =)$ **1.52-fold supersaturation**, or synonymously **a degree of supersaturation of 1.52**. The number of degrees of subcooling or of supersaturation is regarded as the important criterion for the determination of the stage in the continued expansion of the steam at which complete supersaturation would end and a more or less rapid reversion of the fluid to equilibrium, by progressive condensation, would be initiated.

These considerations have developed in some detail the characteristics of complete supersaturation as that phenomenon occurs during and influences the rapid adiabatic expansion of an initially dry or moderately superheated vapor down through the state at which condensation would begin in an equilibrium type of expansion and to the critical pressure. This phase of the subject has its important applications in flow metering and in the design of the nozzles (and blading) of the first or earlier stages of a multi-stage steam turbine. Attention to these items is of obvious importance, but equally so would be an adequate consideration of (a) the further expansion of the vapor beyond the limits of complete supersaturation and in the phase of semi-supersaturation and progressive reversion to equilibrium and (b) the semi-supersaturated expansion of an initially wet vapor. Both of these types of flow are of major interest in steam turbine design. Unfortunately, however, any adequate analyses of them would involve various complex considerations which are regarded as being quite beyond the intended scope of this material,²⁴ and which also may as yet be regarded as somewhat problematical and uncertain.

Example 22. A steam turbine is provided with four first-stage nozzles of the convergent type which are known to have an exit area of 0.15 sq. in. each. The steam in the chest from which the nozzles draw their supply is at 225 lb. per sq. in. by gage and 420° F. Estimate by any suitable means the hourly steam consumption of the turbine if the pressure observed in the wheel chamber to which the nozzles discharge is (a) 150 lb. per sq. in. gage, (b) 115 lb. per sq. in. gage.

²⁴ For a very comprehensive discussion of the various aspects of supersaturation, see Stodola, *Steam and Gas Turbines*, 1927.

116. Flow of a Vapor through an Orifice. — Exactly as was indicated for the flow of a gas through an orifice (Art. 108), an orifice will perform the function only of the convergent portion of a nozzle. Therefore, again, a vapor will expand in the actual zone of the orifice to any discharge region pressure which is greater than or equal to the critical pressure, but not to any lower pressure. Any further expansion must take place beyond the orifice.

It follows that we again encounter the two general conditions of

- (a) *flow with the discharge region pressure greater than the critical*, when computation of the ideal rate of discharge may be based upon expansion in the orifice to the lower pressure, and
- (b) *flow with the discharge region pressure equal to or less than the critical*, when computation of the ideal rate of discharge must be based upon expansion in the orifice only to the critical pressure.

Consequently *if proper attention is given to the selection of the second pressure* (in the light of the above two general conditions) the ideal mass-rate of flow through an orifice may be computed upon the basis of the equilibrium type of expansion simply by the methods of example 17 or, upon the supersaturation basis, by the methods of equations 17 to 17x and example 21.

For a **rounded entrance** orifice the discharge coefficient should be about 0.98 for superheated steam and also for saturated steam *if the flow is computed upon the supersaturation basis*. As has been previously noted, the coefficient may be unity or slightly greater with dry saturated or moderately superheated steam supply *if the flow is computed upon the equilibrium basis*.

For a **sharp-edged** orifice the discharge coefficient exhibits the same wide variation which was discussed in Art. 110, ranging from some 0.60 upwards. However, this type of orifice is frequently employed in commercial steam-flow meters. With these the manufacturer furnishes indicators which are actuated by the pressure difference across the orifice and the scales of which are graduated to read directly the mass-rate of flow. The vagaries of the orifice coefficient are quite adequately taken care of in the production of the indicator scales by calibration tests at the operating conditions under which the meter will be used. Such calibrations of any sharp-edged orifice are easily performed if the fluid is a vapor by reason of the ease of condensing and taking timed weighings of the condensate.

Example 23. For a steam supply at 150 lb. per sq. in. gage and 400° F. estimate by any suitable method the probable mass rate of discharge through a $\frac{1}{4}$ -in. diameter

orifice to a discharge region at a pressure of (a) 100 lb. gage, (b) 75 lb. gage and (c) the atmosphere. Do this both for a rounded-entrance and a sharp-edged orifice.

117. Diffuser and Ejector. — A device operating as a reversed nozzle is known as a **diffuser**. In the foregoing a nozzle has been described as a properly shaped channel through which a fluid may expand from a higher to a lower pressure to produce a high velocity jet. Conversely, a diffuser is a channel so shaped that it will utilize the kinetic energy of a high velocity jet of a lower pressure fluid to discharge the fluid to a region of higher pressure. Referring to the Mollier diagram of Fig. 53, a fluid passing reversibly through a nozzle from pressure P_1 to the lower pressure P_2 would change state along the path ab , at constant entropy, decreasing enthalpy, increasing specific volume, and increasing velocity. A diffuser in which the action was wholly reversible would take the high velocity fluid under the conditions existing at the nozzle outlet, b , and return it along state path ba to its original state and to a region of the higher original pressure P_1 . All of the state changes which occurred in the ideal nozzle would occur in the ideal diffuser in the reverse order.

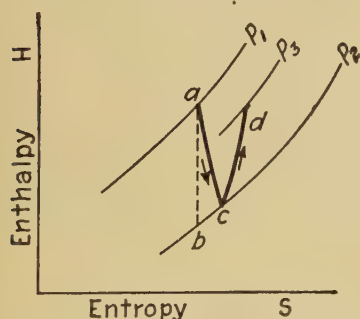


FIG. 53

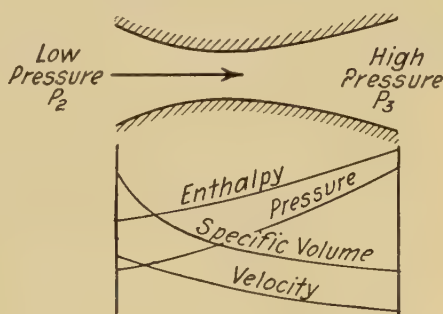


FIG. 54

The contour of the channel required by the diffuser is determined from exactly the same considerations which determine the contour of a nozzle (Art. 106), namely, consideration of the steady-flow energy equation and the Continuity of Flow principle. Parallel features of the two devices would be that, just as expansion in a nozzle to a lower pressure which exceeds about one half of the higher calls for a converging section only, so a compression in a diffuser from a lower pressure in excess of one half of the higher would require only a diverging section. Similarly, if the stream velocity to the diffuser were sufficient to enable discharge to a region the pressure of which exceeded twice that of the lower pressure supply region, the diffuser channel would be initially convergent and then divergent from a throat. The general form of a diffuser for operation under the latter condition is shown in Fig. 54.

Below the diffuser are depicted the decreasing velocity and specific volume and the increasing pressure and enthalpy as the fluid flows from the lower pressure to the higher pressure region.

In Fig. 53 there is also shown the character of the *actual* processes in a serial arrangement of nozzle and diffuser. The line *ac* represents the actual expansion in the nozzle from pressure P_1 to pressure P_2 , and the line *cd* represents the recompression in the diffuser. The paths are both of increasing entropy and so, if in general the recompression may be one to a final enthalpy H_3 equal to the original enthalpy H_1 , the final pressure P_3 must be less than the original pressure P_1 by an amount which depends on the degree of irreversibility of the two processes.

If the deceleration and compression in a diffuser were that of a gas and if it were carried to a negligible exit velocity U_3 the pressure to and against which the stream might be delivered would depend on the pressure, temperature, and velocity of the supply stream. Thus, by a development of the energy equation in a form which is rather parallel to that of equation 7 (Art. 106) and of the footnote of Art. 110, for a gas,

$$\frac{U_2^2}{64.34} = \frac{k}{k-1} RT_2 \left[\left(\frac{P_3}{P_2} \right)^{(n-1)/n} - 1 \right], \quad \text{or}$$

$$P_{3, \max.} = P_2 \left[\frac{k-1}{kRT_2} \frac{U_2^2}{64.34} + 1 \right]^{n/n-1}, \quad (18)$$

in which $P_{3, \max.}$ = maximum discharge pressure against which the stream could be delivered from diffuser after deceleration to a negligible velocity.

P_2 = pressure of the region of the supply stream.

U_2 = velocity of the supply stream, feet per second.

T_2 = supply region temperature, degrees R.

$k = c_p/c_v = 1.40$ for air.

n = the effective P - V exponent of the more-or-less irreversible adiabatic compression process, the value of which exponent would equal k for ideal reversible compression, would exceed k for irreversible compression (see Art. 88) and would be an index of the character of the diffuser performance.

Common applications of the diffuser are found in the downstream section of the Venturi meter (Art. 111) and in centrifugal pumps and compressors. In the latter high velocity streams are created at the periphery of a rotating element and stationary diffusers are employed to utilize the kinetic energy of the streams for delivery to discharge regions of higher pressure.

The diffuser forms an essential component of the **ejector**, which is a device in which the kinetic energy of one fluid is used to pump another fluid from a region of lower to one of higher pressure. In the steam air ejector, Fig. 55, a high velocity jet of steam is produced by the steam nozzle AC . Air is drawn into the ejector from a region of low pressure D , mixes with the jet, and the mixture is compressed in the diffuser EG and discharged into a region of higher pressure beyond G .

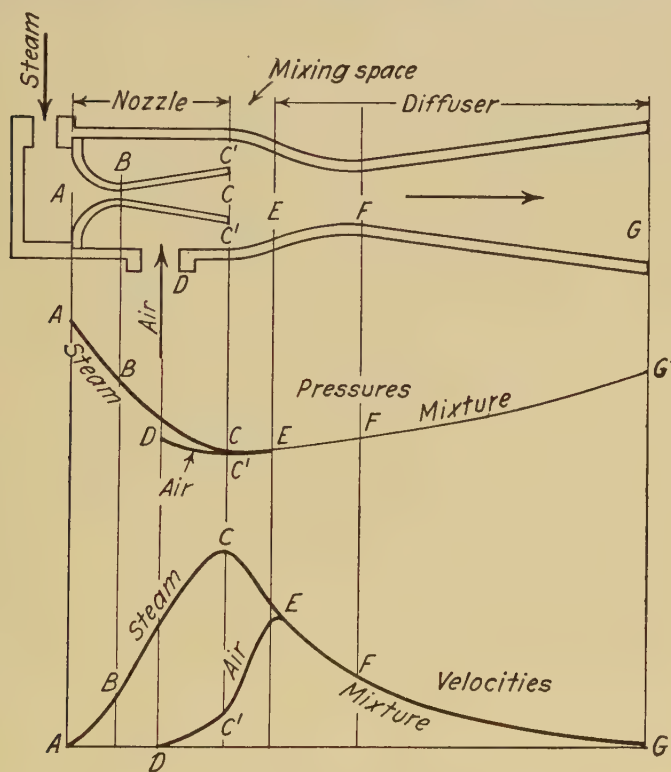


FIG. 55, 55a and 55b

The changes in pressure, velocity, enthalpy and entropy occurring throughout the ejector are shown graphically in Figs. 55(a), 55(b), and 55(c). The steam is supplied to the nozzle at a pressure P_1 considerably higher than the pressure P_3 at which the air is to be discharged. Along AC occurs the increase of velocity in the nozzle. Along DC' is a slight increase in velocity as the air is drawn into the ejector. Between C and some indefinite point E the steam and air mix at nearly constant pressure and with an increase in air velocity and decrease of steam velocity. This mixing is considered to occur by a wholly irreversible

process described as inelastic impact, but with conservation of momentum of the streams of fluid. From E to G the mixture is compressed with decreasing velocity and increasing entropy and enthalpy. The

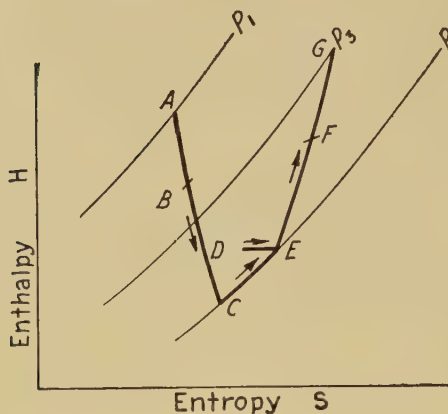


FIG. 55c

diffuser requires a minimum section at F if the absolute pressure at E is less than about half the pressure at G . Because of lack of knowledge of just what occurs during the mixing process and also of the efficiency of the compression process in the diffuser, no satisfactorily exact analysis of the complete ejector has been made. Design is largely by empirical methods. However, it is not difficult to compute the efficiency of an ejector from performance data.

Let M'_s = mass of steam used in unit time.

M'_a = mass of air handled in unit time.

$M'_s(H_1 - H_3)_s$ = adiabatic decrease in enthalpy of steam from nozzle supply pressure P_1 to air discharge (not suction) pressure P_3 .

$M'_a W_a$ = adiabatic work of compression of air from air suction pressure P_2 to air discharge pressure P_3 (see Art. 174 for expression).

Then efficiency of ejector = work required to compress air adiabatically, divided by available energy supplied by steam above discharge pressure, or

$$\text{Ejector Eff.} = \frac{M'_a W_a}{J M'_s (H_1 - H_3)_s} \quad (19)$$

The efficiency computed in this manner corresponds to the efficiency which should be used if the air were compressed in a steam-driven piston or centrifugal air pump directly by the use of mechanical work. The additional kinetic energy obtained in the steam jet by expansion from P_3 to P_2 is not charged to the ejector, since exactly that amount of energy is ideally required to compress the same steam back from P_2 to P_3 in the diffuser.

Example 24. A steam-air ejector is operated with dry saturated steam at a gage pressure of 100 lb. per sq. in. Air is discharged into the atmosphere from a vessel

under a vacuum of 24.4 in. Hg. and at a temperature of 70° F. The barometer is 30.5 in. Hg. Three pounds of steam are used per pound of air discharged. Compute:

- (a) ideal velocity of discharge from steam nozzle.
- (b) ideal velocity of mixture of steam and air at entrance to diffuser.
- (c) efficiency of ejector.

Solution. (a) Atmospheric pressure, $P_3 = 15.0$ lb. per sq. in. abs.

Steam pressure, $P_1 = 115$ lb. per sq. in. abs.

Air pressure in vessel, $P_2 = 3$ lb. per sq. in. abs.

Ideal enthalpy drop in entire nozzle, 115 to 3 lb. = 1189 - 945 = 244 B.t.u.

Corresponding jet velocity = $223.7 \sqrt{244} = 3485$ ft. per sec. (See Art. 114.)

(b) Assume velocity of air at point *c*, just before mixing, to be 200 ft. per sec. Applying principle of equal momenta before and after mixing $(3485 \times 3) \div 200 =$ velocity of mixture $\times 4$. Ideal velocity of mixture = 2664 ft. per sec.

If the mixing occurred without loss of kinetic energy the velocity of mixture would be 3030 ft. per sec. As stated, little is known as to what actually occurs at this point.

(c) The work required to compress the air is computed on the basis of reversible adiabatic compression from a pressure of 3 lb. per sq. in. abs. to a pressure of 15 lb. per sq. in. Applying the method of Art. 174 gives 57,000 ft.-lb. per pound of air. The adiabatic enthalpy drop in the steam nozzle from 115 lb. to 15 lb. is 148.8 B.t.u. per lb. This is the energy available to produce the compression of the *air* in the diffuser. The over-all efficiency of the ejector is then:

$$\frac{\text{Ideal energy required for air compression}}{\text{Ideal energy available in steam nozzle}} = \frac{57,000}{148.8 \times 778 \times 3} \\ = 0.164 = 16.4 \text{ per cent.}$$

In steam condenser applications, the air withdrawn from the condenser is saturated with water vapor at the temperature of the air-vapor mixture, and in computing the work of compression it is this mixture which must be considered (see Art. 100). For high vacuum in condensers it becomes necessary to use two or three stages of ejectors in series, condensing the steam between stages to reduce the amount of steam which otherwise would need to be compressed in the diffusers.

118. Summary, Flow of Steam through the Nozzle, Orifice et Cetera.

—The general energy and continuity relations which apply in the flow, expansion, and acceleration of a fluid through a nozzle or orifice are the same whether the fluid is a gas or a vapor, such as steam. Thus equations 1, 2, 3 and 5 of Art. 105 are universally applicable. Also in the applications of these relations the same characteristic features of a nozzle and the same critical pressure phenomenon become evident for either a gas or a vapor. However, the particularized methods employed in the actual use of the relations may differ in the feature that use may be made of the customary tabular and graphical presentations of vapor properties.

Upon the evidence of results of nozzle computations based on such vapor tables and on the presumption of a wholly reversible expansion,

the critical ratio for steam would seem to vary slightly, ranging from about 0.55 for a fairly highly superheated supply to about 0.58 for a saturated supply.

A further conclusion based upon the same evidence is that a progressive condensation of an initially dry saturated supply would accompany the ideal, reversible adiabatic expansion of the steam through the nozzle. Such a conclusion would predicate a continued stable condition of the vapor during its expansion and, by the use of suitable coefficients, actual nozzle and orifice computations may be made upon that presumption. However the anomalous condition is frequently encountered that the actual mass rate of flow through a nozzle or orifice may equal or slightly exceed the ideal rate when that is computed upon the presumption of the stable expansion, which result would seem to be a rather unprecedented one. Upon such evidence and other of still more conclusive character it has been concluded that, instead of the stable condition, the extremely rapid expansion of a vapor through a nozzle or orifice is characterized by a temporary unstable condition in which condensation is retarded or deferred and an originally dry or superheated vapor will act as a gas until its pressure is well below that at which condensation might normally be expected to begin. A vapor expanding in that unstable manner is said to be *supersaturated*.

In proceeding with computations of nozzle or orifice flow with a dry or superheated steam supply upon the presumption of complete supersaturation relations are employed which are essentially parallel to those used for a gas. Thus there is employed the basic property relation for the frictionless adiabatic expansion,

$$PV^{1.3} = \text{a constant.} \quad (16)$$

Using this P - V relationship in conjunction with the energy and Continuity Equations, expressions for the critical ratio and the mass-rate of flow (with supersaturation) are obtained which are essentially parallel to the corresponding equations for gas flow.

Thus, corresponding to the general gas-flow relations of equations 8 to $8x$, there are developed in equations 17 to $17x$ the parallel but specialized relations for the flow of superheated and supersaturated steam. For the same fluid the critical ratio has the value 0.545.

Results of actual flow rate tests show consistently that the actual rates are slightly less than the ideal rates as so computed, which condition conforms more closely with our sense of the appropriate and affords one substantiation of the presumption that supersaturation exists during the flow through the nozzle.

Adequate information is still lacking at this writing concerning the

extent or degree of persistence of supersaturation with the continued expansion of a vapor to pressures much below the critical.

Whether an analysis upon the basis of stable flow or of unstable flow be employed the same character of critical pressure phenomenon maintains, whence for the orifice there exist the same two distinctive cases of flow as were described in connection with the flow of a gas. These are (a) that in which the discharge region pressure exceeds the critical, when the steam will expand in the orifice to the discharge region pressure, and (b) that in which the discharge region pressure is equal to or less than the critical, when the steam will expand only to the critical pressure in the immediate zone of the orifice. As regards the coefficient of discharge of a steam orifice it is at least to be remarked that for the sharp-edged orifice there is again encountered a wide range of coefficients.

The diffuser is in principle the reverse of a nozzle and is a device by which a lower pressure, high velocity jet may deliver itself at a low velocity at and against a higher pressure. In terms of energy, it acts to transform the kinetic energy of the fluid to increase of enthalpy, ideally doing this without increase of entropy but actually with the usual inescapable entropy increase. The steam-driven air ejector is one of the practical devices which employ the diffuser principle. In the ejector a high velocity steam jet acts to entrap air with the steam and jointly to deliver the two against a pressure which is less than that of the original steam supply but exceeds that of the air supply.

119. Review Questions and Topics. —

1. Review concisely the objectives and accomplishments of Parts I, II and III of the foregoing material.
2. State the distinguishing functions of the nozzle and orifice.
3. (a) Write the steady-flow energy equation in the particularized form in which it is most applicable to the nozzle or orifice.
(b) Write the Continuity Equation.
- (c) Write the relations for evaluating the actual and the ideal velocity attained by an expansion in a nozzle or orifice, expressing the relation in terms of the enthalpy change and the correction factor for initial velocity.
4. (a) What is the distinctive property characteristic in the ideal expansion of a fluid in a nozzle; how may this characteristic be employed to ascertain the final specific volume and the enthalpy change as a result of the ideal expansion of a gas from a given initial state to a specified second (lower) pressure?
(b) Describe the distinctive feature as regards the manner of variation of the successive cross-sectional areas of a nozzle passage which shall properly provide for the progressive expansion of a gas through the nozzle; define the critical pressure.
(c) What type of nozzle will properly provide for the expansion of a gas to a pressure greater than the critical and what factors control the mass-rate of flow per unit exit area of nozzle?
(d) What type of nozzle is required in order to provide properly for the expansion

of a gas to a pressure less than the critical; what factors control the mass-rate of flow per unit throat area of the nozzle; what factors control the requisite area for a given mass-rate of flow?

(e) Discuss the origin and the range of utility of equations 8 to 8c.

5. (a) Define the "critical ratio," describe the general method by which it is ascertained; upon what property of the fluid does it depend and what is its value for air?

(b) Discuss the specific utility of equations 6x, 7x and 8x.

6. Describe and discuss the two distinctive cases associated with the flow of a gas through an orifice and cite the correct relations to be employed in computing the ideal mass-rate of discharge through an orifice in the two cases.

7. (a) What is the "correction factor for approach velocity" and under what general circumstance will it become negligible?

(b) Is the critical ratio influenced by approach velocity?

(c) How may the necessity for approach velocity correction be obviated?

8. (a) Define nozzle efficiency; state the influence of a nozzle efficiency of less than 100 per cent on the specific volume of the fluid, on the attained velocity and on the attained mass-rate of flow.

(b) Define the "coefficient of discharge" of an orifice; give a reasonable range of its value for a rounded-entrance orifice; give a reasonable range of its value for a sharp-edged orifice and discuss briefly the reason for this relatively wide range.

9. Describe the form and manner of use of a Venturi meter and indicate which of the mass-rate relations which have been presented are suitable for use in connection with the computations of the meter. Will the correction factor for approach velocity commonly need be considered in the computations of the meter? What are reasonable values of its coefficient of discharge?

10. Describe the form, principle of operation and manner of use of an impact tube or Pitot tube and indicate which of the velocity relations which have been presented are suitable for use in connection with the tube. What type of procedure must be followed in order to find the average velocity and the mass-rate of flow through a duct by use of the tube?

11. Discuss concisely the similar and the dissimilar features of the computation of the nozzle form for the reversible flow of a vapor, as compared with the computations for the flow of a gas. Summarize the outstanding characteristics of such a steam nozzle. In what manner would the superheat or the quality of the vapor vary during the progress of such an expansion?

12. (a) Describe the phenomenon which is known as supersaturation.

(b) Describe the method of accounting for actual nozzle efficiency in the design of a steam nozzle without accounting for supersaturation. What anomalous condition is found to exist as regards the actual and the ideal mass-rate of flow through a nozzle when the latter is computed without attention to supersaturation.

(c) Outline the general method of nozzle design when complete supersaturation is presumed.

13. Restate the two distinctive cases associated with the flow of a vapor through an orifice.

14. (a) Describe the action and the principles of action of a diffuser.

(b) Describe the action of a steam-driven air ejector.

Symbols and Abbreviations, Chapter XI

- A = cross-sectional area of a flow-channel, sq. ft.
 C_d = coefficient of discharge.
 c_p = specific heat at constant pressure.
 C_v = coefficient of velocity.
 e = efficiency.
 H = enthalpy, B.t.u. per pound mass.
 J = Joule's equivalent, 778 ft-lb per B.t.u.
 k = specific heat ratio (c_p/c_v).
 M' = mass-rate of flow, pounds per second.
 n = a subscript referring to a nozzle (as e_n).
 P = pressure, lb. per sq. ft.
 Q = energy in transition by radiation or conduction (i.e., as heat), B.t.u. per pound mass of a fluid.
 R = gas constant (53.3 for air).
 S = entropy per pound mass and = a subscript denoting constancy of entropy (as in a reversible adiabatic or isentropic state change).
 T = absolute temperature, deg. Rankine (i.e., deg. Fahr. + 460).
 U = velocity, ft. per second.
 V = specific volume, cu. ft. per pound mass.
 W = energy in transition as shaft-work, ft-lb. per pound mass of a fluid.
 x = a subscript designating a property, area et cetera at *any* selected point in a progressive expansion through a flow channel and = quality of a vapor.

CHAPTER XII

STEAM POWER PLANT CYCLES

There have been evolved and are at present available to man two general types of devices or methods for the production of power from the energy stored in the fuels. These are the internal-combustion engine and the type of composite device familiarly exemplified by the steam power plant. Of these the steam plant was developed much the earlier and at this writing it occupies and bids fair to maintain a major position in power generation in circumstances where physical mobility or portability of plant is not essential or where large concentration of power is desired. The thermodynamic principles which control and direct the design or operation of the steam portion of the steam power plant are developed in the following articles.

120. Heat Engine Cycles. — In various of the discussions of Part II (Arts. 39, 41, 44 and 57) attention was directed to certain significant characteristics of the processes which man has been able to devise for acquiring shaft-work from those reservoirs of chemical energy which have been stored by nature in the fuels and which we have learned to release by the process of combustion. Those characteristics were (a) that they all are processes which depend upon the release of the energy *at an elevated temperature* (see Art. 39) and (b) that a temperature engine which operates on a *wholly reversible cycle* is the most efficient device conceivable whereby any available portion of the energy so released may be transformed to work (Art. 46). In this connection it may well be remarked further that all actual heat-power devices operate in a cyclic series of processes, whether they may be *fluid* cycles as in the usual steam power plant or simply *machine* cycles as in the internal-combustion engine (see also Art. 40). The initial release of the energy from the fuel is however non-cyclic.

Of the several wholly reversible cycles only one was described in any detail in the foregoing, the Carnot cycle (Art. 42). That cycle will be recalled to consist of a sequence made up of

- (a) a reversible isothermal expansion of a working fluid and a concurrent reception of energy by the fluid at a constant upper temperature from some high temperature source;

- (b) a reversible adiabatic expansion of the fluid whereby it passes to the lower temperature of the cycle in the most favorable manner conceivable;
- (c) a reversible isothermal compression, at the lower temperature, during which the fluid discards any unavailable energy to the lower temperature receiver; and
- (d) a reversible adiabatic compression by which the fluid is returned to the upper temperature and original state in the most favorable manner.

Although admittedly no wholly reversible process is actually attainable and also although the mechanism of the classical Carnot engine (with its interchangeable cylinder heads, see Art. 42) is fantastic and impracticable, the cycle nevertheless has offered extremely valuable guidance in the selection and development of the actual practicable steam power plant cycles. Thus, as in the Carnot, those real cycles provide (a) for the reception of energy by the working fluid at a more or less constant upper temperature from some higher temperature source, (b) for an expansion of the fluid from the upper to the lower temperature of the cycle, this expansion ideally taking place in a reversible adiabatic manner, and (c) for the rejection of the unavailable energy by the fluid to a lower temperature receiver. In principle the real cycles differ from the Carnot only in the means employed for returning the fluid from the lower to the upper temperature. Their mechanisms also differ distinctly from the Carnot engine mechanism in the feature that the several successive processes of the cycles are performed in separate organs of a system, each part being so devised as to accomplish to best advantage one individual process or function. It is the purpose of the following to describe in some detail and to analyze the basic practical cycles of the present-day steam power plant.

121. The Rankine Cycle. — The Rankine cycle was for many years the sole ideal prototype of actual steam power plants and as such its ideal efficiency was long employed as a standard of performance with which to compare and by which to judge the performance of actual operating plants (and engines), the Carnot cycle having then been regarded as wholly impracticable of approach even in principle. For that reason the cycle will be considered in some detail, this in spite of the fact that at this writing it may be regarded as obsolescent in so far as concerns the ideal cycles of operation of the large modern power plant.

The ideal Rankine cycle is a steady-flow one which operates with a

condensable vapor as the working fluid and consists in principle of four distinct processes. These processes are described in the following paragraphs. To facilitate their description frequent reference will be made to Fig. 56, which is a diagrammatic representation of the sequence and arrangement of the essential pieces of apparatus in which the processes are carried out, and to Figs. 57, 58 and 59, in which the state changes which occur progressively in the cycle are depicted re-

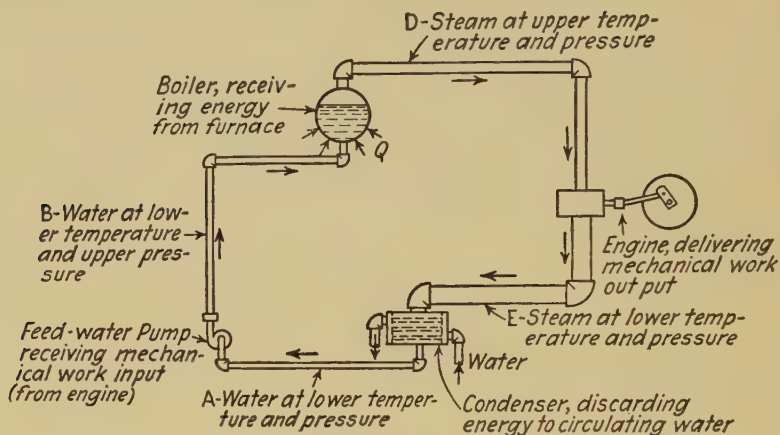


FIG. 56. Elementary (Rankine cycle) steam power plant.

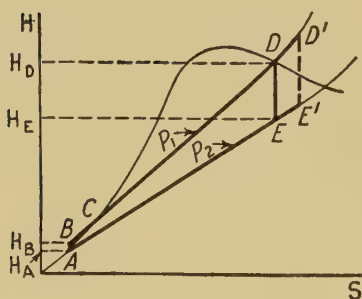


FIG. 57

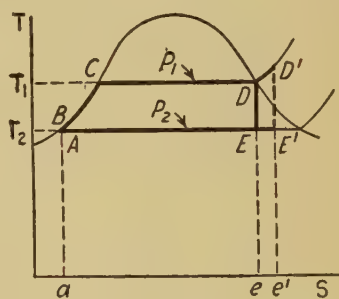


FIG. 58

spectively on H - S , T - S and P - V coordinates. In these diagrams the usual saturated liquid and saturated vapor lines are drawn in order to assist in the orientation of the several processes.

To proceed with the description — let there be selected arbitrarily as a starting point in the cycle the point and state **A** at which the fluid, which in present practice is almost invariably water, exists as liquid at the lowest pressure and temperature of the cycle. In the ideal cycle

the fluid at this state is taken to be *saturated* liquid. Physically this state occurs in the suction line of the **pump**, or pumps, which acts to raise the pressure of the water from this lowest pressure to the highest pressure of the cycle.

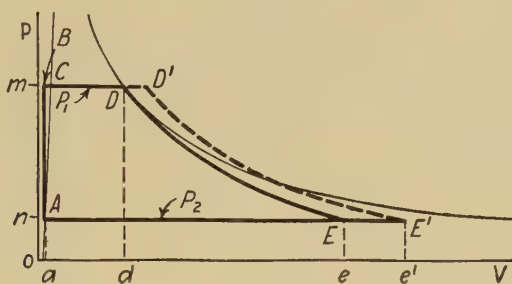


FIG. 59

From this initial state the fluid passes progressively through the following series of processes:

(1) The pressure of the water is raised by a reversible adiabatic compression in the pump from the lower pressure P_2 to the upper pressure P_1 , causing a change from the saturated liquid state A to a *subcooled* liquid state B . During this pumping process there is no change of entropy, as the process is ideally reversible, and also, as the fluid is a liquid, the compression is not accompanied by any appreciable change of temperature, molecular energy, or specific volume (see Art. 63). This latter feature is the outstanding one in which the Rankine cycle differs fundamentally from the Carnot. It is recalled that in the Carnot cycle this compression is performed upon the fluid when it is in such a gaseous or vapor state that the compression accomplishes not only the pressure increase but also a reversible temperature rise from the lowest to the top temperature of the cycle.

Applying the steady-flow energy equation (Art. 32) to the liquid pumping process the equation reduces to the relation,

$$W_{\text{pumps}} = J(H_B - H_A), \text{ ft.-lb. per pound.} \quad (1)$$

The kinetic energy and heat terms of the general steady-flow equation are respectively negligible and not pertinent. In the ideal cycle this work which is required to drive the pumps is regarded as being obtained directly or indirectly from the main engine of the plant.¹

The enthalpy increase through the pump, and thus the pump work, is evidently represented on the H - S diagram of Fig. 57 by the vertical

¹ In actual plants the pump work is almost invariably furnished by independent "auxiliary" engines or motors.

distance between points B and A . In the figure this distance is somewhat exaggerated for purposes of clarity. On the T - S diagram the states A and B are represented by practically coincident points, this by reason of the ideal equality of the entropies and the virtual equality of the temperatures before and after the pump.

The energy equation for the pumping process may be expressed in an alternative form which is useful. Thus, as $H = E + PV/J$ and as the molecular energy (E) and the specific volume (V) of the water are not changed appreciably by the compression,

$$\begin{aligned} W_{\text{pumps}} &= JE_B + P_1V_B - JE_A - P_2V_A \\ &= (P_1 - P_2)V_{f,2}, \end{aligned} \quad (1a)$$

where $V_{f,2}$ = specific volume of saturated liquid of temperature T_2 .

This expression is of particular utility in that it provides a direct means for computing the pump work and also for portraying the work by an area on the P - V coordinates. In Fig. 59 the product $(P_1 - P_2)V_{f,2}$ is represented by the area $mBAN$, which area thus represents the pump work.

(2) The second distinctive process of the Rankine cycle is the warming, vaporizing and perhaps superheating of the fluid, all occurring under a virtually constant pressure. The energy supply for accomplishing these several phases of the cycle is provided by heat conduction through the *heating surfaces* of the **boiler** and the **superheater** (if the latter is supplied) and is furnished directly or indirectly from the primary high-temperature energy source, namely, the hot products of combustion of the fuel in the *furnace*. The individual processes are carried out as the fluid flows progressively through the several devices.

In the feed-water heating portion of the process the water is warmed at P_1 from temperature T_2 up to the saturation temperature which corresponds to that pressure, whence the state of the liquid after completion of the heating is represented by the saturated liquid state point C . If the boiler is presumed to accomplish a complete vaporization of the water the state of the fluid leaving the boiler is represented by the dry saturated vapor state point D . If in addition a superheater is supplied the fluid leaves that device at the superheated state point D' , which point must lie on the P_1 line in the superheat region.

The energy equation for this complete process of energy supply as heat becomes

$$Q_{\text{supplied}} = H_{D \text{ (or } D')} - H_B, \text{ B.t.u. per lb.} \quad (2)$$

The kinetic energy terms are again to be regarded as negligible or as differing by a negligible amount in the ideal cycle and a shaft-work term is not pertinent for this process.

Referring to the figures, the total energy supply as heat is evidently measured directly on the H - S diagram by the vertical distance of point D (or D') above B . Also let it be recognized that for this fluid state change which is accomplished solely by energy supply as heat,

$Q = \int_1^2 T dS$, whence the heat supplied to produce the state change BCD (or $BCDD'$) will be measured on the T - S diagram by the area $aBCDe$ (or $aBCDD'e'$). It will be further recalled that on the same diagram the inherently unavailable portion of the heat energy supply is represented by the area $aAEe$ (or $aAE'e'$).

Although the state change is represented by the path BCD (or $BCDD'$) on the P - V coördinates of Fig. 59 the diagram affords no graphical measure of the amount of energy supplied.²

(3) The next state change in the ideal Rankine cycle is the reversible adiabatic expansion of the fluid from the upper temperature and pressure to the lower. To accomplish this the fluid is delivered to the **engine**³ at constant pressure and expands therein, after which it departs from the engine at a constant lower pressure. It is in this composite delivery, expansion, and rejection process which takes place in the engine that the available portion of the previously supplied heat energy is transformed to shaft-work. In subsequent chapters attention is given to the details of the process as carried out in the various types of engines.

Recalling that the property characteristic of a reversible adiabatic state change is the constancy of the entropy, it is evident that the expansion through the ideal engine shall be represented on the H - S and T - S diagrams by the vertical isentropic line DE (or $D'E'$). The line DE in the P - V diagram portrays the typical appearance of this reversible adiabatic state change on those coördinates.

The energy equation for the engine becomes

$$\begin{aligned} W_{\text{engine}} &= J(H_E - H_D) \quad [\text{or } J(H_{E'} - H_{D'})], \text{ ft.-lb. per pound} \\ &= J(H_1 - H_2)_S. \end{aligned} \quad (3)$$

The kinetic energy terms are again to be regarded as ideally negligible or to differ negligibly and the adiabatic process is by definition one in which no energy enters or departs by radiation or conduction, that is,

² It is of interest to note however that on the P - V diagram the area $omDd$ represents the flow-work passing to the boiler, from the pumps, plus that portion of the heat energy supplied which is transformed to flow-work in the stream departing from the boiler.

³ The term *engine* is here used in a generic sense to designate the main *prime mover* of the plant, whether that be actually a *reciprocating engine* or *turbine engine*.

as heat. Q . The work output of the ideal engine is therefore measured on the H - S diagram by the vertical distance of point D above E . The method of computing the enthalpy at E , given the entropy and pressure, has been considered in Art. 68.

The engine work may not be represented exactly on the T - S diagram. However it will appear subsequently that the net work output of the complete cycle, that is, the engine work less the work which must be returned to drive the feed-water pump, is measured on those coördinates by the area $ABCDE$ (or $ABCD'E'$), and also it will appear that the engine work and the net work of the cycle will in general differ by less than one per cent.

To ascertain the method of representing the engine work on the P - V diagram it is convenient to employ the relations of Art. 36*b*, which evaluates in terms of P and V the total Mechanical Effects in any mechanically reversible steady-flow process:

$$\begin{aligned} - \int_1^2 P dV &= \text{Mechanical Effects in any mechanically reversible process,} \\ &= \frac{U_1^2 - U_2^2}{64.34} + (P_1 V_1 - P_2 V_2) + W, \text{ for reversible steady-flow.} \end{aligned}$$

Solving this relation, but neglecting the kinetic energy term,

$$W_{\text{engine, ideal}} = - (P_1 V_1 + \int_1^2 P dV - P_2 V_2).^4 \quad (3a)$$

In Fig. 59, $P_1 V_1 = \text{area } omDd$,
 $\int_1^2 P dV = \text{area } dDEe$ and
 $P_2 V_2 = \text{area } onEe$.

It follows from the geometry of the figure that

$$\begin{aligned} W_{\text{engine, ideal}} &= - \text{area } mDEn, \text{ or} \\ &= + \int_1^2 V dP. \end{aligned} \quad (3b)$$

It is to be emphasized that this expression for the engine work applies to any ideal engine, whether of the reciprocating or turbine type.

(4) The closing process of the cycle must obviously be a condensation of the *exhaust* steam from the engine, by which the fluid is returned at

⁴ In accordance with the convention employed in this material (see Art. 31*b*) the negative value which results from computations based on these relations indicates that energy is departing from the system as shaft-work. That is obviously the case in the engine.

constant pressure from the state E to the initial saturated liquid state A and during which there occurs the discarding of the inescapably unavailable residue of the previously supplied heat energy. In the ideal cycle and in many practical installations this process is accomplished during passage through a **condenser** in which the energy departure and the condensation are effected by energy transfer as heat conducted from the steam through the condenser structure to *circulating water*. The same state change and energy rejection may be accomplished however in various other practical manners, as by condensation in the atmosphere, in a heating system or in any other process in which the low temperature energy of the exhaust steam may be employed to advantage.

The energy equation for the condensation process becomes

$$Q_{\text{condenser}} = H_A - H_E \text{ (or } H_A - H_{E'}), \text{ B.t.u. per lb.} \quad (4)$$

It follows from the equation that on the H - S diagram the vertical distance between state points E and A measures the energy rejected in the condensation.

On the T - S diagram this energy discard as heat [= $T_2(S_A - S_E)$] is represented by the area $aAEe$ (or $aAE'e'$). Therefore, as the area $aBCDe$ measured the amount of energy supplied from the source as heat, the difference between these two areas, or the area $ABCDE$, measures the *net* amount of work obtainable from the operation of the cycle, that is, the net amount after debiting the engine output with the portion of its work which must be employed to drive the pump. The P - V diagram does not provide a graphic representation of the energy rejected as heat in the condensation.

The cycle is thus completed. It remains to ascertain its thermal efficiency, the factors affecting that efficiency, and the possibilities of other practical cycles of higher inherent efficiencies. Those items will be considered in the following articles.

Example 1. A Rankine cycle type of steam plant is operating with a boiler pressure of 300 lb. per sq. in. abs. and a 20 lb. per sq. in. abs. back pressure. There is no superheater and the steam delivered by the boiler has a 99 per cent quality. Compute the properties of the steam or water at each significant point of the ideal cycle, the ideal feed pump work (in ft-lb. and in B.t.u. per pound of steam), the requisite heat supply and the ideal minimum energy discard per pound of steam.

How many pounds of steam would be required in the ideal Rankine plant per net horsepower-hour output and how many pounds of coal if of a heating value of 13,500 B.t.u. per lb. and if the furnace and boiler efficiency were 70 per cent?

Solution.

Water leaving condenser: $P_A = P_2 = 20 \times 144$; $V_A = V_{f,2} = 0.0168$; $t = 228.0^\circ \text{ F.}$; $H_A = H_{f,2} = 196.1$; $S_A = S_{f,2} = 0.3356$.

Water leaving pumps: $P_B = P_1 = 300 \times 144$; $V_B = V_{f,2} = 0.0168$; $t = 228.0^\circ \text{ F.}$; $H_B = 196.1 + 144 \times (300 - 20) \times 0.0168/778 = 197.0$; $S_B = S_A = 0.3356$.

Steam leaving boiler: $P_D = 300 \times 144$; $V_D = 0.99 \times 1.541 = 1.526$; $t = 417.3^\circ \text{ F.}$; $H_D = 393.9 + 0.99 \times 808.5 = 1194.2$; $S_D = 0.5883 + 0.99 \times 0.9220 = 1.5010$.

Steam leaving engine: $P_E = 20 \times 144$; $t = 228.0^\circ \text{ F.}$; $S_E = S_D = 1.5010$; $x_E = (1.5010 - 0.3356)/1.3960 = 0.835$; $H_E = 196.1 + 0.835 \times 959.9 = 997.6$; $V_E = 0.832 \times 20.10 = 16.73$.

Pump work $= 144 (300 - 20) \times 0.0168 = 680 \text{ ft-lb.} = 0.87 \text{ B.t.u. per lb.}$

Engine work $= 778 (997.6 - 1194.2) = -153,000 \text{ ft-lb.} = -196.6 \text{ B.t.u. per lb.}$

Net work output $= -153,000 + 680 = -152,300 \text{ ft-lb.} = -195.7 \text{ B.t.u. per lb.}$

Energy supplied as heat $= 1194.2 - 197.0 = 997.2 \text{ B.t.u. per lb.}$

Energy discard $= 196.1 - 997.6 = -801.5 \text{ B.t.u. per lb.}$

Steam ideally required per horsepower-hour of plant output $= (33,000 \times 60)/152,300 = 13.00 \text{ lb. (or } = 2545/195.7), = \text{the ideal "steam rate" of cycle.}$

Coal required per horsepower-hour of plant output $= \text{B.t.u. supplied to steam per horsepower-hour divided by B.t.u. supplied to steam per pound of coal} = (13.00 \times 997.2)/(13,500 \times 0.70) = 1.37, = \text{the ideal "fuel rate" of plant.}$

Example 2. Check from the Mollier Chart (and from the Pressure-entropy table if available) the values of H obtained in example 1 at the states leaving the boiler and leaving the engine. Sketch the cycle to approximate scale on H - S , T - S , and P - V coordinates and indicate by distances or areas the various heat and work quantities. Also check the value of H_E by equation 8a of Art. 68.

Example 3. A Rankine cycle type of steam plant is operating with a steam pressure of 275.3 lb. per sq. in. gage and a temperature of 600° F. and with a condenser vacuum of 28.4 in. Hg. (with a 29.9 in. barometer). Compute the pump work, the engine work, the heat supplied to the steam via the boiler, and the energy discarded via the condenser (all per pound of steam) if the plant might operate on the ideal cycle. (0.86; 419.1; 1253.5; 835.3 B.t.u. per lb.)

How many pounds of steam would be required per horsepower-hour of plant output and what would be the fuel requirements of the plant if it were delivering 10,000 hp. with oil fuel of 19,000 B.t.u. per lb. and with a furnace and boiler efficiency of 75 per cent? What power would be required to drive the pumps?

(6.07 lb. per hp-hr.; 5330 lb. per hr.; 20.6 hp.)

122. Thermal Efficiency, Rankine Cycle and Rankine Engine. — It was developed in the preceding paragraphs that for the Rankine cycle (employing the same subscripts as above),

Energy transition as Heat

Supplied to system (via boiler etc.) $= H_D - H_B$

Discarded from system (via condenser) $= H_E - H_A$

Difference $= (H_D - H_B) - (H_E - H_A)$
 $= (H_D - H_E) - (H_B - H_A).$

Energy transition as Work

$$\text{Engine work output} = H_D - H_E$$

$$\text{Work required for pumping} = H_B - H_A$$

$$\text{Net work output from system} = (H_D - H_E) - (H_B - H_A).^5$$

Defining the **thermal efficiency** of a heat engine cycle, or of a heat engine alone, as *the ratio between the amount of work output and the amount of heat energy which must be supplied in order to accomplish that output*, or

$$\text{Thermal efficiency } (e_t) = \frac{\text{Work output from system}}{\text{Heat supplied to system}}, \quad (5)$$

one may readily express specifically the thermal efficiency for any cycle such as the Rankine. Thus, in terms of energy quantities per pound of fluid, and continuing for the moment to employ the subscripts used above,

$$\text{Thermal efficiency, Rankine cycle } (e_R) = \frac{(H_D - H_E) - (H_B - H_A)}{(H_D - H_B)},$$

or, for convenience, expanding the denominator by the addition and subtraction of H_A ,

$$e_R = \frac{(H_D - H_E) - (H_B - H_A)}{(H_D - H_A) - (H_B - H_A)}.$$

Scrutinizing this latter expression it appears that both the numerator and the denominator of the efficiency ratio have been caused to contain the term $(H_B - H_A)$, which is recognized as the amount of the pump work, $(P_1 - P_2) V_f$. Referring to examples 1 to 3 it appears that in those examples this term had a magnitude of less than 0.1 of one per cent of the denominator and of less than 0.5 of one per cent of the numerator. As these relative magnitudes are fairly representative of all conditions where the boiler pressure is moderate, the cycle efficiency is frequently expressed approximately by the ratio

$$\text{Thermal efficiency, Rankine cycle} = (\text{approx.}) \frac{H_D - H_E}{H_D - H_A},$$

in which the pump work obviously is neglected.

⁵ It is axiomatic that for conformity with the First Law the energy quantities of the cycle must "balance." That they have balanced is evident from inspection of this expression and of the preceding one for the difference of the heat supplied and that discarded.

To express these relations in general terms,

$$\text{Thermal efficiency, Rankine cycle} = \frac{(H_1 - H_2)_S - (P_1 - P_2)V_{f,2}/J}{(H_1 - H_{f,2}) - (P_1 - P_2)V_{f,2}/J} \quad (6)$$

$$= (\text{approx.}) \frac{(H_1 - H_2)_S}{H_1 - H_{f,2}}; \quad (6a)$$

where

H_1 = enthalpy of the steam as supplied at P_1 ,
 $(H_1 - H_2)_S$ = decrease of enthalpy by isentropic expansion to P_2 ,

$H_{f,2}$ = enthalpy of saturated water at P_2 ,

$V_{f,2}$ = specific volume of saturated water at P_2 .

The error arising from use of the approximation is usually regarded as negligible except when the modern higher boiler pressures (500 lb. per sq. in. and up) are employed.

It is frequently desired to evaluate the ideal thermal efficiency of an engine alone. If the engine is operating in a plant cycle the prototype of which is the Rankine cycle,⁶ in which the heat requirements for the operation of the cycle *and thus of the engine* would be that as already determined, then

$$\text{Thermal efficiency, Rankine engine} = \frac{(H_1 - H_2)_S}{(H_1 - H_{f,2}) - (P_1 - P_2)V_{f,2}/J} \quad (7)$$

$$= (\text{approx.}) \frac{(H_1 - H_2)_S}{H_1 - H_{f,2}}. \quad (7a)$$

This latter approximation and the approximate equation 6a are seen to be identical. However note that there is no approximation in the numerator of equation 7a.

Example 4. Compute the Rankine cycle thermal efficiencies and the thermal efficiency of the engines of examples 1 and 3. (19.7 per cent; 33.5 per cent.)

123. Factors Influencing Rankine Cycle Thermal Efficiency. — Although, for reasons which will be analyzed in a subsequent article, the thermal efficiency of a Rankine cycle which operates through a given temperature range may not equal that of a Carnot cycle for the same range, it will nevertheless be influenced by the same conditions. Thus, quite as the efficiency of the Carnot cycle $[= (T_1 - T_2)/T_1]$ is increased by increase of the temperature T_1 of the fluid during its energy reception from the source or by decrease of the temperature T_2 during the discarding of the unavailable energy to the receiver, so the Rankine efficiency is increased by increase of the temperature of the steam leaving

⁶ This would be regarded as being the case if no steam were extracted from the engine for purposes of feed-water heating and if reheating were not employed. See also Arts. 125 and 126.

the boiler or the superheater or by decrease of the temperature of the steam passing to the condenser. However the specific degree of effect of such temperature changes differs in the two cycles. Also it is to be noted that by virtue of the property characteristics of a vapor the *average* temperature of the fluid during energy reception may in the Rankine cycle be increased by two alternative means, (a) by increase of the boiler pressure and thus of the saturation temperature during vaporization or (b) by increase in the amount of superheating after vaporization. The vapor (steam) temperature during energy rejection may be reduced only by reduction of the pressure to which the vapor expands in the engine and at which it therefore departs from the engine and is condensed.

The fact of these influences of boiler pressure, of superheater temperatures, and of condenser pressure becomes quite evident from analysis of the H - S diagram of Fig. 57 or the T - S diagram of Fig. 58. However,

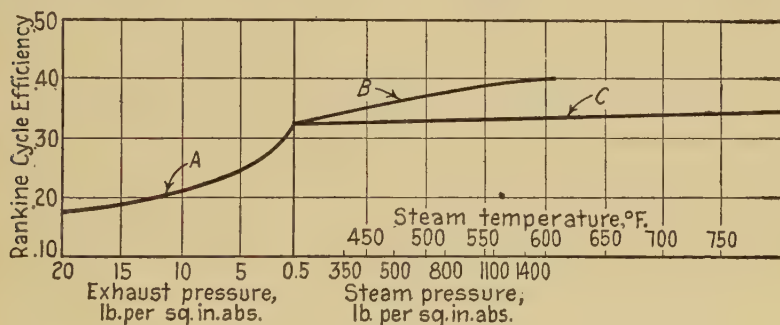


FIG. 60

to illustrate their influence more concretely there have been plotted in Fig. 60 curves showing the change of the efficiency of a Rankine cycle as the various operating conditions are caused to depart individually from some standard set of conditions. The standard conditions for the figure, which conditions have been selected wholly arbitrarily, are those of a steam supply at 215 lb. per sq. in. abs., saturated and 100 per cent quality and an exhaust pressure of 0.5 lb. per sq. in. abs. (= 1.02 in. Hg.). The curve labeled *A* shows the effect of increased exhaust steam pressure; that labeled *B* shows the effect of increased steam pressure (without superheat); and the one labeled *C*, the effect of superheating the steam at the standard supply pressure.⁷

⁷ In the figure a uniform scale of temperature is employed but such a variable pressure scale as to coördinate saturation pressures with their corresponding temperatures. This will serve to indicate more clearly the relative influence of increased temperature as that may be produced by increased pressure or by increased superheat.

Scrutinizing the curves, it would appear from curve *A* that the most effective means of altering (increasing) the efficiency is by variation (decrease) of the exhaust pressure and temperature, and in fact the development in the art which has evolved the modern high-vacuum condensing plant has been a major factor contributing to the great improvements in steam power plant performances. However, it is to be recognized that unfortunately the temperature of the atmosphere or of the sources from which the cooling water for the condenser may be obtained imposes a lower limit on the practicably obtainable exhaust temperature and pressure. With modern condensing apparatus and procedure this lower limit is being very closely approached.

From the evidence of curve *B* the procedure which is next in effectiveness is that of steam pressure increase. For this reason engineering progress has been marked by steadily rising steam pressures. To illustrate, the earlier steam plants employed boiler pressures of some 15 to 30 lb. gage, whereas at this writing pressures of 400 to 600 lb. per sq. in. are common among modern shore plants and 1200 to 1400 lb. are being used in a number of installations. It is obvious that this rise may no more than keep pace with the developments in the art of producing equipment which will safely withstand these high pressures.

The procedure which appears to be of the least theoretical effectiveness is that of superheating. However, superheating is found to give actual benefits in efficiency which exceed the theoretical, this due to several circumstances but primarily to the facts (a) that the existence of moisture in the steam in its passage through a turbine or a reciprocating engine is particularly conducive to loss of efficiency and (b) the appearance of moisture during such passage is either deferred or wholly prevented by the use of superheat (see Fig. 57). As a consequence superheat is universally employed in modern shore plants and at this writing steam temperatures of 650° to 750° F. are common. Higher temperatures have not commonly been employed because of a serious and progressive loss of strength of modern commercial metals (iron and steel and its alloys) with increase of temperature in excess of about 750° to 800° F., whence further advance in the direction of increased superheat must await metallurgical developments.

It is thus to be understood that gains in efficiency may be secured by increased steam pressure, by increased superheat and by increased condenser vacuum, but it is further to be understood that for actual realization of these gains the plant must be designed for operation at the modified conditions and that with an existing plant an attempt at performance betterment by increase of steam pressure, superheat, or

vacuum may easily be futile if the plant is not designed for such operation.

Example 5. Illustrate by use of the H - S diagram the influence (a) of increased steam pressure on efficiency, (b) of increased superheat, (c) of decreased exhaust pressure, and (d) of throttling of the steam supply to the engine.

Example 6. Compute the percentage gain in thermal efficiency of the cycle of example 1 if the exhaust pressure is reduced to 1.5 in. Hg. abs.

Example 7. Compute the percentage gain in thermal efficiency of the cycle of example 3 if (a) the steam pressure is increased to 550 lb. per sq. in. gage, with the same superheat temperature, (b) if the steam temperature is increased to 750° F., with the same steam pressure, (c) if the steam pressure is increased to 550 lb. and the steam temperature to 750° F.

124. Handicaps of the Rankine Cycle; Modern Cycles. — It was remarked in Art. 123 that, for a like temperature range in each, the thermal efficiency of the Rankine cycle may not equal that of the Carnot, it being recalled that the latter cycle shall be one in which the energy supply is delivered to the fluid while its temperature is maintained constant and in which similarly the fluid temperature shall be constant during the discarding of the unavailable energy. The fact of this lesser efficiency is well illustrated by the results of examples 1 and 3. In those examples the Rankine cycle thermal efficiencies were respectively 0.197 and 0.336 (as per the results of example 4), whereas the Carnot efficiency for the temperature ranges of the two cycles are respectively $[(417.3 - 228.0)/(417.3 + 460) =]$ 0.216 and $[(600 - 91.8)/(600 + 460) =]$ 0.479.

A general reason for this lesser efficiency of the Rankine cycle becomes evident from inspection of the T - S diagrams of 61 and 62. Thus, referring to Fig. 61, which is representative of the conditions of example 1, the energy required for the feed-water warming from state B to state C is measured by the area $aBCf$. Of this energy there is available for transformation to shaft-work an amount measured by the triangular area BCF . In distinction, the energy supplied for the vaporization from C to D is measured by the area $fCDe$, and of this energy the portion measured by the rectangular area $FCDE$ is available. The simple geometry of the figure makes quite apparent the relatively greater inherent degree of availability of the energy received at the constant upper temperature T_1 , as compared with that received during the warming from T_2 to T_1 . It is obvious that only that portion which is received at T_1 might be utilized with Carnot efficiency. Thus the

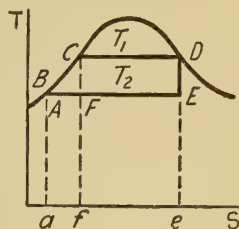


FIG. 61

discover the anomolous circumstance that the same feed heating may be accomplished in such a manner that its deleterious influence is obviated or much reduced. Thus it becomes desirable to investigate further in order to ascertain a more exact or basic cause for the lesser efficiency of the Rankine cycle and to discern the principle or law the respective violation or observance of which in the feed-heating phase will or will not produce the reduced efficiency.

To discern this principle it is necessary only to observe that in the Rankine cycle as described in Art. 121 the cool condensate (at T_2) is introduced directly into the boiler and there mixes with the hot boiler water which is at the saturation temperature corresponding to the upper pressure of the cycle. The thermal irreversibility of this mixing process is obvious (see Art. 21), and the reader should recall (Art. 56*b*) the inescapable increase of entropy and consequent decrease of available efficiency which must result from such an irreversible mixing.⁸ At least one objective of the following may be regarded as that of indicating that serious thermal irreversibility occurring in any process whatsoever in the generation of power by heat engine cycles must appreciably contribute to inefficiency.

It will be recalled that in the Carnot cycle the return of the fluid from the lower to the upper temperature of the cycle is accomplished reversibly by an adiabatic compression. To duplicate the procedure in a

⁸ An alternative arrangement of the Rankine cycle would be one as shown in Fig. 63, in which the feed-water is heated outside of the boiler in a "heater" which is activated by high-pressure saturated steam from the boiler. It is to be taken that just sufficient steam is provided to the heater to accomplish the feed-water heating by the condensation of the steam and that the condensed steam is returned to the boiler. In this arrangement the thermal irreversibility is identical in its deleterious influence but now arises not from mixing but from irreversible *heat transfer* in the heater from the hot condensing steam to the feed-water, the temperature of which ranges from T_2 to the final saturation temperature corresponding to the upper pressure. The thermal efficiency of the cycle in the two arrangements will be identical, as is shown in the following.

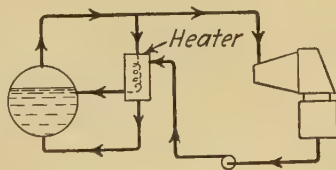


FIG. 63

The energy equation for the heater is, $M_h(H_{g,1} - H_{f,1}) = M_e(H_{f,1} - H_{f,2})$, where M_h = the mass of steam supplied to the heater per unit time and M_e = the remaining mass of steam passing to the engine, condensing and subsequently passing as water to the heater. The boiler would vaporize water in the amount $(M_h + M_e)$ and would supply energy in the amount $Q_b = (M_h + M_e)(H_{g,1} - H_{f,1}) =$ (by the above heater equation) $M_e(H_{g,1} - H_{f,2})$. The engine delivers work in the amount $M_e(H_{g,1} - H_2)S$. Compare the resulting efficiency with that formulated in equation 6*a*.

steam plant it would be necessary that the vapor be withdrawn from the condenser when only partially condensed, as for example in state F of Fig. 61, and then be brought to state C by isentropic compression. Such a procedure is not regarded as practicable in actual steam plant design and operation. Fortunately, however, the return of the cooler condensate to the saturation temperature at the boiler may still be accomplished ideally in a reversible but wholly different manner.

To illustrate, this ideal reversible return might be obtained or closely approached in principle if (a) a progressive warming of the condensate were effected by heat conduction in a large number of successive, steam-actuated feed-water heaters in each of which the temperature of the condensing steam exceeded only infinitesimally the temperature of the water, whereby the thermal irreversibility of the Rankine cycle might be avoided, and if (b) the steam employed in each heater had been brought to the pressure and temperature at which it condensed in a wholly reversible manner. To make this idealized conception more concrete

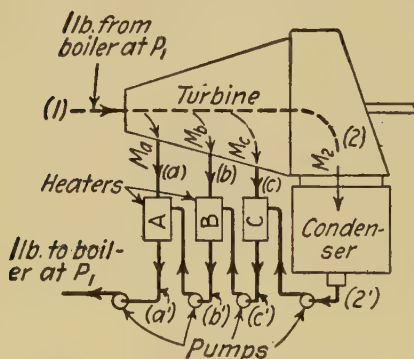


FIG. 64. Regenerative feed-water heating.

let us observe the actual manner in which the general idea is applied in modern steam power plants, and then see subsequently how the practical procedure might ideally be so extended as to permit the approach to complete reversibility and to Carnot efficiency.

The practical cycle is known as the **regenerative (or "stage") feed-heating cycle**. Except as the feed pump arrangement et cetera may be modified for purposes of convenience or simplicity, the regenerative plant is arranged as shown

in Fig. 64. The distinctive characteristics of the cycle as portrayed in the figure are:

- (a) the extraction or "bleeding" of steam from several intermediate points in the engine, these being points at which expansion of the steam has proceeded to several successive pressures which are intermediate between the supply and exhaust pressures;
- (b) the continued expansion of the remaining steam through the engine to the condenser and its condensation therein; and
- (c) the delivery of the bled steam to heaters through which the

feed-water is passed successively and in which it is warmed progressively by energy furnished by the condensation of the bled steam.

From the first characteristic it appears that the cycle is applicable to plants in which the prime mover is a multi-stage reciprocating engine, from which steam may be bled between stages; or when a turbine is used it needs only be so designed and constructed as to permit steam extraction at points along the casing after various degrees of expansion. The number of points employed in an actual turbine plant is dictated by considerations not only of thermal efficiency gains but also of first costs, operating conditions and structural complications. Frequently three points are employed, with two or four not uncommon. In order to accomplish the basic purpose of the cycle (the minimization of the thermal irreversibility associated with the feed-water heating phase) the pressures at the extraction points should in general be so selected that the saturation temperatures of the steam as bled shall be about uniformly distributed as between the saturation temperature in the boiler and that in the condenser.

Further characteristics of the cycle as shown in Fig. 64, but with the cycle idealized to some degree, are:

- (d) that the condensed steam from any heater shall leave that heater at the saturation temperature corresponding to the heater pressure;
- (e) that the water passing to and through a heater shall be warmed to the saturation temperature corresponding to the heater pressure;
- (f) that the condensed steam and warmed water from any heater may thus join and be pumped jointly to the heater of next higher pressure;⁹
- (g) that the expansions of the steam in the engine shall be ideal isentropic ones.

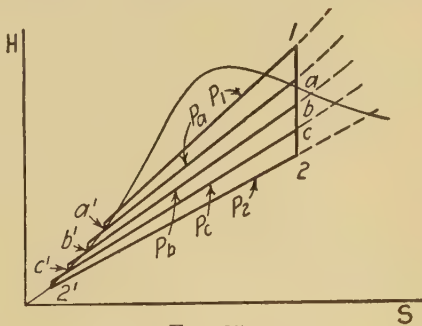


FIG. 65

The several significant states of the supplied and bled steam, the exhaust steam and the water are depicted on H - S coordinates in Fig. 65,

⁹ See also footnote 11.

like letters being used to designate the states of the fluid and the various corresponding points in the cycle of Fig. 64 at which those states exist.¹⁰ The amount of steam supplied to the engine in any given time will be regarded as relatively unity in the following energy analysis; that bled to the first heater will be designated as M_a ; that to the second heater, M_b ; that to the third heater, M_c ; and that finally passing to the condenser, $M_2 (= 1 - M_a - M_b - M_c)$.

The following energy equations for each of the several successive heaters and for the complete plant will serve to indicate the manner of ascertaining the requisite distribution of the steam to the heaters and to the condenser and also the manner of determining the ideal work output of the turbine and the necessary energy supply via the boiler, and thus the cycle efficiency.

For heater A the energy equation becomes, per pound of steam to the engine,

$$M_a H_a + (1 - M_a)[H_{b'} + (P_a - P_b)V_b/J] = H_{a'}.$$

Neglecting the feed-pump work item $(P_a - P_b)V_b$, and recognizing that for the idealized heater $H_{b'} = H_{f,b}$ and $H_{a'} = H_{f,a}$, the energy equation may be written approximately as

$$M_a H_a + (1 - M_a) H_{f,b} = H_{f,a}; \quad \text{or} \quad M_a = \frac{H_{f,a} - H_{f,b}}{H_a - H_{f,b}}. \quad (8)$$

For the remaining heaters, under similarly idealized conditions and again neglecting the feed-pump work terms:

For heater B ,

$$M_b H_b + (1 - M_a - M_b) H_{f,c} = (1 - M_a) H_{f,b}; \quad \text{or} \\ M_b = (1 - M_a) \frac{H_{f,b} - H_{f,c}}{H_b - H_{f,c}}. \quad (9)$$

For heater C ,

$$M_c H_c + (1 - M_a - M_b - M_c) H_{f,2} = (1 - M_a - M_b) H_{f,c}; \quad \text{or} \\ M_c = (1 - M_a - M_b) \frac{H_{f,c} - H_{f,2}}{H_c - H_{f,2}}. \quad (10)$$

For the engine,

Work output per pound of steam supplied

$$= (H_1 - H_a) + (1 - M_a)(H_a - H_b) + (1 - M_a - M_b)(H_b - H_c) \\ + (1 - M_a - M_b - M_c)(H_c - H_2). \quad (11)$$

¹⁰ Note that the quantitative values of H or S at any point in the H - S diagram are values for *one pound* of the fluid, whereas any individual pound originally coming to the engine of the regenerative cycle subsequently divides and its parts exist simultaneously at different states in various parts of the system. For that reason the figure may not be regarded as an exact graphic representation of the cycle.

For the boiler (plus superheater),

$$\text{Heat supplied per pound of steam formed} = (H_1 - H_{f,a}). \quad (12)$$

To illustrate the manner of analysis of the cycle and to verify the expectation of improved thermal efficiency as compared with that of a Rankine cycle the following example is solved in detail.

Example 8. Steam is supplied in a turbine plant at 300 lb. per sq. in. abs. and 520° F. The condenser pressure is 1 in. Hg. abs. Bleeders for stage feed-heating are provided at points along the turbine after expansion to pressures of 107 and 29 lb. and 10.2 in. Hg. abs. Find (a) the amount of steam required in each heater per pound of steam to the turbine, (b) the work output from the turbine per pound of steam supplied, (c) the energy supplied in the boiler and superheater per pound of steam produced, (d) the thermal efficiency of the plant, and (e) the pounds of steam generated per horsepower-hour output. Compare the results of items (b) to (e) with those items for a straight Rankine cycle operating with the same steam supply and condenser pressures.

Solution.

At 300 lb. and 520° F., $H_1 = 1268.5$. After isentropic expansion from this state to 107 lb., 29 lb., 10.2 in. Hg. and 1 in. Hg. the enthalpies are respectively 1175.8, 1078.1, 967.8 and 849.9 B.t.u. per lb. The enthalpies of saturated water at those pressures are, respectively, 303.5, 216.8, 130.1 and 47.1 B.t.u. per lb.

Per pound of steam to the turbine the amounts bled to the several heaters, the work output from the turbine, the heat supplied et cetera are:

$$M_a = \frac{303.5 - 216.8}{1175.8 - 216.8} = 0.0905 \text{ lb.}$$

$$M_b = (1 - 0.0905) \frac{216.8 - 130.1}{1078.1 - 130.1} = 0.0832 \text{ lb.}$$

$$M_c = (1 - 0.0905 - 0.0832) \frac{130.1 - 47.1}{967.8 - 47.1} = 0.0746 \text{ lb.}$$

Total steam bled = 0.248 lb.; steam passing to condenser = 0.752 lb.

Work output of turbine = $(1268.5 - 1175.8) + 0.0905(1175.8 - 1078.1) + 0.8263(1078.1 - 967.8) + 0.752(967.8 - 849.9) = 361.5 \text{ B.t.u.}$

Heat supplied via boiler and superheater = $(1268.5 - 303.5) = 965.0 \text{ B.t.u.}$

Thermal efficiency of cycle = $361.5/965.0 = 0.375 = 37.5 \text{ per cent.}$

Steam generated per horsepower-hour = $2546/361.5 = 7.04 \text{ lb.}$

For a Rankine cycle:

Work per pound = $1268.5 - 849.9 = 418.6 \text{ B.t.u.}$

Heat per pound = $1268.5 - 47.1 = 1221.4 \text{ B.t.u.}$

Thermal efficiency = $418.6/1221.4 = 0.343 = 34.3 \text{ per cent.}$

Steam per horsepower-hour = $2545/418.6 = 6.09 \text{ lb.}$

Scrutinizing the results of this typical example the very interesting features appear that although the steam to be supplied to the turbine per horsepower-hour is greater in the regenerative cycle than in the Rankine (although only 87 per cent as much passes to the condenser)

the thermal efficiency of the regenerative cycle is 3.2 per cent greater. This represents an improvement of 9.3 per cent over the Rankine performance.¹¹

This betterment of efficiency is occasioned basically by the lesser temperature differences and consequent lesser degrees of thermal irreversibility associated with the heat transition from the cooler steam to the coldest water, from the moderately warm steam to the partially warmed water, et cetera, combined necessarily with an optimum realization of the available energy of the several masses of heater steam prior to their extraction for the purposes of feed-heating.¹² It would now seem reasonable to reassert the principle which was stated earlier in the article, that (for a cycle operating with saturated steam supply) if the number of extraction points and heaters were so increased as to approach infinity, whereby the temperature differences between the heater steam and the water in any individual heater might approach zero, then the feed-water heating would in the same degree approach reversibility and the efficiency of the cycle would approach the Carnot efficiency. Obviously such an arrangement would be absurdly impractical but, also obviously, the benefits of stage feed-heating are very real and worth while.

The foregoing has served to indicate in particular the benefits of reduction in the degree of thermal irreversibility in the feed-heating phase of a cycle. It should also serve to validate certain broader but very significant further deductions, that

¹¹ In the usual regenerative plant as actually constructed the series of pumps represented in Fig. 64 is not regarded as desirable and is dispensed with by conducting the drains of any heater to the steam space of the heater of next lower pressure or to a common hot-well, or is otherwise disposed of in such manner as appears most desirable to the designer. By such arrangements the feed-water may be delivered through the series of heaters by one or two pumps, but a slightly different distribution of the steam to the various heaters is obtained and some increase of thermal irreversibility, throttling loss, and decrease of plant efficiency is incurred.

¹² The design and construction of many simpler steam plants ashore and most marine plants has been such that some feed-heating is accomplished in a single heater which is supplied with steam at or somewhat above atmospheric pressure. The steam used is that exhausted from various smaller engines which are employed for driving the pumps, et cetera. Such feed-heating has certain practical objectives and would also have some influence toward improved plant efficiency if the auxiliary engines were efficient but, unfortunately, that influence has too frequently been more than neutralized by the use of extremely inefficient auxiliaries. In the more modern plants in which regenerative heating is employed it is a common practice to drive the auxiliary machinery by electric motors the energy for which is obtained very efficiently from a single larger turbo-generator or from the main turbo-generators of the plant.

- (a) *The presence in a plant of any process whatsoever in which energy is transferred as heat from a fluid at higher temperature to a region at lower temperature must invariably tend to reduce the thermal efficiency of the plant as a whole.*
- (b) *The magnitude of this efficiency reduction is increasingly greater as the temperature difference is large.*
- (c) *The loss is minimized as means are provided whereby the hot fluid shall first be caused to deliver the available portion of its energy as work by efficient expansion in an engine, with a concurrent temperature drop to such a temperature that the heat transition may take place with a temperature differential which is as small as is practicable or as economic conditions will permit.*

These deductions are simply recognitions of the practical aspects of the Carnot Principle and of the Second Law and of the losses associated with thermal irreversibility. The importance of the reduction of mechanical or fluid friction (mechanical irreversibility) is to an extent self-evident to everyone and has long been recognized. The equal importance of the reduction of thermal irreversibility has too frequently been overlooked. In this connection it may be remarked that the efforts toward the increase of the temperature of the steam supply in the steam power plant may be regarded alternatively as an effort toward the reduction of the degree of thermal irreversibility associated with the heat transition from the extremely hot furnace and furnace gases to the steam. This highly irreversible phase of the steam plant process constitutes in a sense the major handicap toward the attainment of steam plant efficiencies approaching those of the internal-combustion engine. In the latter engine such a phase does not exist because the hot gases themselves act as the working fluid of the machine.

Example 9. Compute the ideal thermal efficiency for a turbine plant operating on the regenerative cycle with the steam supply at 500 lb. gage and 627° F. and with steam extraction for feed-heating at 120 lb. gage and at atmospheric pressure. The condenser pressure is 1.0 in. Hg.

The progressive analysis of the regenerative cycle according to the schedule of equations 8 to 12 is rather the most illuminating method. However, for a three-heater installation equations 9 and 10 may be combined with equation 8 to give the following independent expressions:

$$M_B = \left(\frac{H_a - H_{f,a}}{H_a - H_{f,b}} \right) \left(\frac{H_{f,b} - H_{f,c}}{H_b - H_{f,c}} \right) \quad (9a)$$

$$M_C = \left(\frac{H_a - H_{f,a}}{H_a - H_{f,b}} \right) \left(\frac{H_b - H_{f,b}}{H_b - H_{f,c}} \right) \left(\frac{H_{f,c} - H_{f,2}}{H_c - H_{f,2}} \right) \quad (10a)$$

Throughout these relations the previous approximation is made of neglecting the feed-pump works. With the arrangement of heaters, heater drains, and pumps as shown in Fig. 64 the individual and total pump work items would be, in foot-pounds per pound of steam generated,

$$(1 - M_A - M_B - M_C)(P_c - P_2)V_{f,2} + (1 - M_A - M_B)(P_b - P_c)V_{f,c} + (1 - M_A)(P_a - P_b)V_{f,b} + (P_1 - P_a)V_{f,a}. \quad (13)$$

For the data of example 8 the individual and total pump-works would be $(8 + 47 + 173 + 495 =) 723$ ft.-lb. or 0.93 B.t.u. This quantity is 0.25 per cent of the turbine output and 0.1 per cent of the heat supply and so it may frequently be regarded as relatively negligible.

126. Reheating and Reheating-Regenerative Cycles. — The reheating cycle is one in which an original superheated steam supply is partially expanded through the high-pressure portion of an engine and is then returned to the boiler-room and resuperheated, after which the expansion is completed through the low-pressure section of the engine. The general arrangement of the cycle is portrayed in Fig. 66, and in Fig. 67 the successive states of the working fluid throughout the cycle are indicated on H - S coordinates.

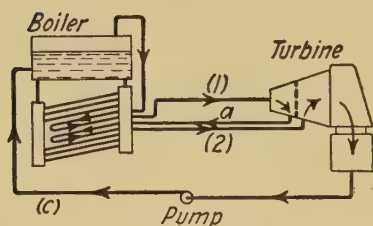


FIG. 66. Reheating cycle.

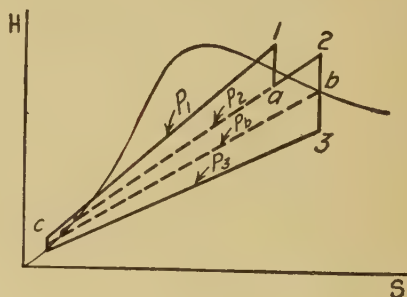


FIG. 67

Evidently the arrangement offers a method whereby advantage may be taken of the higher efficiency possibilities which result inherently from increase of the average temperature at which the working fluid receives its energy from the source. Alternatively it may be regarded as providing for a dual realization of the more highly available energy received during superheating (see also Art. 123 and Fig. 62). A further practical benefit of reheating also arises from the evident deferring or even the avoidance of condensation during the expansion. The appearance of moisture during expansion has a particularly serious influence on the fluid friction losses and the consequent efficiency of the turbine

type of engine. The cycle has its greatest utility in those more modern plants in which boiler pressures of 1000 lb. per sq. in. or more are employed, whereby the resuperheating may be done at steam pressure of perhaps 350 lb. or more and thus on steam which still is of quite moderate specific volume. Impracticably large return piping and reheaters are thereby avoided. The major disadvantage of the arrangement obviously lies in this very necessity for repiping the steam to the fire-room and for providing the second superheater.¹³

The energy analysis of the ideal cycle is very like that for the Rankine, differing only in the necessity for accounting for the additional energy supplied in the reheater and for recognizing the two-stage character of the expansion. Thus, employing the symbols which are used in the diagrams,

$$\begin{aligned}\text{Total energy supplied} &= (H_1 - H_c) + (H_2 - H_a) \\ &= (H_1 - H_a) + (H_2 - H_{f,3}) \\ &\quad - (P_1 - P_3)V_{f,3}/J.\end{aligned}\tag{14}$$

$$\text{Total engine work} = (H_1 - H_a) + (H_2 - H_3).\tag{15}$$

$$\text{Pump work} = (P_1 - P_3)V_{f,3}/J.\tag{16}$$

$$\text{Net work} = (H_1 - H_a) + (H_2 - H_3) - (P_1 - P_3)V_{f,3}/J.\tag{17}$$

Thermal efficiency of cycle

$$= \frac{(H_1 - H_a) + (H_2 - H_3) - (P_1 - P_3)V_{f,3}/J}{(H_1 - H_a) + (H_2 - H_{f,3}) - (P_1 - P_3)V_{f,3}/J}\tag{18}$$

$$= \frac{(H_1 - H_a) + (H_2 - H_3)}{(H_1 - H_a) + (H_2 - H_{f,3})} \text{ (approximately).}\tag{18a}$$

Observe that for the ideal cycle the values of H_a and H_3 are those after isentropic expansion from states 1 and 2 respectively.

Example 10. For an ideal reheating cycle with steam supplied at 1200 lb. per sq. in. abs. and 700° F., reheating at 250 lb. to 600° F. and exhaust at 1 in. Hg., compute for the ideal cycle (a) the total energy supplied per pound of steam, (b) the engine work, (c) the pump work, (d) the net work, (e) the thermal efficiency of the engine and of the cycle and (f) the steam required per horsepower-hour output of engine.
(1412.2; 577.0; 3.6; 573.4; 0.406; 4.41.)

¹³ To avoid the first of these disadvantages it has at times been suggested that the reheating be done through the agency of high-pressure superheated steam which is taken directly from the main steam supply and is delivered to the heater through which the medium-pressure steam is passed. This procedure serves only to realize the benefits accruing from the reduction or possible avoidance of condensation during the second expansion. Any thermodynamic gain arising from the resuperheating is obviated by the thermal irreversibility of the heat transfer in the heater, or alternatively by the failure to secure the "available" energy from the steam which has been so diverted to the heater.

In a plant in which reheating is employed it is advantageous also to use several stages of regenerative feed-water heating, the first heater usually taking its steam from the line which returns to the reheater and the subsequent heater or heaters bleeding their supply after the reheated steam has partially completed its expansion. In Fig. 67 the states of the heater-supply steam would be represented by points such as *a* and *b* respectively. By direct analogy with the methods of Art. 125 and example 8, analysis and computations may readily be made to ascertain the amount of steam required for the several heaters. Thus, for a two-heater installation and neglecting the feed-pump work,

$$\text{Steam to first heater} = M_a = \frac{H_{f,a} - H_{f,b}}{H_a - H_{f,b}}, \text{ lb. per pound of steam to engine};$$

$$\text{Steam to second heater} = M_b = (1 - M_a) \frac{H_{f,b} - H_{f,3}}{H_b - H_{f,3}}.$$

It follows that

Work per pound to engine =

$$(H_1 - H_a) + (1 - M_a)(H_2 - H_b) + (1 - M_a - M_b)(H_b - H_3) \quad (19)$$

Heat supplied per pound to engine =

$$(H_1 - H_{f,a}) + (1 - M_a)(H_2 - H_a). \quad (20)$$

Example 11. For the reheating cycle of example 10, except provided with regenerative feed-water heaters at the reheating pressure (250 lb. abs.) and at 20 lb. abs., compute the items called for in the specified example. (Eff. = 0.445.)

127. Multiple-Fluid Cycles. — In view of the abundant supplies of

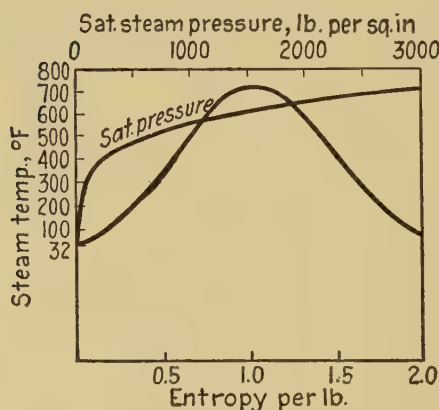


FIG. 68

water which nature has fortunately provided we shall very probably continue to use water vapor as a primary fluid medium in our temperature engine cycles so long as we continue to employ vapor cycles. However, it is to be recognized that water vapor possesses certain inherent disadvantages for the purpose. One of these has been suggested in Art. 124 and will be discussed further and others will be cited in the following. It may perhaps not be amiss in this connection to philosophize briefly concerning the characteristics which might be possessed by a more favorable fluid. For these discussions reference

be possessed by a more favorable fluid. For these discussions reference

may be made with advantage to the combined T - S and T - P diagram for water as that appears in Fig. 68.

The particular characteristics of a fluid which affect most definitely its desirability are perhaps the following:

(a) *Temperature and Pressure at Saturation.* Mention has already been made of the thermodynamic advantage of employing high saturation temperatures, due to the fact that the majority of the energy reception by a vapor occurs during vaporization, whereby the vaporization temperature is outstandingly effective in establishing the average upper temperature of the vapor cycle. However, attention has also been directed to the practical difficulties in the use of high saturation temperatures with water vapor due to the rapid increase of the saturation pressure of water with material increase of temperature (see the T - P curve of Fig. 68). Obviously a more favorable fluid would be one for which the saturation pressures would not increase so abruptly at higher temperatures. Of the various fluids which have been investigated with reference to this characteristic, mercury is the only one which at this writing is employed in practice. There appears a brief description of the mercury vapor cycle in a subsequent paragraph.

Water has an analogous but opposite disadvantage at the lower temperature end of the vapor cycle. At our lowest continuously available temperatures (of the atmosphere and of natural water supplies) the saturation pressure of water vapor is less than 1 lb. per sq. in. abs., whence it becomes necessary that high-vacuum condensing equipment shall be provided in order that the thermodynamic benefits of low receiver temperature may be realized. A more favorable fluid might be one for which the saturation pressure at atmospheric temperatures should exceed in some degree the normal atmospheric pressure, whereby the bother and expense of condensing under vacuum might be avoided. Such a fluid would probably exhibit the additional advantage of moderate specific volume at normal receiver temperatures. At this writing no such fluid has been employed in practice, but the introduction of such a fluid is not at all inconceivable.

(b) *Specific Heat of the Liquid and Latent Heat of Vaporization.* It has been remarked before that unless recourse is had to regenerative feed-heating the liquid of the vapor cycle must be warmed irreversibly from the lower or condensing temperature to the upper or vaporizing temperature, with consequent decrease in the attainable efficiency of the cycle (see Art. 125). The magnitude of the unfavorable influence of the feed-heating phase of the non-regenerative cycle is somewhat in proportion to the relative amount of energy received during the warming of the liquid as compared to that received during vaporization.

Water has the disadvantageous characteristic of a rather high specific heat and a markedly increasing specific heat with increasing temperature, combined with a marked decrease of the latent heat of vaporization with increase of temperature. All fluids may be expected to exhibit this characteristic to some degree but some do so to a less degree than water. A more favorable fluid would be one which possessed a low specific heat of the liquid, and thus a sharply rising saturated liquid line on T - S coördinates,¹⁴ and also a relatively large latent heat of vaporization, whereby the unfavorable influence of the feed-heating phase would be minimized without the need for the plant complications associated with regenerative feed-heating.

A fluid possessing such characteristics would exhibit saturated liquid and vapor lines perhaps similar to those in Fig. 69. A further characteristic of such a fluid would be the approximately uniform entropy of the saturated vapor, whereby isentropic expansion would not incur excessive condensation.



FIG. 69

One fluid, diphenyl oxide, which has been actively proposed for power generation purposes exhibits the joint characteristics of moderate vapor pressure and a saturated vapor line which *recedes* with decreasing temperature and pressure. However, it does not possess the advantage of a relatively low specific heat of the liquid, and also the recession of the saturated vapor line may be shown to introduce a thermodynamic loss.

Mercury has the advantage of a low pressure at high temperatures and a high latent heat compared to its specific heat but it condenses rapidly on expansion.

At this writing the ideal fluid remains to be found, and may easily remain so. If discovered it must have the further practical characteristics of no corrosive action on ferrous metals and no poisonous action on operating personnel, non-inflammability, the property of "wetting" metallic surfaces so that adequate heat transmission rates may be attainable, resistance to decomposition, et cetera.

It is possible and practicable to utilize certain of the advantages of a fluid and in some degree suppress its disadvantages by the joint use of

¹⁴ Recall that $d'Q = c dT$ and also $= T dS$. Therefore $T/c = dT/dS$, which is the slope of the liquid line on T - S coördinates. Thus a low specific heat means a steep slope. See Art. 55.

two or more fluids in a cascade arrangement of cycles. In such an arrangement, which would be known as a multiple-fluid cycle, advantageous pressure-temperature characteristics may be utilized by employing a fluid of relatively lower vapor pressure in the higher temperature section of the composite cycle and a fluid of higher vapor pressure in the lower temperature section. A compound cycle of this character which uses the two fluids mercury and water is in successful use at this writing. The general scheme of arrangement of the apparatus of this binary-fluid cycle is shown in Fig. 70. Observe that the energy rejected from the condensing mercury forms the energy source for the vaporizing water. In Fig. 71 there is shown a T - S chart for 9 lb. of mercury and for 1 lb. of water. These are about the requisite proportions between the rates of mercury and water circulation in the cycle, due to the relative latent heats being such that the heat delivered in condensing one pound of mercury after expansion through the mercury turbine is about one-ninth of the heat required to evaporate one pound of water.

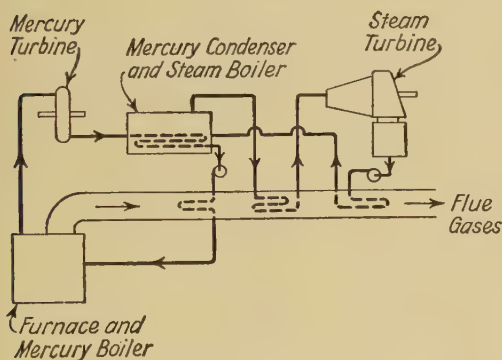


FIG. 70. Binary-fluid (mercury-water) cycle.

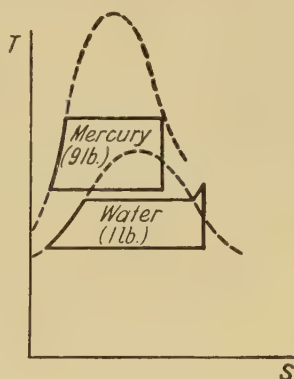


FIG. 71

128. Type Efficiency. — In order to measure the effectiveness with which a cycle might ideally utilize the temperature range encountered in the cycle and thus to designate the degree to which the ideal cycle efficiency approaches the Carnot efficiency for that temperature range there is employed a figure of merit which is known as the **type efficiency** of the cycle. This is defined as the ratio between the ideal thermal efficiency of a cycle and the thermal efficiency of a Carnot cycle operating between the extremes of temperature found in the cycle under consideration, or

$$\text{Type efficiency} = \frac{\text{Ideal thermal efficiency of cycle}}{(T_1 - T_2)/T_1}. \quad (21)$$

Of the several cycles considered in the foregoing the Rankine cycle will exhibit the lowest type efficiency,¹⁵ with the simple reheating cycle showing a slightly better value and the regenerative cycles giving the nearest approach to the Carnot performance.

Example 12. From the results of examples 4, 8, 10 and 11 compute the type efficiencies of the several cycles of those examples.

129. Actual Engine Performance; Engine Efficiency and Steam Rate. — It is axiomatic that no actual engine can attain a performance equal to that of an ideal engine operating on the same cycle, this because of the numerous irreversible actions which will inescapably occur in any real process. Heat will be radiated or otherwise dissipated from higher to lower temperature levels without full realization of its available energy; and mechanical irreversibility will occur due to fluid friction, throttling and turbulence and to the friction of moving parts of the engine mechanism, et cetera. The result must be less actual work output per pound of fluid than that ideally obtainable in the ideal cycle upon which the engine operation is based.

In order to evaluate definitely and informatively the relative effectiveness of an actual engine and the degree to which it approaches ideal performance there is employed a figure of merit which is known as the **engine efficiency**¹⁶ and which is defined as the ratio between the output actually obtained per pound of steam supplied to the engine and the output ideally obtainable upon isentropic expansion of that steam, or

Engine efficiency

$$= \frac{\text{Actual output per pound of steam supplied to engine}}{\text{Work ideally obtainable by isentropic expansion}}. \quad (22)$$

The actual output of an engine is necessarily determined by test. Such a test requires a determination of the actual power output of the engine and the mass of steam supplied per unit time. Also in order that the ideal work output per pound may be computed it is further necessary to determine the state of the steam as supplied, the vapor pressure at the engine exhaust, the pressures at which steam is extracted and the amounts extracted if regenerative feed-water heating or resuperheating

¹⁵ In certain instances the type efficiency may be somewhat misleading as an index of the effectiveness of the cycle. Thus the type efficiency of a Rankine cycle without superheat will exceed that of a Rankine cycle with superheat if the pressures in each are the same, although the thermal efficiency of the latter is the greater.

¹⁶ There has existed an unfortunate multiplicity of terms which have been used to designate this efficiency. Such are "efficiency ratio," "relative efficiency," "Rankine efficiency," and "turbine efficiency." Engine efficiency is the term employed in the Power Test Codes of the American Society of Mechanical Engineers.

is employed, and finally the state of the steam as it returns to the engine if reheating is used.

The power delivered to the piston face of a reciprocating engine may be measured with more or less accuracy by the *engine indicator*, the shaft horsepower output of any engine may likewise be measured by a suitable *dynamometer*, or for a direct-connected engine-generator set the electrical power output may be determined by proper electrical instruments. The mass-rate of steam supply may be determined by condensing and weighing the steam or by various types of steam flow-meters or calibrated orifices. The state of the steam is determined by its absolute pressure and its quality (using a calorimeter) or its temperature, if superheated. The various pressures are found by gages or manometers.

By reason of the variety of manners and positions which may be selected in designating the *output* of an engine a like variety of engine efficiencies may be assigned to a given machine. Each output mentioned above is successively less than the preceding one by reason of the progressive energy dissipation through mechanical and electrical losses. It is obvious that due attention must be given to this condition in quoting values of engine efficiency or in interpreting quoted values.

Presuming that adequate test data are available on the actual steam requirements and the power output of an engine, these data may be combined with the factors 2545 (B.t.u. per horsepower-hour) or 3413 (B.t.u. per kilowatt-hour) and the numerator of equation 22 may thereby be evaluated. Thus

Actual output per pound of steam supplied, B.t.u.

$$= \frac{\text{power output, hp. (or kw.)} \times 2545 \text{ (or 3413)}}{\text{steam supplied, pounds per hour}}. \quad (23)$$

The denominator of equation 22, that is, the output ideally obtainable per pound of steam supplied, may be computed for engines operating according to the conditions of the several types of cycles quite as indicated (a) in Art. 121 (Eq. 3) for an engine with complete and continuous expansion such as occurs in a Rankine cycle, (b) in Art. 125 (Eq. 11) and Art. 126 (Eq. 19) for an engine with steam extraction or (c) in Art. 126 (Eq. 15) for an engine with reheating only — *except that* in the analysis of the performance of an engine with steam extraction the masses of steam extracted shall be taken as the *actual* masses per pound supplied, as determined by test.¹⁷ It is recalled that in the equations as developed in the above references the masses were those associated with the carrying out of the idealized feed-water heating.

¹⁷ This procedure is that as established by the A.S.M.E. Power Test Code.

The ratio between the steam supplied per hour and the power output is known as the **steam rate** of an engine and is commonly expressed in terms of the *pounds of steam supplied per horsepower-hour (or kilowatt-hour)*, or

$$\text{Actual steam rate} = \frac{\text{pounds of steam supplied per hour}}{\text{output in hp. (or kw.)}}. \quad (24)$$

Introducing this term in equation 23,

Actual output, B.t.u. per lb.

$$= \frac{2545 \text{ (or 3413)}}{\text{actual steam rate, lb. per hp-hr. (or kw-hr.)}}; \quad (23a)$$

whereby the engine efficiency may be expressed more concretely in the form,

$$\begin{aligned} \text{Engine efficiency} &= \frac{2545/\text{actual steam rate, lb. per hp-hr.}}{\text{output ideally obtainable, B.t.u. per lb.}}, \text{ or} \\ &= \frac{3413/\text{actual steam rate, lb. per kw-hr.}}{\text{output ideally obtainable, B.t.u. per lb.}}.^{18} \quad (22a) \end{aligned}$$

Example 13. Compute the actual steam rate and the engine efficiency of an engine operating with continuous expansion (without extraction) and with the steam supply and exhaust pressure conditions of example 1 if 20,000 lb. of steam are required per hour to deliver 1000 hp. (0.646.)

Example 14. For an actual turbine operating with the load, steam supply conditions and exhaust pressure of example 3, compute the steam rate, the work output per pound of steam and the engine efficiency if test showed the actual steam required to be 80,000 lb. per hour. (0.758.)

¹⁸ The performance of an ideal engine may similarly be expressed in terms of an **ideal steam rate**, in pounds per hour per unit power output. In fact this ideal steam rate has been computed in several of the illustrative examples of the foregoing articles, obtaining its magnitude by dividing the output equivalent of a horsepower-hour (or kilowatt-hour) by the ideal work output per pound of steam supplied, or

$$\text{Ideal steam rate, lb. per hp-hr. (or kw-hr.)} = \frac{2545 \text{ (or 3413)}}{\text{ideal output, B.t.u. per lb.}}. \quad (24a)$$

For an engine from which steam is *not* extracted for feed-heating or other purposes equation 24a may be employed to express the ideal output in terms of an ideal steam rate and that expression may be introduced in equation 22a, giving the further alternative relation for expressing engine efficiency in terms of actual and ideal steam rates, or

$$\text{Engine efficiency, non-extraction engine} = \frac{2545/\text{actual steam rate}}{2545/\text{ideal steam rate}} = \frac{\text{ideal steam rate}}{\text{actual steam rate}}.$$

This relation is not applicable in the case of an extraction type of engine due to the fact that the amounts actually extracted do not need necessarily to conform to the amounts ideally required for regenerative feed heating.

Example 15. Compute the engine efficiency of a turbine operating on the regenerative cycle schedule of example 8 if test data showed the actual steam requirements of the turbine to be 12 lb. per kw-hr. and the rates of steam extraction to the successive heaters to be 0.088, 0.080 and 0.078 lb. per pound of steam supplied to the turbine. (0.784.)

Example 16. For an actual turbine operating with reheating and under the conditions of example 10, compute the engine efficiency if test showed the steam rate to be 8.25 lb. per kw-hr. (0.716.)

Example 17. Does information concerning the steam rate alone afford a sufficient basis for judging the performance of an engine? If not, why not?

130. Actual Thermal Efficiency and Heat Rate. — The engine efficiency and the steam rate are more particularly figures of merit for judging the performance of an engine alone and in comparison with its best conceivable performance when operating under the imposed conditions of steam supply and exhaust pressure. Two other current and highly useful figures which are applied both to the engine and to the entire plant, and which are figures of perhaps more definite economic interest to an operator, are the **actual thermal efficiency** and the **actual heat rate**.

The actual thermal efficiency is a figure which in its basic significance is quite parallel to the *ideal* thermal efficiency of a cycle or an engine, which was broadly defined in Art. 122 (Eq. 5) as the fraction $\frac{\text{work output from system}}{\text{heat supplied to system}}$. However, in evaluating this actual efficiency for an operating engine or engine-generator unit or for a complete operating plant the useful output may in practice be variously regarded as the *indicated* power output of a reciprocating engine, or as the *shaft*-power output of any engine, or as the *electrical* power output of an engine-generator unit, or as the *net* electrical output at the bus-bars of a complete plant. Also in ascribing a value to the energy supply that quantity may be taken variously as the energy supplied as heat (at the boiler), that supplied as chemical energy in the fuel, et cetera. It is thus apparent that a considerable variety of interpretations may be applied to the term — without need for confusion, however, if due care is employed in making the interpretation. In any individual case and irrespective of its particular interpretation the thermal efficiency may invariably be regarded as a ratio between the useful output delivered and the energy supplied for the procuring of that output, or

$$\text{Actual thermal efficiency} = \frac{\text{Useful output from system}}{\text{Energy supplied to system}}. \quad (25)$$

It is obvious that for concrete evaluation of the efficiency actual test data must be available. In utilization of such data it is commonly

more convenient to express it in terms of a *specific* output or supply, that is, an output (or supply) *per unit time, per unit mass of steam* supplied to an engine or *per unit mass of fuel* supplied to a plant. It will be recalled that such a procedure was followed in the preceding article and that in equation 23a of that article the specific output per pound of steam to an engine was expressed by the relation

$$\begin{aligned} &\text{Actual output, B.t.u. per pound of steam to engine} \\ &= \frac{2545 \text{ (or 3413)}}{\text{actual steam rate, lb. per hp-hr. (or kw-hr.)}}. \end{aligned} \quad (23a)$$

A parallel relation for the complete plant, in terms of the fuel rate, is
Actual output, B.t.u. per pound of fuel to plant

$$= \frac{2545 \text{ (or 3413)}}{\text{actual fuel rate, lb. per hp-hr. (or kw-hr.)}}. \quad (23b)$$

Either of these relations may be employed directly as the numerator of the efficiency fraction of equation 25.

Several distinctive practices are followed in interpreting the denominator of the efficiency fraction in the various circumstances in which efficiency is to be determined. These circumstances and practices may be broadly classified as follows:

(A) *Engine without steam extraction.* For such an engine it is the established practice to regard the energy supply which is assignable to each pound of steam and is chargeable against the engine as the enthalpy difference between that of *saturated liquid at the temperature of the engine exhaust* and that of the steam as supplied to the engine. If the engine steam is returned for reheating the foregoing energy is evidently to be increased by the enthalpy increase during reheating. It is to be observed that for this case no concern is given to the actual temperature at which the condensate may be leaving the condenser, this in order to avoid penalizing the engine for any subcooling of the condensate which may occur by reason of unfavorable characteristics of the condenser. For this type of engine the efficiency thus becomes

$$\begin{aligned} &\text{Actual thermal eff. of} \\ &\text{non-extractive engine} = \frac{2545 \text{ (or 3413)}/\text{steam rate}}{H_1 - H_{f,2}(+\Delta H \text{ in reheating})}. \end{aligned} \quad (25a)$$

(B) *Engine with steam extraction.* For this type of engine it is difficult to assign an invariable relation which shall evaluate the energy supply chargeable to each pound of steam, this due to a variety of purposes other than feed-heating to which the bled steam may be directed and which may or may not influence the steam requirements or the heat requirements of the engine. For an engine in which the bled

steam is used only for stage feed-heating it is sometimes taken that

$$\text{Actual thermal eff., engine with stage heaters} = \frac{2545 \text{ (or 3413)}/\text{steam rate}}{H_1 - H_{\text{water leaving last heater}}}. \quad (25b)$$

It is to be remarked that use of this relation can not avoid penalizing the engine for subcooling of the condensate from the condenser or the heaters.

(C) *Complete power plant.* For the complete power plant it is the established practice to regard the chemical energy released by a complete combustion of the fuel as the energy supply. That energy when evaluated per pound of fuel is known as the **calorific value** or **heating value** of the fuel. It is determined by calorimetric measurements made upon representative samples of the fuel. Introducing this quantity in the efficiency fraction,

$$\text{Actual thermal eff., complete plant} = \frac{2545 \text{ (or 3413)}/\text{fuel rate}}{\text{heating value of fuel, B.t.u. per lb.}}. \quad (25c)$$

The **heat rate** of an engine or of a plant is defined as the amount of energy actually to be supplied *per horsepower-hour (or per kilowatt-hour) of useful output*. This may readily be expressed in terms of the steam rate, or the fuel rate, and the energy supply per pound of steam or of fuel, or

$$\begin{aligned} \text{Actual heat rate, B.t.u. per hp-hr. (or kw-hr.)} &= \frac{\text{steam rate of engine} \times \text{energy supplied per pound of steam, or}}{\text{fuel rate of plant} \times \text{heating value, B.t.u. per pound of fuel.}} \end{aligned} \quad \begin{aligned} (26a) \\ (26b) \end{aligned}$$

Scrutinizing equations 25a, 25b and 25c it is evident that the heat rate is thus the effective denominator of those efficiency relations, the numerator being regarded as the constants 2545 or 3413. These are recalled to be the B.t.u. equivalent of the horsepower-hour and the kilowatt-hour respectively.

Example 18. Compute the actual thermal efficiencies and the heat rates of the engines of examples 13 and 14.

Example 19. Compute the actual thermal efficiency and heat rate of the engine of example 16 if the steam leaving the engine enroute to the resuperheater is dry and saturated.

Example 20. Compute the actual thermal efficiency and heat rate of a plant which is delivering 1 kw-hr. per each 1.10 lb. of coal supplied, the fuel having a calorific value of 13,000 B.t.u. per lb.

131. The Energy ("Heat") Balance.—In the thermodynamic design of a steam power plant and in the analysis of its actual operation

attention is very commonly given to what is known as the "heat" balance of the plant. By this is meant, more exactly, a comprehensive energy analysis and energy accounting for the purpose of ascertaining the ultimate destination of all portions of the original energy supply in the fuel and also for the more important purpose of correcting in so far as may be practicable any wastage of available energy.

Two viewpoints are available in the development of such an energy analysis. On the simpler but less valuable basis concern is given only to the distribution of the energy as regards its physical location and destination, ascribing losses only as energy may obviously be dissipated by radiation to the atmosphere from hot pipes or surfaces, in the hot stack-gases from the furnace, by conduction to the circulating water or by mechanical friction in the moving parts of machines. Because the development of such an accounting is dependent only on the Conservation of Energy Principle it may be designated as a **First Law Energy Balance**.

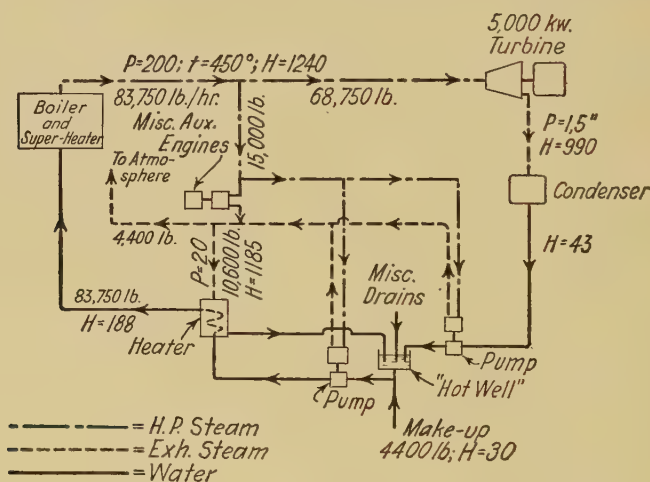


FIG. 72. Conventional diagram giving data for 1st. Law Energy Balance.

The evolution of a First Law Balance must evidently proceed from specific information concerning the type, arrangement and interconnections of all steam-using equipment which is installed in the plant and from concrete data or estimates concerning operating conditions and operating efficiencies obtaining in the numerous individual units which comprise the plant. From such information a detailed book-keeping account may be developed which shall record specifically the amount and character of all energy entering or departing from each device and shall permit a summarization portraying the direction, mag-

nitude and destination of all component parts of the energy stream enroute through the plant.

Such a summarization of the energy account may, as usual, be presented to advantage graphically. Either one of two general types of procedure is commonly employed in the graphic representation. One is illustrated in Fig. 72, in which are depicted diagrammatically the primary components of a simple steam power plant, the interconnecting pipes joining the several devices, the direction of flow between them, the

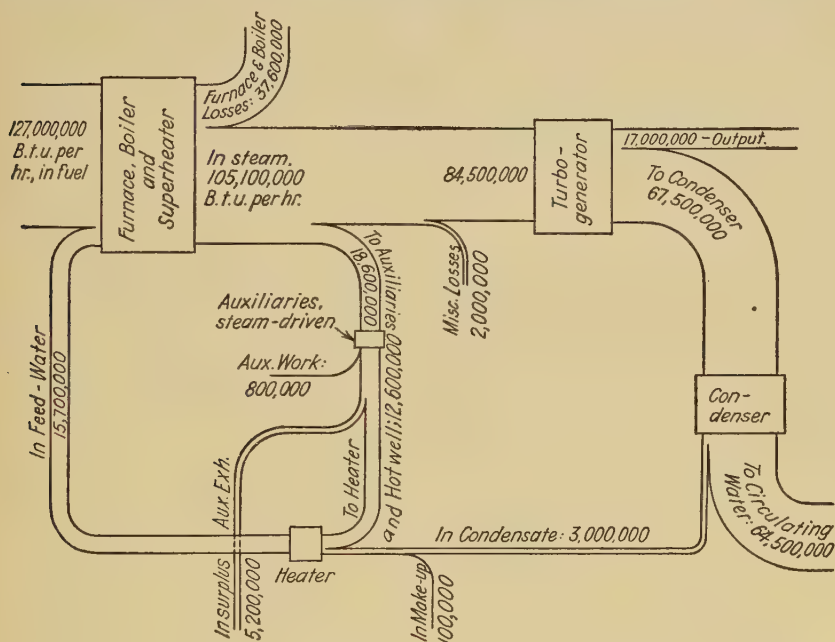


FIG. 73. Conventional diagram portraying 1st. Law Energy Balance.

amount of fluid flowing per hour, and pertinent properties of the fluid enroute. In the figure there is intentionally presented a plant character and performance which are *not* representative of better modern practice, this in order that certain unfortunate features of the plant may the better be recognized. It is observed that the plant is so devised that the various *auxiliary* engines exhaust at about atmospheric pressure and that the exhaust steam from those engines is condensed as far as possible in a heater in which some degree of feed-water heating is thus accomplished (see footnote 12).

The second method of presentation of the First Law Balance appears in Fig. 73. In that figure the character of the interconnections between parts or the fluid properties at specific points are not emphasized but

instead there is shown graphically the relative magnitude of the energy quantities at each significant stage in the energy stream. The energy magnitudes are in general expressed in terms of the total enthalpies of the mass of fluid passing a given section in a unit time. The enthalpies are those with respect to a relative zero of enthalpy for water at 32° F., and in that feature the diagram may be to an extent misleading as the fluid does not and practically could not attain that temperature at any point in the cycle. This procedure is followed because it is the conventional and more convenient one. The figure portrays with maximum clarity the general directions of the energy stream through the plant, the manner of distribution of the energy enroute and the unfortunately small proportion of the initial energy from the fuel which actually is made available as work output.

It is to be observed that in Fig. 72 the combined steam exhaust from the various auxiliary engines is shown to be in excess of that which can practicably be condensed in the feed-water heater, whereby there exists a surplus of auxiliary exhaust which must be rejected to the atmosphere. This condition would obviously be an inefficient one.¹⁹ To avoid its existence, as well as to avoid the opposite circumstance of an exhaust steam supply which is insufficient for adequate feed-water heating, is one frequent objective of the development of an energy balance in the design of a plant. Also, in recognition of the necessity of assuring a proper balance between the amount of auxiliary exhaust steam and the amount which can profitably be employed for the purpose of feed-water heating, the notion of energy balancing and the terms *heat-balance design*, *heat-balance arrangement*, *heat-balance diagram*, et cetera, are frequently employed in a somewhat limited sense referring only to the arrangement and character of the auxiliary machinery as that influences the proper balance between the exhaust steam supply and the possibility for its utilization.

The maintenance of such a suitable balance under all normal operating conditions in the large and complex power plant becomes a very nice problem indeed and one offering many varieties of solution. It becomes still more important that the designer adequately recognize the requirement of avoiding to the maximum practicable limit the *dissipation of available energy to unavailable* by reason of any serious degree of irreversibility in any of the numerous energy transition processes which take place in the plant. A valuable device for the detection and avoid-

¹⁹ In a marine plant such surplus auxiliary exhaust would commonly be condensed in an auxiliary condenser, in the main condensers, or perhaps in the evaporators, in order to avoid fresh-water wastage. The inefficiency would still persist, however.

ance of such irreversibilities is what is known as a **Second Law Energy Balance**.

The general procedure in the development of a Second Law Balance is first that of determining the energy quantities throughout the energy stream, quite as in the First Law Balance, and then the separation of the energy passing each significant point in the stream into its inherently unavailable and ideally available portions. It is felt that the general discussions in Part II of this material and the particular discussions in Chapter VII should serve adequately to delineate the basic methods whereby the specific computations of the available and unavailable energies may be carried out. For that reason the methods will not be considered in detail here. However the results of such a subdivision of the energy stream of Fig. 73 are presented in Fig. 74.

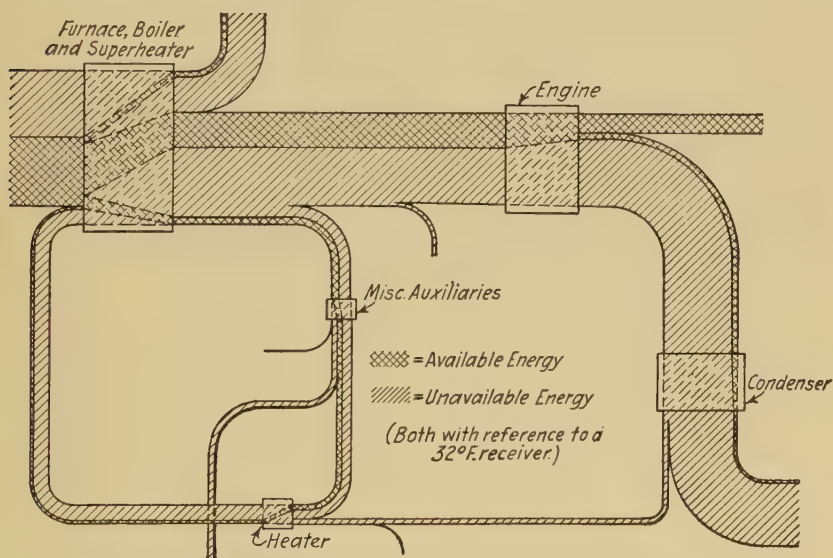


FIG. 74. Diagram portraying 2nd. Law Energy Balance.

The features which are perhaps of more outstanding significance in the figure are (a) the pronounced dissipation of available energy which occurs in the energy transition from the high temperature furnace region to the relatively moderate temperature steam²⁰ and (b) the quite

²⁰ Refer also to the consideration given to this feature in connection with the general deduction offered in Art. 125 and in the discussions of Art. 56a. Unfortunately a determination of the available portion of the energy released in the combustion of fuel is difficult, somewhat uncertain due to variable specific heat and dissociation phenomena and also dependent on local furnace and combustion conditions. For a computation based on an assumed set of furnace conditions see a paper by W. L. DeBaufre, A.S.M.E. Transactions, 1925.

appreciable dissipation of available energy and consequent loss of overall plant efficiency which results from inefficient auxiliary machinery and thermally irreversible heat transmission occurring in the feed-water heating equipment.

An alternative procedure for depicting a Second Law energy analysis would be a similar graphic representation, not of the actual energy quantities or rates, but of the entropy of the various energy quantities, showing the progressive increase of the total entropy as the energy proceeds through the plant. It is evident that from the definition of entropy as a measure of the unavailability of the energy the resulting diagram would virtually be a representation of the unavailable portions of the energy as those appear in Fig. 74. Such a diagram would offer the advantage of showing rather directly and obviously the points of pronounced available energy dissipation, those points being the points of material entropy increase.

132. Summary. — The Carnot cycle is one of the several concepts of wholly reversible temperature-engine cycles by which energy that is being supplied at an elevated temperature may be let down to a lower temperature and in the process the maximum conceivable portion of that energy may be converted to work. Also this cycle, notwithstanding its impracticability and unattainability, is in certain features the prototype of the various more or less modern steam power plant cycles and in all cases it acts as an invaluable guide in the selection and development of those cycles.

Of the actual steam plant cycles the Rankine cycle is of initial concern not only because of its simplicity, practicability and historical importance but also because the other more complex cycles may in a sense be regarded as refinements of the Rankine. The ideal Rankine cycle consists in general of (a) the warming, constant temperature vaporization and, perhaps, additional superheating of the working fluid of the cycle at a constant upper pressure, this through the provision of energy from the *source* to the fluid whereby the fluid is brought from the lower to the upper temperature limit of the cycle, (b) the isentropic expansion of the fluid to the lower temperature and pressure limits of the cycle, (c) the complete condensation of the fluid by the emission of energy at the lower temperature and pressure, this energy departure constituting the rejection of the unavailable energy to the *receiver* and (d) the return of the condensed fluid to the upper pressure by reversible adiabatic compression, whereupon the fluid cycle may be repeated. These processes take place successively in the boiler and superheater, the engine, the condenser and the feed pump. The most fundamental difference between this cycle and the Carnot lies in the irreversible

manner of the return of the condensed fluid from the lower to the upper temperature of the cycle.

The thermal efficiency of any ideal cycle is the ratio between the net useful energy output of the cycle as work and the energy input from the source, this energy being practically invariably supplied as heat conducted through the boiler and superheater. The ideal thermal efficiency of any engine alone is the ratio of the engine work output to the energy input to the cycle as heat. For the Rankine cycle, expressing all energy quantities in B.t.u. per pound of fluid,

$$\text{Energy ideally delivered as work from engine} = (H_1 - H_2)_S. \quad (3)$$

$$\text{Energy ideally required for feed-pumping} = (P_1 - P_2)V_{f,2}/J. \quad (1)$$

$$\begin{aligned} \text{Net useful energy delivered by cycle} \\ = (H_1 - H_2)_S - (P_1 - P_2)V_{f,2}/J. \end{aligned}$$

$$\begin{aligned} \text{Energy supplied from source, as heat} \\ = (H_1 - H_{f,2}) - (P_1 - P_2)V_{f,2}/J. \end{aligned} \quad (2)$$

$$\text{Energy rejected to receiver, as heat} = (H_2 - H_{f,2}). \quad (4)$$

$$\begin{aligned} \text{Thermal efficiency of ideal cycle} \\ = \frac{(H_1 - H_2)_S - (P_1 - P_2)V_{f,2}/J}{(H_1 - H_{f,2}) - (P_1 - P_2)V_{f,2}/J}. \end{aligned} \quad (6)$$

$$\text{Thermal efficiency, engine alone} = \frac{(H_1 - H_2)_S}{(H_1 - H_{f,2}) - (P_1 - P_2)V_{f,2}/J}. \quad (7)$$

In equations 2, 6 and 7 the feed-pump work item, $(P_1 - P_2)V_{f,2}/J$, may frequently be disregarded with negligible degree of error. The ideal steam rate of the cycle equals 2545 (or 3413), that is, the B.t.u. equivalent of a horsepower-hour (or a kilowatt-hour), divided by the ideal work output in B.t.u. per pound of steam.

From consideration of the Carnot cycle and its efficiency fraction, $(T_1 - T_2)/T_1$, it is evident that the efficiency of a temperature-engine cycle may be increased by increase of the temperature at which the fluid receives its energy from the source, or by decrease of the temperature to which the fluid expands reversibly in the engine and at which therefore it discards the unavailable residue of energy to the receiver. In a vapor cycle such as the Rankine the temperature of energy rejection is established primarily and practically by that of the atmosphere or of the natural supplies of water in the streams, lakes, or seas. The attainment of suitable cyclic efficiency requires that the fluid shall expand as closely as practicable to this temperature. With water as the working fluid, this requires that a very low pressure (high vacuum) be maintained in the condenser. In order to gain efficiency by increase of the temperature of energy reception in the Rankine vapor cycle the tem-

perature of vaporization may be raised by increase of vapor pressure or the average temperature of energy reception may be raised moderately by superheating after vaporization is completed.

For a like temperature range in each the Rankine cycle may not attain the thermal efficiency of a wholly reversible cycle such as the Carnot. For a cycle operating without superheat this is in one sense due to the lower average temperature of the liquid during warming from the condensing to the vaporizing temperature. From an alternative and perhaps more fundamental viewpoint the reduced efficiency is due to thermal irreversibility associated with the liquid-warming phase of the cycle. This irreversibility may be reduced by the use of *stage* or *regenerative* feed-heating and the cyclic efficiency may thereby be sensibly bettered. The improvement of efficiency resulting from superheating may be doubly realized by the expedient of *resuperheating* the vapor after partial expansion through the engine. The benefits of high vaporization temperatures may be obtained without the disadvantage of high attendant pressures by the use of a fluid of lower vapor pressure than water. All of these expedients are employed in modern power plant practice.

The regenerative feed-heating cycle employs a method of progressive liquid warming in a series of heaters which are activated by steam extracted from the engine after more or less complete expansion therein. Thermal irreversibility is thus reduced by accomplishing the first portion of the warming with lower temperature steam the available energy of which has been well realized in the engine, by similarly accomplishing the next step in the warming of the liquid by medium temperature steam, and so on. This particular method of efficiency betterment by reduction of thermal irreversibility in the feed-heating is simply illustrative of the gains invariably to be secured by the reduction of any irreversible features. The cycle is best analyzed in detail by writing and solving the energy equation for each successive heater in order from the highest pressure heater down. Thereby the relative amount of steam to be extracted for each heater may be computed, whereupon the available energy output obtainable in each successive portion of the engine may be determined.

The reheating cycle may effect an ideal efficiency betterment only if the resuperheating of the partially expanded steam is accomplished by energy supplied directly from the primary source (the hot furnace gases) and if the resuperheating serves to raise the average temperature of energy reception by the fluid from that source. The analysis of the cycle is quite like that of the Rankine cycle except in the necessity for recognizing the two-stage character of the expansion in the engine and

for properly accounting the additional energy supplied to the fluid in the resuperheater. The cycle lends itself well to the addition of regenerative feed-heating.

Binary fluid cycles are employed for the purpose of permitting high average temperatures of energy reception from the source without the handicap of difficultly high fluid pressures in the higher temperature portion of the cycle. This is accomplished by employing in that portion of the cycle a fluid which has favorable vapor pressure-temperature characteristics.

Type efficiency is defined as the ratio between the ideal thermal efficiency of any specific cycle operating between certain extremes of temperature and the efficiency of a Carnot or other wholly reversible cycle operating between the same temperature limits. It is thus an index of the relative effectiveness of an ideal cycle.

Engine efficiency is a figure of merit employed to express the degree to which an actual engine approaches the best conceivable efficiency for an ideal engine operating under the same conditions as the actual engine. It may be conveniently evaluated in terms of the actual output per pound of steam supplied and the work output ideally obtainable per pound. The output per pound may in turn be evaluated to advantage by the ratio 2545 (or 3413) divided by the actual steam rate, lb. per hp-hr. (or kw-hr.), the steam rate obviously being determined by test. The work output ideally obtainable from each pound of steam depends upon the character of the cycle in which the actual engine is operating, but the method of its computation is in all respects similar to the methods employed in the analyses of the ideal engine cycles, except that in the case of an engine with steam extraction consideration shall be given to the actual and not the ideal masses of steam extracted from the engine or continuing through the engine.

The actual thermal efficiency of an engine or of a complete plant is perhaps expressed and computed to best advantage by the quotient of the useful energy output per pound of steam to the engine or of fuel to the plant divided by the corresponding energy supply to or in each pound. The useful output per pound may be determined readily from experimental data on the steam rate or the fuel rate. The energy input per pound of fuel is its calorific or heating value. The energy input per pound of steam which may properly be charged against an engine varies with the type of engine and its operating conditions. The heat rate of an engine or of a plant is a term designating the amount of energy supplied per unit useful output, being commonly expressed in B.t.u. supplied per hp-hr. or per kw-hr. useful output. The heat rate becomes simply 2545 (or 3413) divided by the thermal efficiency.

The Energy Balance, or as it frequently is designated, the Heat Balance, of a complete plant is in principle a book-keeping account and a graphic representation of the course and destination of all portions of the energy which is supplied to the plant. In a First Law Balance attention is limited to the quantitative aspects of the energy distribution, whereas a Second Law Balance involves a further separation of the individual energy quantities into their available and unavailable components. A First Law Balance serves to depict the points of energy departure from the system, and thus to detect and minimize the more obvious losses; the Second Law Balance assists in detecting the less obvious points of available energy dissipation as caused by any character of irreversibility.

133. Review Questions and Topics.—

1. (a) What is the primary characteristic of all schemes as yet devised by man for the acquisition of work energy from the chemical energy stored in the fuels?

(b) What primary characteristic must obtain in order that the optimum conceivable efficiency might be secured in transforming the chemical energy (in part) to work?

(c) Outline the general characteristics in which the actual vapor cycles parallel the Carnot cycle.

2. (a) Outline the several distinctive processes which go to make up the Rankine cycle, describing briefly the types of devices in which each process is carried out and formulate the energy relations which pertain to each.

(b) Depict the Rankine cycle on P - V , T - S and H - S coordinates and indicate how all pertinent energy quantities associated with the several processes of the cycle may be portrayed on these diagrams.

3. Define thermal efficiency and formulate relations expressing the ideal thermal efficiency of a Rankine cycle and of an engine operating in a Rankine cycle, expressing the efficiency in terms of the requisite properties of the working fluid at the several distinctive points in the fluid cycle.

4. (a) In what specific features does the temperature range of any cycle influence the cycle efficiency?

(b) How may one vary the temperature of energy reception of the fluid in a vapor cycle such as the Rankine and how the temperature of unavailable energy rejection? Discuss briefly the relative merits of these several methods as regards their influence on cyclic efficiency and as regards their ease of accomplishment.

5. (a) In what particular phase of the Rankine cycle lies the reason for an efficiency which must inherently be less than that of the Carnot? What is the fundamentally undesirable feature of this phase?

(b) Discuss briefly but analytically the reasons for efficiency betterment by the use of superheat.

(c) As between increase of the extreme upper temperature of a vapor cycle by increase of vaporization temperature or by superheating, which is the more effective towards efficiency betterment, and why?

6. (a) Describe the physical arrangement and the manner of operation of an ideal regenerative feed-heating cycle.

(b) Depict on H - S and on T - S coordinates the states of the working fluid at all significant points of a regenerative feed-heating cycle.

(c) Write the energy equations for the successive heaters of a regenerative cycle, for the engine and for the boiler and superheater.

(d) What fundamental characteristic of the regenerative cycle accounts for its increased efficiency? What general principles are thereby illustrated?

7. (a) Describe the physical arrangement and the manner of operation of an ideal reheating cycle.

(b) Depict on H - S and on T - S coordinates the states of the working fluid at all significant points of a reheating cycle.

(c) Write the energy equation for the boiler, superheater and resuperheater and the equation for the engine of a reheating cycle and express the thermal efficiency of the cycle.

(d) Describe the manner of combining the reheating and the regenerative feed-heating cycles.

8. (a) What are the inherent disadvantages of water as the working fluid of a vapor cycle, giving attention to both the upper temperature portion of the cycle and the lower temperature portion?

(b) Describe the physical arrangement and the manner of operation of a binary fluid cycle.

9. Define type efficiency.

10. (a) Define engine efficiency and steam rate.

(b) Discuss briefly the character of test data necessary for ascertaining the ideal and the actual output of an engine.

(c) Express the engine efficiency of an engine in terms of its steam rate and the useful output ideally attainable, B.t.u. per pound.

(d) Describe briefly the various bases upon which the steam rate may be expressed.

11. (a) Express the actual thermal efficiency of an engine in terms of its steam rate and the requisite energy supply per pound of fluid, B.t.u. per pound. State the standard basis for evaluating the energy supply per pound in the case of the non-extractive type of engine.

(b) Define heat rate.

(c) Define the thermal efficiency and heat rate of an entire plant and indicate the general manner of determination of these figures.

12. (a) Describe the general character of a First Law Heat Balance and discuss its utility and its deficiencies.

(b) Describe the general character of a Second Law Heat Balance and indicate how it supplies the deficiencies of a First Law Balance.

Symbols and Abbreviations, Chapter XII

e = efficiency.

f = a subscript designating the saturated liquid state.

g = a subscript designating the dry saturated vapor state.

H = enthalpy, B.t.u. per pound of fluid.

J = Joule's equivalent, 778 (ft.-lb. per B.t.u.).

M = proportional mass of vapor passing to a particular heater in a regenerative cycle.

P = pressure (absolute), pound per square foot.

Q = energy transferred as heat, B.t.u. per pound of fluid.

S = entropy per pound mass, and also a subscript designating constancy of entropy during a process.

T = absolute temperature, degrees Rankine (= deg. Fahr. abs.).

U = linear velocity, feet per second.

V = specific volume, cubic feet per pound.

W = energy in transition as shaft-work, foot-pounds per pound of fluid.

CHAPTER XIII

THE RECIPROCATING STEAM ENGINE

In the preceding chapter there were considered several basic conventional vapor cycles by which the available portion of the energy supplied to the fluid in the upper temperature range of the cycle might be transformed to shaft-work. In each of the cycles the **engine** was designated as the device in which work output was effected during the passage of the fluid from the upper temperature to the lower, but it will have been noted that no limiting condition was imposed as to the physical character of the mechanism which formed the engine. The sole requisite of the ideal engine was that the state change of the fluid in the engine should be a reversible adiabatic one.

The engine mechanisms which are employed at present are practically always one or the other of two general types, the **turbine engine** and the **reciprocating engine**. The turbine form of engine was conceived in principle many centuries prior to the reciprocating engine and at present occupies rather the outstanding position in power generation projects where weight or space limitations are of concern and where large power concentration is essential. However, the reciprocating engine was the first to receive commercial development. Also that development, as initiated by James Watt in the latter part of the eighteenth century, marked the inception of and made possible the remarkable era of industrial and commercial activity which we now enjoy. The thermodynamic principles underlying the development, design, and operation of the reciprocating engine are considered briefly in the following.

134. The Ideal Reciprocating Engine. — The reader is presumably familiar with the general form of the reciprocating engine mechanism but for convenience an idealized diagram of a single-stage, single-acting¹ engine is shown in Fig. 75. The mechanism consists of the familiar cylinder and piston, with admission and exhaust valves located in the cylinder head. These valves are actuated by the moving parts of the engine in such a manner that

¹ A *single-acting* engine is one in which steam is admitted to only one side of the piston. This is in distinction to the more usual *double-acting* engine arrangement in which valves, et cetera, are provided at each end of the cylinder and steam is admitted alternately to each side of the piston.

- (a) by opening the admission valve during an initial portion of the travel of the piston from its head-end position the space between the piston and the cylinder head shall be put in communication with the high-pressure steam supply region and a suitable amount of steam shall thereby be admitted at constant pressure,
- (b) by closure of both the admission and the exhaust valve the previously admitted steam shall be isolated within the cylinder and shall be permitted to expand as the piston completes the travel to the farther end of its stroke, with ideally an isentropic and complete expansion of the steam to the pressure and temperature of the lower-pressure exhaust steam region,
- (c) by opening the exhaust valve during the major portion of the return stroke of the piston the lower temperature steam shall be rejected to the atmosphere or to the condenser, in an amount equal to that originally taken in during admission, and
- (d) by closure of both the admission and the exhaust valve the steam remaining in the cylinder shall be isolated and during the remainder of the return stroke of the piston shall ideally be compressed isentropically to the temperature and pressure of the steam supply, whereupon steam admission may again be caused to take place upon the next stroke of the piston.

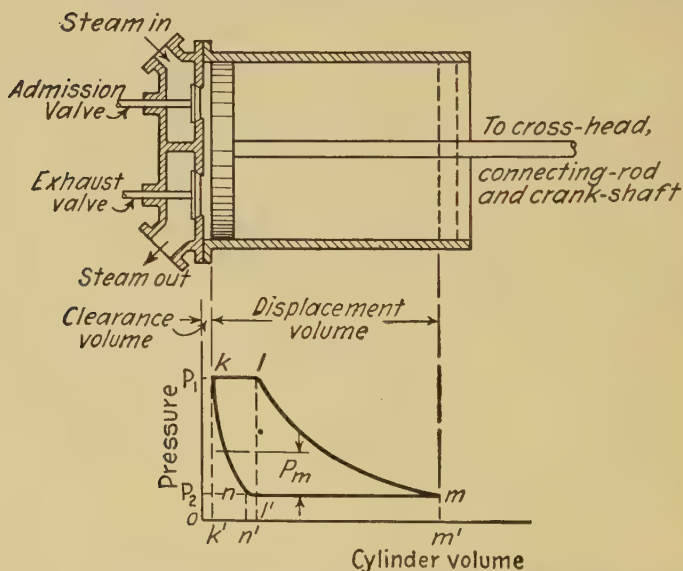


FIG. 75. Conventional representation of reciprocating steam engine cylinder, with indicator (pressure-volume) diagram.

The reciprocating piston acts through the piston rod, cross-head and connecting rod to produce a rotative motion of the crank-shaft and thereby the shaft-work delivery.

The events thus taking place in the complete mechanical cycle of operations of the ideal engine are depicted on pressure-volume coördinates in the lower diagram of the figure. Referring to that diagram, the volume of the space between the piston and the cylinder head (including the volume of the valve ports, et cetera) when the piston is at the extreme *left* or *head* end of its stroke is known as the **clearance volume**. The provision of such a clearance space is an obvious practical necessity. The total *increase* of volume within the cylinder during the travel of the piston to the other end of its stroke is known as the **piston displacement** per stroke. It is customary to express the relative magnitude of the clearance volume as a fraction or percentage of the piston displacement.² We shall designate that fraction by the symbol C . From the figure,

$$C = \frac{v_k}{v_m - v_k}$$

The point in the piston stroke at which the admission valve is caused to close is commonly known as the **point of cut-off**, and the ratio between the piston travel to the point of cut-off and the total length of its stroke is known as the **cut-off ratio**. For this ratio we shall employ the symbol $C.O.$. Referring to the figure,

$$\text{Cut-off ratio } (C.O.) = \frac{v_l - v_k}{v_m - v_k}.$$

Further, the ratio between the total volume within the cylinder at the extreme *right* end of its stroke, after the expansion of the fluid, and the volume at cut-off and thus at the beginning of expansion, is known as the **expansion ratio**. For this ratio we shall employ the symbol R . It will be convenient to have a relation between the cut-off ratio ($C.O.$), the expansion ratio (R) and the clearance (C). Such a relation may be written by inspection from the figure. Thus, if the displacement per stroke ($v_m - v_k$) is regarded as relatively unity,

$$\begin{aligned} R &= \frac{v_m}{v_l} = \frac{v_k + (v_m - v_k)}{v_k + (v_l - v_k)} \\ &= \frac{C + 1}{C + C.O.}, \quad \text{or} \end{aligned} \tag{1}$$

$$C.O. = \frac{1 + C}{R} - C. \tag{1a}$$

² In actual engines the clearance will range from about 2 to 15 per cent of the piston displacement.

Note that, from equation 1a, for a hypothetical engine of zero clearance volume the ratio of expansion would be simply the reciprocal of the cut-off ratio.

The ideal work output of the engine in one complete mechanical cycle may be analyzed to advantage by further reference to the pressure-volume diagram. Considering individually the several phases of the cycle:

Work during Admission. During admission the force acting upon the face of the moving piston is ideally the product P_1A , where P_1 is the pressure of the steam as supplied to the engine and A is the net area of the piston. The work done at the piston face, and ideally delivered to the shaft, equals the above force times the distance $\bar{k}\bar{l}$ (Fig. 75) through which it acts, or work = $P_1A\bar{k}\bar{l}$. But $A\bar{k}\bar{l} = v_l - v_k$, which latter is the volume increase in the steam space during admission and thus the volume of the steam which is admitted. Indicating the specific volume of the steam supplied as V_1 , the volume of the steam admitted equals MV_1 , where M is the mass of the steam admitted. Thus, summarizing,

$$\begin{aligned}\text{Work output during admission} &= P_1A\bar{k}\bar{l} \\ &= P_1(v_l - v_k) \\ &= MP_1V_1\end{aligned}\quad (2)$$

where P_1 = pressure of steam supplied to engine, pounds per square foot

V_1 = specific volume of steam supplied, cubic feet per pound.

M = mass of steam supplied during admission, pound.

In the diagram of Fig. 75 the work output during admission is represented by the area $k'kll'$. It is pertinent to note that the work obtained during admission in the ideal reciprocating engine is thus equal to the flow-work energy coming to the engine via the steam line (see Art. 31, paragraph *d*).

Work during Expansion. At the point of cut-off there is in the engine cylinder the freshly admitted mass of steam M together with the mass of steam M_c which has remained in the clearance space from the preceding cycle and had ideally been compressed to the same state as the steam supply. Upon cut-off the composite mass expands reversibly and adiabatically to the lower or exhaust pressure and temperature of the vapor cycle in which the engine operates. As the expansion is a non-flow state change within the cylinder the work done must be wholly at the expense of the internal energy of the fluid, or

$$\text{Work output during expansion} = J(M + M_c)(E_1 - E_2)_S, \quad (3)$$

where M_c = mass of steam carried over in the clearance volume.
 $(E_1 - E_2)_S$ = change of internal energy of the steam during isentropic expansion of the steam from the state as supplied to the exhaust pressure P_2 , B.t.u. per pound.

Referring to Art. 36, paragraph *a*, it is recalled that the work output during the ideal expansion is alternatively measured by the $\int_{v_l}^{v_m} P dv$. In Fig. 75 this integral is represented by the area $l'lm m'$.

Work during Exhaust. During the exhaust portion of the return stroke of the piston steam at pressure P_2 must be delivered from the cylinder in an amount equal to that supplied during admission, else an accumulation of steam within the cylinder would result. For that delivery work must in effect be done upon or returned to the engine system. By an analysis parallel to that employed above for ascertaining the work output during admission,

$$\begin{aligned}\text{Work input during exhaust} &= P_2 A \overline{mn} \\ &= P_2 (v_m - v_n) \\ &= M P_2 V_2,\end{aligned}\tag{4}$$

where P_2 = exhaust pressure, pounds per square foot,

V_2 = specific volume of steam after isentropic expansion of the steam from the state as supplied to the pressure P_2 , cubic feet per pound.

This work is represented graphically in Fig. 75 by the area $n'nm m'$. It is again pertinent to note that the work required for ejection of the steam from the ideal reciprocating engine is thus in fact the flow-work energy departing from the engine via the exhaust line.

Work during Compression. For the non-flow recompression of the steam remaining in the clearance, work is required in an amount equal to the increase of internal energy associated with this isentropic compression, or

$$\text{Work input during compression} = J M_c (E_1 - E_2)_S.\tag{5}$$

Again referring to Art. 36, it appears that the work required for the ideal compression is represented graphically in Fig. 75 by the area $k'knn'$.

Net work output during Complete Cycle. Summarizing the foregoing individual expressions associated with the several phases of the complete engine cycle of operation,

Net work output during complete cycle

$$\begin{aligned}
 &= MP_1V_1 + J(M + M_c)(E_1 - E_2)_s - MP_2V_2 - JM_c(E_1 - E_2)_s \\
 &= MJ\left[\left(\frac{P_1V_1}{J} + E_1\right) - \left(\frac{P_2V_2}{J} + E_2\right)\right] \\
 &= MJ(H_1 - H_2)_s.
 \end{aligned} \tag{6}$$

This expression should be and is identical with the relation already developed in Art. 121, Eq. 3, for the work output (per pound of steam) of *any* engine operating in accordance with the requirements of the Rankine cycle.

It follows from the above analyses that the net work output per cycle is represented graphically in Fig. 75 by the following summation of area:

$$k'kl' + l'mm' - n'nm' - k'knn',$$

or the enclosed area $klmn$. In terms of the calculus this area is the $\left(\int_{P_2}^{P_1} v \, dP\right)$ for the entire enclosed figure. This conclusion is again in conformity with the conclusion of Art. 121, Eq. 3b.

It will be observed that for the ideal complete-expansion engine the work output per pound of steam supplied is wholly uninfluenced by the clearance volume as the clearance steam is simply expanded and recompressed reversibly between the supply steam and exhaust steam states.

135. The Engine Indicator Diagram. — The practical counterpart of the pressure-volume diagram of Fig. 75 is the familiar **card** or **diagram** obtainable from an actual operating engine by use of the **engine indicator**. The reader is assumed to be acquainted with that instrument and with its manner of operation, by which is obtained a progressive graphic record of the pressures actually existing within the *indicator* cylinder and presumably existing within the *engine* cylinder at all points of the several piston strokes.

The pressures actually recorded on the indicator diagram are in effect the *gauge* pressures (pressures relative to the atmospheric pressure which exists outside of the indicator piston), and actual distances along the card are (presuming an accurate *reducing motion*) directly proportional to the distances moved by the piston from its end-of-stroke or dead-center positions, and thus are proportional to the volumes displaced by the piston in its travel. If however, on the actual diagram, a volume axis is drawn at a scalar distance below the atmospheric line equal to the atmospheric pressure, and if a pressure axis is drawn perpendicular to the atmospheric line at a scalar distance beyond the clearance end of the card equal to the clearance volume of the actual engine, then

the indicator diagram becomes a pressure-volume diagram which is the essential equivalent of the diagram of Fig. 75. Scalar distances above the volume axis measure absolute pressures and scalar distances from the pressure axis measure the total volume (including clearance) of the steam in the cylinder. The scalar length of the original card measures the piston displacement per stroke.

In Art. 134 it was indicated that areas on the pressure-volume diagram of Fig. 75 measure the work output (per engine cycle) for the *ideal* reciprocating engine. In this connection note that it may properly be anticipated that the various operations of the *actual* engine will depart appreciably from the ideal. There will be fluid friction and turbulence during the entry and departure of the steam through the valves, energy transfer as heat will take place between the steam and the cylinder walls, expansion within the cylinder may not be carried down completely to the exhaust pressure, mechanical friction will exist between the moving parts of the engine, et cetera. If it is recalled further in this connection that in Art. 36 it was emphasized that area on pressure-volume coördinates may be regarded as a measure of energy transition as mechanical work only for mechanically reversible processes, then the evident necessary conclusion would be that the pressure-volume record of the actual indicator diagram may not be taken as a measure of the actual *shaft*-work output of an engine. That is quite true but it is also the case that, while the steam is actually within a reciprocating engine cylinder and thus is registering its pressure in the indicator, its various state changes take place without extreme turbulence or fluid friction and under internal conditions of virtual mechanical reversibility, whereby it may correspondingly be concluded that the indicator diagram area may properly be regarded as a measure of the mechanical *work accomplished at the piston face* by the action of the fluid pressure upon that face.³ The diagram has therefore had wide application in the measurement of the work and power delivered at the piston face, that is, the so-called **indicated** work or **indicated** power of the engine.

Determination of the net work and power actually delivered to the engine shaft has so commonly been determined in laboratory practice by brake-dynamometer tests that the shaft power is very generally designated as the **brake** power of the engine. As the difference between

³ This conclusion becomes incorrect directly as excessive rate of fluid flow and fluid friction exists in the connections between the engine cylinder and the indicator cylinder. This and other phenomena make the utility of the indicator and the accuracy of the indicator diagram questionable in the case of engines with rotative speeds in excess of several hundred revolutions per minute.

the indicated and the brake power measures the energy dissipated by mechanical friction in the moving parts of the engine that difference is commonly known as the **friction power**. The ratio between the two is known as the **mechanical efficiency** of the engine.

Intimately associated with the indicator diagram and its use in the measurement of the indicated power of an engine is the index known as the **Indicated mean effective pressure** (I.m.e.p.). In a limited sense this pressure is simply the scalar mean ordinate of the indicator diagram and may be obtained from that diagram by dividing its area by its length and multiplying the quotient by the pressure scale of the diagram. In a broader sense *the indicated mean effective pressure is that equivalent pressure which, if acting unopposed upon the piston face during the power stroke of a cycle, would produce the net amount of work output actually delivered per cycle.* The indicated work per cycle is therefore evaluated by the relation

$$\begin{aligned}\text{Indicated work per cycle, ft-lb.} &= \text{I.m.e.p.} \times A \times L \\ &= P_m \times A \times L,\end{aligned}\tag{7}$$

where P_m = abbreviation for the mean effective pressure, pounds per square foot,

A = piston area, upon which the pressure acts, square feet,

$P_m A$ = mean effective force acting on piston face, pounds,

L = length of piston stroke, feet.

As the product of the piston area and the length of stroke is the piston displacement per stroke the relation may be expressed in the useful alternative form

$$\text{Indicated work per cycle, ft-lb.} = P_m \times \text{piston displacement per stroke, cu. ft.}\tag{8}$$

As noted above, in equation 7 P_m is strictly speaking the mean effective pressure in pounds per square foot, L is the stroke length in feet and A is the piston area in square feet. However in practice it is quite customary to make the computation in the unit of pounds per square inch for the pressure and the unit of square inches for the area. The results are identical.

The power delivered at the piston face and expressed in horsepower becomes by definition $\frac{1}{33,000}$ th of the product of the work per cycle and the cycles per minute, or, in the familiar expression,

$$\text{Indicated horsepower} = \frac{P_m L A N}{33,000},\tag{9}$$

where N is the number of complete engine cycles per minute. This power relation may be expressed alternatively in terms of the average *piston speed*, which is the distance S traveled by the piston per minute, or

$$\text{Indicated horsepower} = \frac{P_m A S}{33,000 K}, \quad (9a)$$

where S = average piston speed, feet per minute (equals $2 \times L \times \text{r.p.m.}$).

K = number of strokes per complete cycle.

The indicated m.e.p. finds a direct utility as an index by the use of which one may conveniently judge the relative effectiveness of a given cylinder size and piston displacement for power production; the higher the m.e.p. obtained the more effectively (but not at all necessarily the more efficiently) one may regard the engine piston displacement to be utilized. A rather hybrid but similarly useful index is one known as the **brake m.e.p.** This figure may perhaps be defined to best advantage as that value which is obtained if the actual shaft-work or power as determined by test is substituted in equations 7, 8, or 9, instead of the indicated work or power, and the equations are solved for the corresponding m.e.p. The brake m.e.p. does not have a direct physical significance as does the indicated m.e.p., but its utility is equally great or greater, and particularly so for engines of such speed that the use of an indicator is impracticable. The ratio between the brake m.e.p. and the indicated m.e.p. is evidently the mechanical efficiency of an engine.

For the ideal complete-expansion engine the indicated m.e.p. may be expressed in terms of the steam properties and the cut-off ratio. Thus, from equations 8 and 6, together with Fig. 75,

$$\begin{aligned} \text{I.m.e.p., ideal complete-expansion} &= \frac{MJ(H_1 - H_2)_S}{\text{engine (lb. per sq. ft.)} \quad (v_m - v_k)} \\ &= \frac{MJ(H_1 - H_2)_S}{(v_l - v_k) \left(\frac{v_m - v_k}{v_l - v_k} \right)} \\ &= \frac{MJ(H_1 - H_2)_S}{(M V_1)(1/C.O.)} \\ &= J(C.O.) \frac{(H_1 - H_2)_S}{V_1}. \quad (10) \end{aligned}$$

It needs be observed that for an engine with complete isentropic expansion of a given fluid a definite expansion ratio R must accompany a given supply and exhaust pressure, or specifically, $R = (V_2/V_1)_S$. In accordance with equation 1a it follows that a definite cut-off ratio must further accompany a given steam and exhaust pressure and a given

clearance. Therefore for the use of equation 10 a value of *C.O.* must be employed which is determined by equation 1a with attention to the pressures and clearance.

A variety of applications of the foregoing general relations may be made. A few are illustrated in the following example.

Example 1. For an ideal complete-expansion reciprocating engine operating on dry steam at 119 lb. per sq. in. abs. with 30 in. Hg. exhaust pressure and with 5 per cent clearance, compute the following items.

- (a) *Expansion ratio.* For the complete-expansion engine the expansion ratio is not only the volume ratio v_m/v_l (Fig. 75) but evidently is also the ratio between the specific volumes of the steam at the end of and at the beginning of the isentropic expansion. Thus

$$R = (V_2/V_1)_S = 23.7/3.75 = 6.32.$$

- (b) *Required cut-off.* By equation 1a,

$$C.O. = \frac{1+C}{R} - C = \frac{1.05}{6.32} - 0.05 = 0.116 = 11.6 \text{ per cent.}$$

- (c) *Work per pound of steam admitted* = $J(H_1 - H_2)_S$
 $= 778(1189.7 - 1037.0) = 778 \times 152.7 = 118,800 \text{ ft.-lb. per lb.}$

- (d) *Steam-rate* = $2545/152.7 = 16.7 \text{ lb. per hp-hr.}$

- (e) *Indicated m.e.p.* By equation 10,

$$\text{I.m.e.p.} = 778 \times 0.116 \times \frac{152.7}{3.75} = 3680 \text{ lb. per sq. ft.} = 25.6 \text{ lb. per sq. in.}$$

- (f) *Piston area required for a 100 I.hp. single-acting engine operating at a piston speed of 600 ft. per minute* =, by equation 9a,
 $(66,000 \times 100)/(3680 \times 600) = 2.99 \text{ sq. ft.}$

Example 2. Compute the above items for a complete-expansion engine with the same steam and exhaust pressure conditions but with 10 per cent clearance and note the influence of the increased clearance on the I.m.e.p. and on the required piston displacement. (16.35 lb. per sq. in.; 4.7 sq. ft.)

136. Incomplete-expansion Engine. — It will have been observed that the ideal engine of Fig. 75 was one in which a *complete* expansion from the supply to the exhaust pressure was accomplished in the cylinder during the expansion phase of the engine cycle. Despite the idealism of this complete expansion it is found in practice to be disadvantageous because of an immoderately large piston displacement and consequent engine size and weight which are required for a given power output, and also because the use of the excessive cylinder size acts to aggravate a serious irreversible energy transfer as heat which occurs between the steam and the cylinder walls and to increase the mechanical losses of the engine. The effect of these evils is reduced in practice by the simple expedient of not attempting to continue the expansion in the cylinder to the exhaust pressure, instead terminating the expansion

within the cylinder after a pressure has been reached that is appreciably in excess of the exhaust region pressure and then permitting the steam in the cylinder to expand irreversibly through the exhaust valve to the discharge region.

A pressure-volume diagram corresponding to the above procedure is depicted in Fig. 76 by the diagram $klxmn$. The opening of the exhaust valve at x results in a rapid pressure drop xm by reason of the escape of a portion of the steam through the valve. The diagram as thus shown is to be compared with either of the two dotted diagrams in which complete expansion is indicated. In the one of these the cut-off is shown sufficiently advanced to secure complete expansion in the cylinder but at the cost of a marked loss of power from the given cylinder size. In the other the displacement is shown sufficiently increased to permit the complete expansion but with only a minor increase of power and at the cost of a large increase in the required piston displacement. The effective result in either is essentially the same.

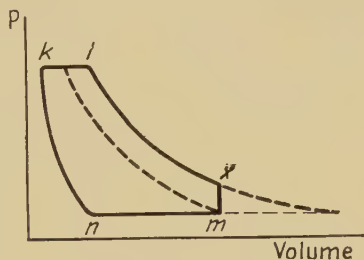


FIG. 76

It was remarked above that by reduction of the relative cylinder size or, expressed alternatively, by retarding the cut-off the magnitude of the irreversible heat transfer losses between the steam and the cylinder walls is found to be materially decreased. Also the relative magnitude of the irreversible friction losses in the moving parts of the engine are jointly reduced in the smaller and lighter engine. In distinction, it is evident that the pressure drop and the corresponding losses due to irreversible throttling of the steam through the exhaust valve are increased by increased cut-off or, alternatively, reduced cylinder size. In the steam engine the composite effect of these several losses is found to reach a minimum at a condition of cut-off which is such as to give a sensibly incomplete expansion in the cylinder. Because of this it is customary to investigate the performance of an engine which is real in the feature of incomplete expansion but ideal in the feature that the expansion is presumed to be adiabatic — without heat transfer between the steam and the cylinder. A comparison between the thermal efficiency of an actual engine and the efficiency of this semi-ideal engine is regarded as a more informative index of performance than would be a comparison with the ideal complete-expansion engine.

For the incomplete-expansion engine the clearance, the cut-off ratio and the expansion ratio have each the same significance as in the case

of the complete-expansion engine. Also they still are mutually related by the same expression as already developed in equation 1.

The computation of the work output of the incomplete-expansion cycle becomes unfortunately quite complex if it is attempted for the cycle *with clearance*.⁴ It is therefore customary to limit the analysis to the case of a hypothetical engine without clearance. The pressure-volume diagram of that engine appears in Fig. 77. The cycle analysis is as follows.

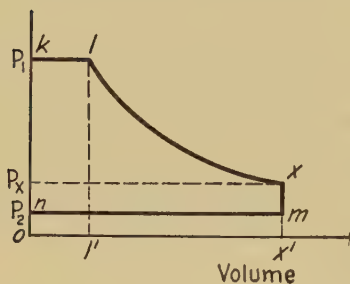


FIG. 77

The work output of the engine cycle may as usual be measured by the indicator diagram area $klxmn$. Quite as with the complete-expansion engine the work during admission is represented by the area under the line kl , or the area $okll'$, and equals the quantity MP_1V_1 ft.-lb. The work during the isentropic expansion within the cylinder to pressure P_x is represented by the area $l'lxx'$ and equals $MJ(E_1 - E_x)_S$. From x to m

a portion of the steam escapes through the exhaust valve, doing no work within the cylinder but leaving a residue M_m which needs be ejected during the exhaust stroke against pressure P_2 . The volume of this residue must evidently equal the volume of the original M lb. at state x . Thus the work required during the ejection becomes $(M_m P_2 V_m =) MP_2 V_x$, where V_x is the specific volume attained after isentropic expansion from the state of the steam as originally supplied to the pressure P_x . Therefore

$$\text{Work output of cycle} = M[P_1 V_1 + J(E_1 - E_x)_S - P_2 V_x],$$

or by adding and subtracting the quantity $MP_x V_x$,

$$= MJ \left[(H_1 - H_x)_S + \frac{(P_x - P_2)V_x}{J} \right], \quad (11)$$

⁴ The complication arises from the condition that with incomplete expansion the clearance steam is in general not in such a state that the recompression would return it to the initial state.

⁵ It is of interest to note in this connection that, although inaccurately arrived at, the same result might be obtained by conceiving an entire cyclic series of non-flow state changes to be effected in an engine cylinder whereby there shall be secured a P - V diagram which is equivalent to the indicator diagram of Fig. 77. Thus the line kl would be taken to represent a constant-pressure warming and vaporization of a mass of fluid from a saturated liquid at T_2 to a vapor at P_1 by heat energy supply in the amount $M(H_1 - H_{f,2})_P$ (see Art. 67). The line lx would properly represent the same non-flow isentropic expansion to pressure P_x which we have seen applies to the ideal cycle of the actual engine. The line xm would be taken to represent a constant-

where H_x = enthalpy after isentropic expansion to the exhaust release pressure P_x .

V_x = specific volume after isentropic expansion to exhaust release pressure P_x ; and equals V_1 /cut-off ratio ($C.O.$).

P_x = exhaust release pressure and equals the pressure after isentropic expansion from the initial state through an expansion ratio equal to the reciprocal of the engine cut-off. (A pressure-entropy table is particularly useful for ascertaining this pressure.)

As the indicated work output of the engine, per pound of steam, must also equal the difference of enthalpy between the steam coming to and leaving the engine (except as the kinetic energies might be appreciable), or work output of cycle = $MJ(H_1 - H_2)$, it follows that

$$H_2 = H_x - \frac{(P_x - P_2)V_x}{J} \text{ [vs. } H_1 - (H_1 - H_2)_S \text{ for the complete-expansion cycle].}^6 \quad (12)$$

The T - S diagram of Fig. 78 and the H - S diagram of Fig. 79 each shows the character of the state change occurring in the engine. The line lx represents the isentropic expansion from P_1 to P_x and the non-

volume state change accomplished by an energy departure as heat in the amount $M(E_x - E_m)V$ (see Art. 66). The line mn would be taken to represent a constant-pressure recondensation at P_2 , by energy departure as heat in the amount $M(H_m - H_{f,2})P$. The work output would equal the *net* amount of energy supplied as heat, or

$$\text{Work output} = MJ[(H_1 - H_{f,2}) - (E_x - E_m) - (H_m - H_{f,2})] = MJ(H_1 - E_x + E_m - H_m).$$

Adding and subtracting $P_x V_x$ to the bracketed term and rearranging,

$$\text{Work output} = MJ[H_1 - H_x + (P_x - P_2)V_x/J].$$

Although the analysis is not at all representative of the actual processes of the real engine it is nevertheless significant that the same result is secured.

⁶ The requisite enthalpy H_2 as it finally leaves the engine may be developed upon an alternative basis by analyzing individually the occurrences from x to n . From x to m there occurs the throttling flow of a part $(M - M_m)$ of the steam from the cylinder through the exhaust valve and through the exhaust line. The energy source for this process is solely the internal energy of the steam originally in the cylinder, or ME_x . From m to n additional energy is supplied for ejection of the remaining steam by the returning piston in the amount $M_m P_2 V_m$, which however must equal $M P_2 V_x$ as $M_m V_m = M V_x$. The total energy departing in the steam in its ultimate passage through the exhaust line is MH_2 . Thus the energy equation for the composite process is $MJE_x + M P_2 V_x = MJH_2$. Adding and subtracting the quantity $M P_x V_x$ from the left side of the equation,

$$MJE_x + M P_x V_x - M P_x V_x + M P_2 V_x = MJH_x - M(P_x - P_2)V_x = MJH_2,$$

which is as found above.

but only the irreversible expansion through the exhaust valve at the end of the stroke. Although the efficiency resulting from this procedure is clearly very poor various smaller auxiliary machinery such as small pumps, et cetera, are built without provision for expansion, the low efficiency being acceptable by reason of the extreme simplicity and low cost of the machine. Such an engine would be known as a **non-expansion** engine.

The indicator diagram of the engine is rectangular, as shown in Fig. 80. For such an engine, regarded as ideal in the feature of no heat transfer between the steam and the cylinder walls,

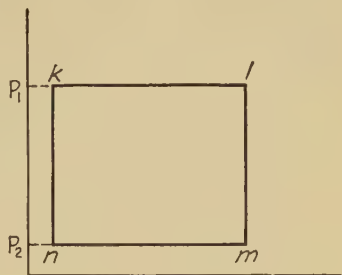


FIG. 80

$$\text{Work output per cycle} = M(P_1 - P_2)V_1. \quad (14)$$

Thus the enthalpy of the exhaust steam is

$$H_2 = H_1 - \frac{(P_1 - P_2)V_1}{J}.$$

The I.m.e.p. will be $(P_1 - P_2)$.

Example 4. For the steam supply and exhaust pressure condition of examples 1 and 3 compute the same items as called for in example 3, except that the engine shall operate with 100 per cent cut-off and no expansion. (35.1 lb. per hp-hr.)

138. Actual Cylinder Action. — In the treatment of the engine cycles in the preceding articles certain ideal conditions were assumed to exist. Admission was considered to occur at the exact beginning and release at the exact end of the stroke. Compression occurred at such a point as to bring the compression pressure exactly to the initial pressure at the opening of the admission valve. Valves were assumed to open wide and close instantaneously, so there would be no rounding of the corners of the indicator card and no drop in pressure between inside and outside of cylinder due to wire-drawing through partly open valves. There was assumed to be no heat interchange between the steam in the cylinder and the cylinder walls or piston, and no heat radiation from the cylinder to the surrounding atmosphere. There was assumed to be no leakage of steam past the piston or through closed valves.

In the actual engine all of the above ideal conditions are departed from to a greater or less degree. It is desirable to have admission occur slightly in advance of the end of the return stroke. Release occurs somewhat before the end of the outward stroke. Compression is not carried quite up to admission pressure, thus causing an irreversible pressure

drop as steam enters at admission. As the valves do not reach their full opening instantly the corners of the card are rounded by the throttling action, called wire-drawing, through partially opened valves. Leakage past the piston and through closed valves cannot be wholly prevented, and some radiation of heat from the cylinder will occur in spite of good insulation.

By far the most serious departure of the engine from ideal conditions is that caused by irreversible heat interchange between the steam and metallic surfaces of the cylinder with which the steam is in contact. During the exhaust stroke the cylinder walls and piston face are exposed to the relatively cool low-pressure outgoing steam and therefore take on a relatively low surface temperature. When the hotter high-pressure steam is admitted to the cylinder and meets the cooler metallic surfaces a transfer of heat occurs from the steam to the cylinder walls which results in the condensation of an appreciable portion of the steam admitted. This phenomenon is commonly referred to as **initial condensation**. During the expansion after cut-off the steam temperature is falling but the wall temperatures continue to rise because of the heat flow into them from the hotter steam, whence surface condensation will persist till a point in the expansion is reached where the steam and wall temperatures are equal. Beyond this point in the expansion and as the steam temperature continues to fall the walls, which are now hotter than the steam, re-emit irreversibly to the cooler steam some of the heat delivered to them during the initial condensation period, causing a re-vaporization of some of the moisture in the cylinder. This transfer of heat from the wall to the steam continues after release and during the exhaust stroke, so that practically all of the energy previously stored in the cylinder walls by initial condensation is carried out of the cylinder by the exhaust steam, but without accomplishing a commensurate amount of work in the cylinder.

Considerable research has been conducted to measure the exact temperatures of the steam and the cylinder walls throughout the cycle and to determine the effect of initial condensation on the efficiency of the cycle. Also the relation between the quality of steam at cut-off and the form of the expansion curve has been the subject of investigation. Detailed analysis of these and other researches will not be given here. However, it is to be remarked that there is evidence that in extreme cases fifty per cent or more of the entering steam may be condensed at the point of cut-off.

The following summary represents briefly the generally accepted ideas regarding the cylinder action and the aforementioned losses in the reciprocating steam engine.

(A) *The losses of available energy in the steam engine* are due to the following causes, arranged in the approximate order of magnitude of loss:

- (1) Initial or surface condensation.
- (2) Incomplete expansion and early release.
- (3) Throttling of steam flowing through partly open valves.
- (4) Leakage of steam past piston and closed valves.
- (5) Friction of moving parts.
- (6) Heat loss by radiation and conduction.
- (7) Compression and early admission.

(B) *Initial or surface condensation* is increased by the following conditions:

- (1) Early cut-off and large ratio of expansion.
- (2) Large pressure and temperature range in a single cylinder.
- (3) Small cylinder diameters, because the exposed surface is large in comparison with the volume.
- (4) Steam supply saturated or wet.
- (5) Long valve passages and the use of the same passage for admission and exhaust steam.

(C) *Conditions favorable to reduction of initial condensation* are:

- (1) Late cut-off and small expansion ratio (but this increases loss due to incomplete expansion in simple engines).
- (2) Superheat.
- (3) Separate valves for admission and exhaust, and short valve passages.
- (4) High compression.
- (5) Compounding (Art. 140).
- (6) Uniflow principle (Art. 141).

The manner in which the losses of available energy affect the indicator card area, and therefore the work delivered by the engine, is shown in a general way by the indicator diagram of Fig. 81, on which are indicated areas which represent approximately certain of the losses. It should be borne in mind that the indicator card does not show truly the state of the steam at all points on the card because of the variable quantity of steam present in the cylinder at the various points represented on the card, but it does truly show the work delivered to the face of the piston.

The action of the steam in the cylinder may be further illustrated by the T - S and H - S diagrams of Figs. 82 and 83. In these diagrams dry saturated steam is assumed to be supplied to the engine in the state

indicated at k in both diagrams. The constant entropy line kc shows the state change which would occur if the process in the cylinder were adiabatic and reversible. The line kl shows the process of initial condensation at nearly constant pressure while the steam is entering and up

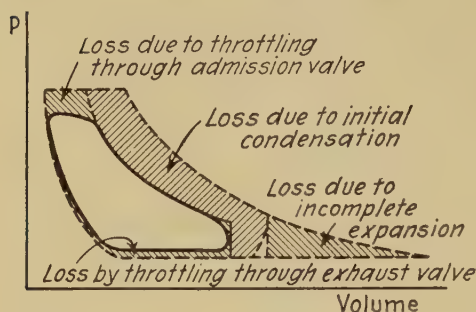


FIG. 81

to the point of cut-off, l . Further surface condensation during the expansion after cut-off occurs along la . At a the steam and cylinder walls are at the same temperature; and so, as expansion continues, re-evaporation occurs along the line ax . The process from release at x to the end of the stroke at m is largely throttling, with re-evaporation

continued. During the exhaust stroke re-evaporation continues for the steam remaining in the cylinder, along the line mn . The final condition of the steam in the exhaust pipe, just outside the engine, is represented at point n . On account of the irreversible actions, such as friction and throttling through the valves, areas on the T - S diagram do not necessarily represent heat interchange during the various parts of the process occurring within the cylinder. Also on account of the heat interchange

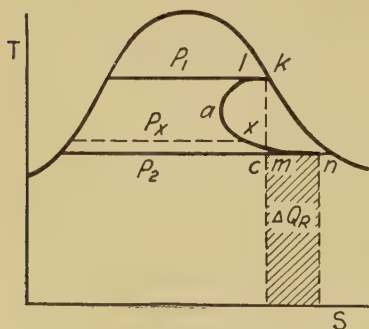


FIG. 82

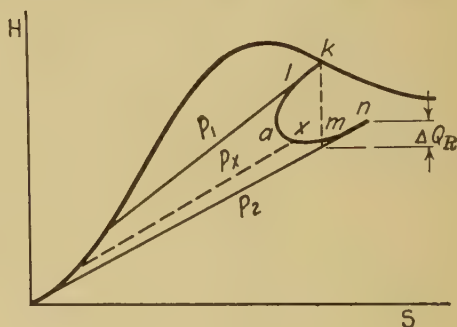


FIG. 83

known to occur between the steam and the cylinder wall differences of H between successive points on the expansion line as shown on the Mollier diagram do not necessarily represent work done by the engine, although the final change of enthalpy will measure the *indicated* work. These limitations of the T - S and H - S diagram, when applied to the steam-engine processes, must be adequately recognized. The dia-

grams are useful for depicting the state changes but discretion is to be used in their interpretation.

Example 5. An actual single-acting steam engine, of 18 in. bore $\times$ 18 in. stroke, operating with dry steam supply at 119 lb. per sq. in. abs. and 30 in. Hg. abs., exhaust pressure gave on test the following performance data:

Indicator diagram: length, 3.2 in.; area, 2.31 sq. in.; pressure (or *spring*) scale, 60 lb. per sq. in. per inch; cut-off, 25 per cent.
 Speed 200 r.p.m.; torque at shaft 2230 lb-ft.;
 Steam required per hour, 2900 lb.

Compute the following:

I.m.e.p. and I.hp.; Brake hp. and brake m.e.p.

Actual steam rate, thermal efficiency and heat rate (see Arts. 129 and 130).
 (34,500 B.t.u. per b.hp-hr.)

Engine efficiency referred (a) to an ideal complete-expansion engine cycle and
 (b) to an ideal incomplete-expansion engine cycle with the given cut-off
 (see examples 1 and 3). (0.487; 0.508.)

Enthalpy of the engine exhaust, B.t.u. per lb. (1102.)

Mechanical efficiency of engine. (0.85.)

139. Engine Governing. — Referring to Art. 134 it will be recalled that for an ideal complete-expansion engine of given clearance a given state of the steam supply and a given exhaust pressure would permit one and only one expansion ratio, one corresponding value of the I.m.e.p. (Eqs. 10 and 1) and therefore one obtainable work output per cycle for an engine of given piston displacement per stroke (Eq. 8). It follows that a variation of the power output of such an engine might be obtained only by variation of the number of engine cycles per unit time (that is, by variation of the engine speed) and that the engine output would be directly proportional to the speed.

In contrast to this condition, except in marine practice, it is commonly necessary that an engine shall be able to deliver a wide variety of power output while continuing to operate at practically uniform speed.⁷ It is therefore necessary further that a like variety of mean effective pressures may be obtainable, and this under conditions of virtually constant supply and exhaust pressure. This result is accomplished in practice in either of two ways. The one is by a simple **throttling** and consequent pressure and temperature reduction of the steam supply by a throttle-valve which is automatically operated by the engine **governor** and which acts to move toward closure as the engine speed tends to increase above the desired speed by reason of a reduction of the external loading imposed on the engine, and vice versa. In addition to the pres-

⁷ Even in a marine engine the required power output is not directly proportional to the desired speed but rather to approximately the third power of the rotative speed of the shaft.

sure and temperature reduction so effected there is further a reduction of the mass-rate of flow of the steam to the engine.

The other method is by a variation of the point in the piston stroke at which cut-off is caused to occur, load reduction and its consequent tendency toward an increase of the engine speed acting through the governor to advance the point of cut-off, and vice versa. The action is only that of a variation of the amount of steam admitted per cycle.

The general influence of the two methods on the form, area and m.e.p. of the indicator diagram is illustrated in Fig. 84, where the obtaining of a reduced area and reduced work output per cycle is effected in diagram *A* by throttling and in diagram *B* by advanced cut-off.

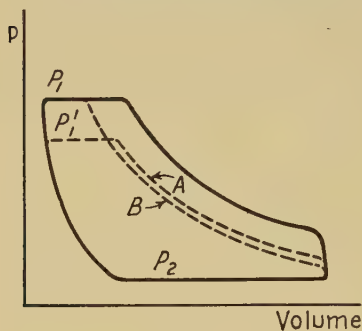


FIG. 84

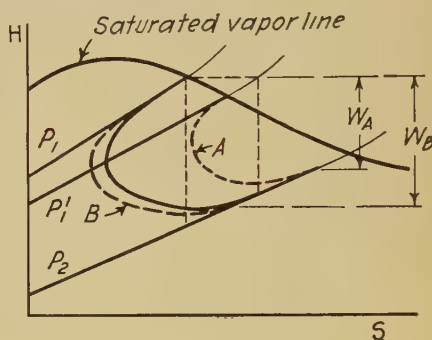


FIG. 85

The manner of influence of the two methods of governing on the work output *per pound of steam* and thus on the indicated thermal efficiency of the engine may similarly be illustrated by the two paths *A* and *B* on the *H-S* diagram of Fig. 85. All paths are so shown as to indicate the effect of the initial condensation. The figure shows that a throttling, as from P_1 to P_1' , acts to reduce materially both the amount of energy ideally available for work delivery and also the amount actually deliverable. This is quite as would be anticipated from the irreversibility of the throttling process. It is found in practice that the deleterious effect of the throttling is in some degree alleviated by a reduced initial condensation which results from the slight superheating of the steam which results from the throttling.

The effect of the advance of cut-off is from one aspect that of reducing the incompleteness of the expansion and thus of reducing the loss due to the irreversibility of the throttling through the exhaust valve, but this benefit may be largely nullified by an increase of the initial condensation which is found to accompany the advance of cut-off. However, the method of governing by cut-off variation affords in general distinctly the better efficiency results throughout the usual range of engine loading.

140. Compounding. — The prior discussions have had to do primarily with an engine arrangement in which the full pressure and temperature drop from supply to exhaust is accomplished by expansion in a single cylinder, that is, in a single-stage or **simple engine**. This is in distinction to the common arrangement of two or more cylinders of progressively increasing diameter in which the denser high-pressure steam supply is first admitted to the smaller cylinder, is expanded therein to some intermediate pressure and is ejected, and then is admitted to and further expanded in the cylinder of next size. Such a procedure is known as **compounding** if a two-stage expansion is employed and as **multiple-expansion** if two or more stages are used.

Multiple-expansion is used chiefly in order that profitable advantage may be taken of a higher over-all expansion ratio, that is, a high steam pressure with a low exhaust pressure or a reasonably high vacuum. The effect of the multiple-expansion is to reduce the serious loss due to initial condensation which we have seen would in general accompany the use of high expansion ratio in a simple reciprocating engine. To discern the manner by which this reduction of condensation is effected observe that if, for example, an expansion ratio of 25 is desired, a cut-off of approximately 4 per cent would be required if produced in a simple engine. If produced in a compound engine, the high pressure cylinder with a cut-off of approximately 20 per cent will give 5 expansions, and the low pressure cylinder with the same approximate cut-off will give 5 expansions, making a total of 5×5 or 25 expansions in the engine. Thus the pressure range *and therefore steam temperature range* in each cylinder is materially less than it would be in a single cylinder. The combination of smaller temperature range and later cut-off brings about the circumstance that there is less surface condensation in the two cylinders of the compound engine than would occur in the one cylinder of a simple engine with the same expansion ratio. If a high pressure range calls for an extremely large expansion ratio, the expansions may be divided into three or four stages in series, these engines being called triple and quadruple expansion engines respectively.

It is also of importance to note that losses of available energy in a simple engine due to leakage, fluid friction, or initial condensation are rejected with the steam to the condenser and are wholly lost to the engine. In multiple-expansion engines, however, energy so discarded from a higher stage becomes at least partly available to obtain work in a lower stage. Multiple-expansion also affords the opportunity for inter-stage reheating (Art. 126) or for regenerative feed heating (Art. 125).

Although the fundamental purpose of compounding is increase of thermal efficiency, multiple-expansion also brings about certain mechani-

cal advantages. In a simple engine the cylinder would have to be as large as the low pressure cylinder of the multiple-expansion engine, and at the same time strong enough to withstand the maximum pressure. In the multiple-expansion engine, the large low pressure cylinder need be constructed to carry only the relatively low pressures existing in that cylinder, and only the small high pressure cylinder need be designed to withstand the highest pressure. Also the cylinders and cranks of the multiple-expansion engine may be distributed in any desired manner to produce a more uniform turning moment on the shaft than could be attained with a single cylinder.

141. The Uniflow Engine.—The essential purpose of the uniflow engine is to produce in a simple engine the efficiency of a compound engine by mitigating the phenomenon of initial condensation. Referring to the sketch of Fig. 86, the engine is double-acting, with mechanically operated admission valves at each end of the cylinder. The piston is very long, being almost as long as the stroke, and this necessitates that the cylinder be about twice the length of the stroke. In place of exhaust valves at the ends of the cylinder, there is one central exhaust belt, connected to a circle of exhaust ports extending entirely around the cylinder and located exactly in the longitudinal center of the cylinder. There are no mechanically operated exhaust valves, the exhaust ports being uncov-

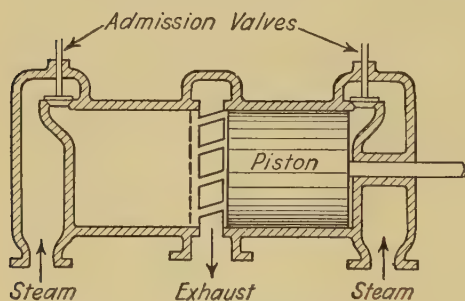


FIG. 86. Cylinder arrangement of uniflow engine.

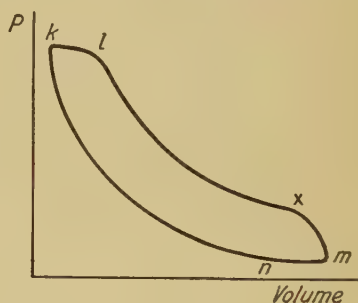


FIG. 87

ered for release and covered for compression by the movement of the piston over the exhaust ports. This central exhaust port, opened and closed by the movement of the long piston, is the characteristic feature of the uniflow principle. Other features are poppet-type admission valves of ample size, steam-jacketed heads, and small clearance. A typical indicator card of a uniflow engine is shown in Fig. 87. The release and compression occur at the same point in the piston movement, since the uncovering and covering of the central exhaust port by the

piston causes these events. The cut-off is early, in order to realize the benefit of a large expansion ratio.

The chief advantage of the uniflow engine, that is, the reduction of initial and surface condensation, is brought about by the following conditions. The ends of the cylinder with which the high-temperature entering steam comes in contact are kept hot by the steam-jacketed heads, by the high temperatures reached during compression, and, particularly, by the condition that the exhaust steam does not pass out of the engine over the surfaces at the end of the cylinder but at the center surfaces, which are as far removed as possible from either end. The center of the cylinder in the exhaust belt region is kept cool by the exhausting steam, since no high temperature steam ever comes in contact with this region. Thus in general, those surfaces of the cylinder with which high temperature steam must come in contact are kept hot and those surfaces exposed to low temperature steam remain at a low temperature. There are at all points moderate temperature differences between the steam and metallic surfaces and therefore little surface condensation. With initial condensation largely eliminated, short cut-off may be used and the desired high expansion ratio employed without the necessity for compounding.

142. Summary. — The reciprocating engine and the turbine are the two outstanding practical mechanisms which have been devised for transforming the available portion of the energy which has been supplied to a vapor, such as steam, into work output during and by the passage of the vapor from the upper to the lower temperature of a vapor cycle.

A reciprocating engine which shall be ideal in the feature of attaining the maximum conceivable thermal efficiency would be one so arranged (a) that the admission of steam from the supply region might occur without pressure drop through the admission valve and without heat transition from the steam to the engine cylinder or piston, (b) that the expansion of the steam within the cylinder after closure of the admission valve might be adiabatic and reversible, or isentropic, and shall continue down to the temperature and pressure of the region to which the steam will subsequently be ejected, (c) that the ejection of the steam to the exhaust region might occur without pressure drop through the exhaust valve and without heat transition between the cylinder and the steam, and (d) that the recompression of any residual steam remaining in the clearance volume should be adiabatic and reversible and thereby should return the clearance steam to the temperature and pressure of the fresh supply of steam. During admission work output is deliverable in the amount $P_1 V_1$ ft.-lb. per pound of steam admitted and during the

non-flow isentropic expansion additional work is deliverable in the amount $J(E_1 - E_2)_S$ ft.-lb. per pound. For the exhaust work is required in the amount $P_2 V_2$ ft.-lb. per pound of steam which had been admitted. The reversible expansion and recompression of the clearance steam involves no *net* delivery nor supply of work energy. The work output of the engine cycle thus becomes:

$$\begin{aligned} \text{Work, ft.-lb. per lb.} \\ \text{of steam admitted} &= P_1 V_1 + J(E_1 - E_2)_S - P_2 V_2 \\ &= J(H_1 - H_2)_S. \end{aligned} \quad (6)$$

The indicator diagram is a graphic record of the pressures at progressive positions of the piston and thus at progressive magnitudes of the volume of the steam space in the cyclic operation of the engine. As the state changes and processes carried out by the fluid when *within* the cylinder are mechanically reversible in the ideal engine and virtually so in the actual engine the area of the diagram may be taken to measure the work done during the cycle of operation. The ratio between the work done per cycle and the piston displacement per stroke is known as the mean effective pressure. The indicated m.e.p. may conveniently be determined directly from the indicator diagram of an actual engine by finding the mean scalar ordinate of the diagram. The m.e.p. is utilized both as an informative measure of the relative amount of work obtained per cycle for a given piston displacement and also for purposes of power computation. The familiar relation for the latter purpose is

$$\text{Hp.} = \frac{P_m L A N}{33,000}. \quad (9)$$

The term *brake* m.e.p. is also used in this connection, in which case the horsepower is the so-called brake or shaft horsepower delivered by the engine.

For a complete isentropic expansion of the vapor in the engine from a given initial state to a given exhaust pressure there is only one corresponding expansion ratio and, at a given clearance ratio, only one corresponding cut-off ratio and indicated m.e.p. This m.e.p. is in general distinctly lower than would be practically advantageous. In order to obtain a greater and more suitable m.e.p. and power output with an engine of given piston displacement the cut-off may be caused to occur later in the piston stroke. However, complete expansion may not then be attained at the end of the stroke and so, even under the most ideal circumstances otherwise, there must occur an irreversible expansion of the steam through the exhaust valve to the exhaust region upon

opening of the valve. Nevertheless on account of its greater practicability incomplete expansion is virtually the rule in reciprocating engine operation and it is customary to evaluate the optimum thermal efficiency of an engine which is ideal in all features except that of incomplete expansion. For such an engine

$$\begin{array}{l} \text{Work output, ft.-lb. per lb.} \\ \text{of steam admitted} \end{array} = J \left[(H_1 - H_x)_s + \frac{(P_x - P_2)V_x}{J} \right]. \quad (11)$$

Maximum work output from a given cylinder is obtainable if no expansion of the steam is permitted but instead admission is continued throughout the stroke. This procedure realizes very little of the available portion of the internal energy of the steam. However it has been employed frequently in smaller auxiliary machinery.

The action of the real engine departs materially from the ideal for various reasons but perhaps the item of major difference is the rapid condensation of the steam during admission, which results from an irreversible heat transfer from the hotter steam to the cooler cylinder walls and piston face. This condensed steam eventually re-evaporates during the latter part of the expansion stroke and during the exhaust stroke but with a minor or even zero benefit toward the obtaining of work delivery. The action is only that of increasing the unavailable energy departing in the exhaust steam.

For obtaining a variation of the work output per cycle of an actual engine and permitting at the same time a control of the number of cycles per minute, or speed of the engine, a governing action may be accomplished either by throttle control of the incoming steam pressure and quantity or by variation of the point of cut-off. The former method is the simpler but the latter is in general the more efficient method.

To reduce the serious loss due to initial condensation during admission the steam expansion may be caused to take place progressively through a series of cylinders, the mean temperature of any individual one of which thus differs less markedly from the temperature of the steam which enters it. This process of multiple-expansion also offers various mechanical advantages.

The uniflow engine arrangement is also a device for reducing the initial condensation effect but doing so without the disadvantages of a multiplicity of cylinders. It obtains this effect in general by a cylinder arrangement such that those surfaces of the cylinder with which high temperature steam must come in contact are kept hot and those surfaces exposed primarily to low temperature steam remain at a lower temperature.

143. Review Questions and Topics. —

1. (a) Describe the sequence of events in an ideal complete-expansion reciprocating engine.

(b) Define clearance volume and clearance ratio, piston displacement, cut-off and cut-off ratio, expansion ratio. State the relation obtaining between the expansion ratio, the clearance ratio and the cut-off ratio.

(c) Analyze the several phases of the engine cycle, indicating the work quantities associated with each phase and the net work output of the complete cycle.

2. (a) Interpret the conventional engine indicator diagram in terms of pressure, volume, and work per engine cycle. Why or when may the diagram area be taken as a measure of the work delivered per cycle at the piston face?

(b) Define indicated mean effective pressure in terms of the work per cycle and the piston displacement and also state how the I.m.e.p. may be obtained directly from the indicator diagram.

(c) Quote the relations for computing the indicated horsepower from data on the I.m.e.p., the engine cylinder dimensions and the frequency of the cycle repetition.

(d) Define brake m.e.p.

(e) Quote the relation expressing the I.m.e.p. obtainable from an ideal complete-expansion engine, given the initial state of the steam supply, the exhaust pressure and the cut-off ratio. Re-express the relation in terms of clearance and expansion ratios instead of the cut-off ratio.

3. (a) Define incomplete expansion as ascribed to an ideal reciprocating engine and indicate the reason for the consideration of such a cycle.

(b) Quote the relation expressing the net work output per cycle and per pound of steam admitted for an ideal incomplete-expansion reciprocating engine.

(c) Illustrate on T - S and H - S coordinates the character of the state change of the steam during the entire expansion of an incomplete-expansion engine.

4. Describe the characteristic processes of a non-expansion engine and quote the relation expressing the work output ideally obtainable from such an engine.

5. (a) Describe the features in which the actions in an actual reciprocating engine depart from those of an ideal engine. What feature incurs the most serious departure?

(b) Illustrate on T - S and H - S coordinates the character of the state changes of the steam during the admission, expansion, and ejection in an actual engine.

6. (a) What is the need and purpose of engine governing? Define throttle governing and variable cut-off governing and compare as regards their effect on thermal efficiency.

(b) Illustrate on P - V , T - S and H - S coordinates the effects of the several types of governing.

7. Define or describe multiple-expansion and state its purpose and advantages.

8. Describe the general character of the uniflow engine and state its purpose and advantages.

Symbols and Abbreviations, Chapter XIII

A = area (projected) of piston face, square feet.

B.hp. = brake or shaft horsepower.

B.m.e.p. = brake or shaft mean effective pressure, pound per square foot.

C = ratio between clearance volume and piston displacement, or clearance ratio.

$C.O.$ = ratio between piston travel to cut-off and total travel per stroke, or cut-off ratio.

E = internal energy, B.t.u. per pound.

H = enthalpy ($E + PV/J$), B.t.u. per pound.

I.hp. = indicated horsepower.

I.m.e.p. = indicated mean effective pressure, pound per square foot.

J = Joule's equivalent (778), foot-pounds per B.t.u.

K = number of piston strokes per complete cycle.

L = length of piston stroke, feet.

M = mass of steam admitted per cycle, pounds.

M_c = mass of steam remaining in clearance, pounds.

N = number of complete engine cycles per minute.

P = absolute pressure, pounds per square foot.

P_m = mean effective pressure (also m.e.p.), pounds per square foot.

R = ratio between cylinder volume at end of piston stroke and volume at cut-off, or expansion ratio.

S = average piston speed, feet per minute, and as subscript, denotes constancy of entropy during expansion.

T = absolute temperature, deg. Rankine (Fahr. abs.).

v = volume of mass M , cubic foot.

V = specific volume, cubic feet per pound.

CHAPTER XIV

STEAM TURBINE

The second and at present the more popular of the two general types of vapor-driven prime movers is the turbine engine. Curiously enough the turbine was originally conceived many centuries before the reciprocating engine, but it was not developed into a commercially practical machine until the latter part of the last century. Since that time its development and adoption have been so rapid that it virtually has supplanted the reciprocator in those steam plants where a considerable concentration of power is desired in a relatively small weight and space. This concentration the turbine is able to accomplish by reason of its inherently high rotative speed — a characteristic which also adapts it particularly well for use in driving the alternating-current generator and the various types of modern centrifugal machinery.

144. General Characteristics and Classifications. — It will be recalled that in the reciprocating engine the energy delivery as work is obtained through the immediate action of a force upon the face of a

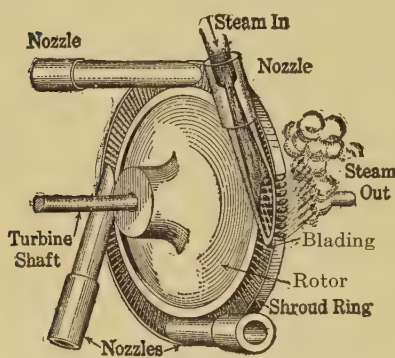


FIG. 88. Simple (impulse) turbine, with casing removed.

moving piston, which force is wholly that due to the static pressure of the vapor within the engine cylinder. In contrast to this the primary moving part of the turbine is a revolving member known as the **rotor**. Upon the rotor are mounted rows of **blades** and against these blades there are directed high velocity fluid jets which have been produced by a steady-flow expansion of the fluid through **nozzles** which are attached to a stationary casing or **cylinder**. The rotor is thereby im-

pelled to move against the resisting force which is exerted by the external *load* on the rotor shaft, with consequent delivery of energy as shaft-work. An inherent characteristic of the turbine is therefore a preliminary conversion of the available energy of the fluid into mechanical kinetic energy, with a subsequent conversion of the kinetic energy into

work. The general arrangement of the nozzles and rotor in one simple type of turbine is indicated in Fig. 88.

In the usual modern turbine of moderate or larger power the shaft and rotor axis and the cylinder are horizontal and, as in the figure, the rows of blades project radially from the periphery of the rotor. The blades are slender metallic strips, crescent-shaped in cross-section and so spaced that a channel is formed between adjacent blades of a row so as to accommodate the flow of the fluid between the blades. The nozzles are so located in the casing that the radial distance of their axes from the shaft axis is the same as the distance to the mid-length of the blades. They are also so formed as to direct the jets in a direction more or less tangential to the plane of a blade row. The general path of the fluid flow through the turbine is a helix which encircles and progresses along the axis. Such a turbine is said to be of the *horizontal, axial flow* type.

Other alternative forms and arrangements which the casing, nozzles and blading may take are quite varied indeed and thus numerous classifications of turbine types may properly be made upon the basis of one or another particular characteristic of form or of utility or operating conditions. However, as the present purpose is solely a consideration of thermodynamic principles, an initial division will be made only into the two classes which differ rather more in principle than in details of form or use. These classes are known as (a) the **impulse** turbine, in which fluid pressure drop and expansion are caused to take place solely within the stationary nozzles, and (b) the **reaction** type, in which the moving blades are so formed that a portion of the pressure drop through the turbine shall also take place within the channels formed by those blades.¹ It would therefore appear that the essential differentiation between the two classes would lie in the character of the moving blades. That is true but other characteristic differences exist as reflections of the primary difference. The basis of the classification and the accompanying characteristics will be made more evident in the subsequent articles.

145. General Energy Relations. — To appreciate the general principles and energy relations which underlie the actions in a turbine and also to recognize more clearly the basic characteristics in which the impulse type differs from the reaction type it will be desirable initially to consider the energy relations in a simple element formed by one stationary nozzle and the passageway or channel between two adjacent

¹ According to the nomenclature of physics and mechanics these class names are in a sense misnomers. They are in current use, however, and although some inappropriateness is recognized they should cause no confusion.

blades in the row of blades directly after the nozzle. This channel is thus in and in effect forms the path of the stream leaving the nozzle. A cross-section through such an element as it would appear in the usual type of turbine described above is represented diagrammatically in Fig. 89, diagram *A* being more representative of the blade cross-section for an impulse turbine and diagram *B* of the blade section for a reaction turbine.

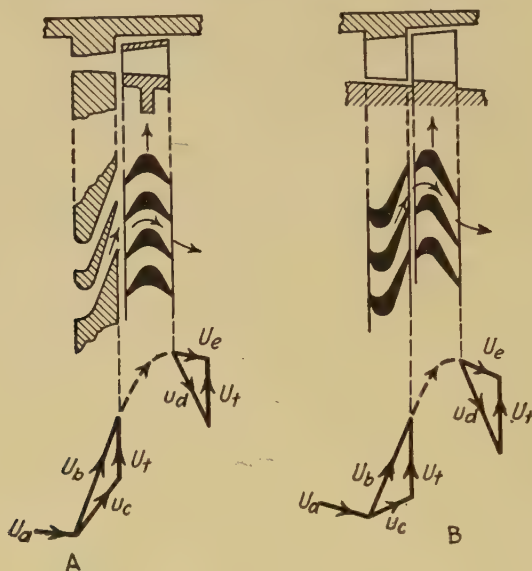


FIG. 89

In the following analyses individual consideration must be given to the several *absolute* and *relative* velocities of the fluid stream as it passes progressively through the nozzle and blading. To designate those velocities the following system of symbols will be employed:

U_a = the *absolute* velocity with which the stream enters the stationary nozzles.

U_b = the *absolute* velocity of the jet leaving the nozzle.

u_c = the velocity of the fluid past a point on the surface of the moving blades at their entrance end, that is, the *relative* velocity of the fluid entering the blade channel.

u_d = the *relative* velocity of the fluid stream at the exit end of the channel formed by the moving blades.

U_e = the *absolute* exit velocity of the stream after leaving the blades.

U_t = the *absolute* tangential velocity of the moving blades.

Making the reasonable assumption that the flow through the moving blades is virtually adiabatic, the energy equation as written between the fluid streams approaching and leaving the moving blades becomes

$$\frac{(U_b)^2}{64.34} + JH_b = \frac{(U_e)^2}{64.34} + JH_e + W_{\text{out}}, \quad \text{or}$$

$$\text{Work output, ft-lb. per lb.} = \frac{(U_b)^2 - (U_e)^2}{64.34} + J(H_b - H_e). \quad (2)$$

This equation is mutually applicable to impulse blading or to reaction blading, as well as to ideal or to actual flow. Particular application to each type of blading will be made shortly but for the present two general and significant conclusions are evident, to wit:

(a) The work output by the blading is increased as the absolute exit velocity U_e is reduced. A corollary of this conclusion is that if the exit velocity and the corresponding kinetic energy of the stream leaving the moving blades are not utilized in a subsequent portion of the turbine it represents a direct loss of available work output. Such a loss would be known as a *leaving loss*.

(b) The work output by the blading is also directly dependent on the change of state of the fluid in passing through the blade channel or, specifically, on its change of enthalpy, $H_b - H_e$.

Equation 2 is written wholly from the viewpoint of one observing the flow to and from the blading from a stationary location such as a point on the casing. A third and useful energy relation may be written between the entrance end (section c) and the exit end (section d) of the channel between the blades, if one adopts the viewpoint of an observer mounted on the moving blades themselves. To such an observer the blade channel would appear as a stationary passage through which the fluid is flowing, quite as the flow in the stationary nozzle would appear to a stationary observer. For this flow within the blade channel the energy equation would become

$$\frac{(u_c)^2}{64.34} + JH_c = \frac{(u_d)^2}{64.34} + JH_d,$$

or

$$\frac{(u_c)^2 - (u_d)^2}{64.34} = J(H_d - H_c). \quad (3)$$

In this relation, except as appreciable shock and turbulence might occur as the fluid passes from the nozzle exit (section b) to blade entrance (section c) or similarly from sections d to e , H_b would equal H_c and H_d would equal H_e . Unquestionably such shock may readily exist and may become a quite material item under conditions of improper blade

design or of failure to operate a turbine under its designed conditions. However, it may be minimized by proper design and operation, and so we shall neglect it for the present; then we may write equation 3 in the alternative form:

$$\frac{(u_c)^2 - (u_d)^2}{64.34} = J(H_e - H_b). \quad (3a)$$

Combining equations 2 and 3a there is obtained a relation which will be of subsequent utility, if *shock losses entering and leaving are negligible*,

Work output in blading, ft-lb. per lb.

$$\begin{aligned} &= \frac{(U_b)^2 - (U_e)^2 - (u_c)^2 + (u_d)^2}{64.34} \\ &= \frac{[(U_b)^2 - (u_c)^2] + [(u_d)^2 - (U_e)^2]}{64.34}. \end{aligned} \quad (4)$$

Quite as equation 3 was written between the entrance and exit sections of the blade channel, so it might similarly be written to include terms for any mid-section. Designating such a section by the symbol x ,

$$\frac{(u_c)^2}{64.34} + JH_c = \frac{(u_x)^2}{64.34} + JH_x = \frac{(u_d)^2}{64.34} + JH_d. \quad (3b)$$

Considering very briefly the conditions which might be anticipated at a section x located at the bend of the channel, it may be noted that the diverting of the stream by the channel curvature must act to crowd the fluid toward the concave surface, with a consequent excess of pressure in the fluid adjacent to that surface in comparison to the pressure in the fluid against the opposite or convex surface. (The concave surface is known as the *face* of the blade, the convex surface is the *back* of the adjoining blade.) It is this phenomenon of unequal pressure on the two sides of the channel which essentially causes or enables the blades and rotor to move against the resisting force imposed by the load which is being carried by the shaft. In the event of high stream velocity within the channel and abrupt curvature the effect may be an actual compression of the fluid to average pressures materially greater than that at the channel entrance, this followed by a reëxpansion in the after portion of the channel. Adequate recognition of such phenomena would be of vital importance in the careful design of blading contours but further attention will not be given them here because of the complex and uncertain character of the analysis.

Example 1. Considering a given *stage* of an impulse turbine, the stage consisting of a group of stationary nozzles and a row of moving blades, presume the steam to come to the nozzles with negligible velocity at 136 lb. per sq. in. abs. and 410° F.

and to expand through a 20 lb. pressure differential in the nozzles. From these data compute:

(a) the actual exit enthalpy, specific volume, and jet velocity obtained if the nozzle efficiency is 0.94;

(b) the work output ideally obtainable if the stream passes through the moving blades without ultimate change of pressure or state and departs from those blades with a negligible absolute leaving velocity;

(c) the work output obtainable if the absolute leaving velocity is 171 ft. per sec. and, due to friction and turbulence, there is an increase of enthalpy of the steam (at constant pressure) of 0.95 B.t.u. per lb. while enroute through the blading.

Plot to approximate but fairly large scale on H - S coordinates the states of the steam at nozzle entrance, at nozzle exit and at blading exit and show the general character of the progressive state changes.

Solution (partial):

$$(a) \quad H_a = 1226.7; S_a = 1.6186; p_b = 116.$$

$$(H_b)_S = 1212.6; (H_a - H_b)_S = 14.1; (H_a - H_b)_{\text{actual}} = 14.1 \times 0.94 = 13.3.$$

$$H_{b, \text{actual}} = 1226.7 - 13.3 = 1213.4; V_{b, \text{actual}} = 4.106.$$

$$U_{b, \text{actual}} = 223.7 \sqrt{13.3} = 816 \text{ ft. per sec.}$$

$$(b) \quad W_{\text{ideal}} = \frac{(816)^2 - 0.0}{64.34} + 0.0 = 10,340 \text{ ft-lb. per lb.}$$

$$(c) \quad W_{\text{actual}} = \dots = 9150 \text{ ft-lb. per lb.}$$

Example 2. Considering a given stage of a reaction turbine, the stage consisting of a row of stationary nozzles and a row of moving blades, presume the steam to come to the stationary nozzles with a negligible velocity and at the same 136 lb. abs. and 410° F. as in example 1. Presume however that the design is such that the steam expands through a 10 lb. per sq. in. pressure drop in the stationary nozzles and an additional 10 lb. in the moving blades. Upon these data compute:

(a) the actual exit enthalpy and jet velocity obtained if the nozzle efficiency is 0.95; (575 ft. per sec.)

(b) the work output obtainable if the absolute velocity leaving the moving blades is 210 ft. per sec. and if enroute through those blades the enthalpy of the steam decreased by 6.5 B.t.u. per lb. by reason of the expansion. Plot on the same diagram as developed for example 1 the steam states at nozzle entrance, nozzle exit and blade exit and again show the general character of the progressive state changes. (9500 ft-lb. per lb.)

146. Impulse Turbine Blading; Energy Analysis.—The fundamental characteristic of the impulse turbine and the one in which impulse turbine blading differs from reaction turbine blading lies in the feature that the turbine is so constructed and the blade channels are so designed that *the pressure at the exit from any given row of moving blades may be and shall be the same as that at the entrance to the row.* In turbine construction this condition is in fact insured by the provision of communicating openings through the rotor disks which carry the blades, but proper blading design must be employed in order that the condition may be met efficiently.

Associated with this condition of equal exit and entrance pressure is the condition that, although the flow through the moving blades is practically adiabatic, some increase of entropy due to turbulence and friction can not conceivably be avoided. Referring again to Fig. 90, in which the nozzle state change was represented by the line ab , this means that the state of the fluid leaving the blades would be representable by a point on the pressure line P_2 but one of increased entropy, such as the state point e_i . Evidently H_e must exceed H_b , whence in equation 2, $(H_b - H_e)$ must be negative or, alternatively, the work output per pound in the moving blades is adversely affected by the mechanical irreversibility in the blading. This is quite as would be anticipated. Optimum performance and maximum work would obviously be obtainable only by ideal reversible flow through the channel, without ultimate increase of entropy or enthalpy. In the figure there is also indicated the character of the state change of the fluid as it progresses through the blade channel. The reason for the slight pressure rise which is shown was discussed in the last paragraph of the preceding article.

Equations 3 or 3a show that for the optimum condition of isentropic flow through the blade channel, with H_e equal to H_b , u_d must likewise equal u_c . The state of the fluid and the relative velocities at both entrance and exit section thus being identical, the blade contour might ideally be such that the channel areas at both sections should be the same, whereby the blades might be symmetrical with respect to a center-line midway between the entrance and exit edges. Such symmetry or an approach to symmetry is thus rather characteristic of impulse turbine blading, in contrast to a more marked dissymmetry which is characteristic of reaction blading.

From the same equations (3 or 3a) it is further evident that for the *actual* condition of an increase of enthalpy through the blading the relative exit velocity u_d must be less than the relative velocity at entrance u_c . Furthermore, if H_e (or H_d) exceeds H_b (or H_c) and the pressure is the same at both states, then the specific volume at exit V_d must correspondingly exceed the volume at entrance V_c . Recognizing these two conditions and applying them in the Continuity Equation, $M'V = AU$, to ascertain the requisite relation between the channel area at blade entrance and exit it develops that

$$\frac{A_d}{A_c} = \frac{V_d u_c}{V_c u_d} \quad (5)$$

As we have seen that both V_d/V_c and u_c/u_d must exceed unity it becomes evident that to accomplish the flow without pressure drop the formation of the actual impulse blade must needs be such that the cross-

sectional area of the channel at exit A_d shall somewhat exceed the area at entrance A_c . In actual turbine construction this requirement frequently is met by a progressive increase of the blade length from entrance to exit edges.

It is customary to evaluate the degree of irreversibility of the flow through the blade channel indirectly, that is, in terms of a ratio between the relative velocities at exit and entrance, or the ratio u_d/u_c . This ratio is known as the **blade velocity coefficient** and will be represented by the symbol ψ (psi). Thus

$$\text{Blade velocity coefficient, } \psi, = \frac{u_d}{u_c}. \quad (6)$$

With successful design of the blade channel and favorable condition, values of 0.9 or greater are attained for this coefficient, with values decreasing materially for adverse conditions and poor design. Among the factors which predominantly influence the coefficient are the breadth of the blades, the radius of curvature of the blade face, the form and dimensions of the cross-section of the blade channel, the thickness of the blade edges and of the edges of the nozzle partitions, and the axial clearance between the nozzle exit and the blade entrance.

Example 3. If the enthalpy and specific volume of the steam entering the moving blades of example 1 were respectively 1213.4 B.t.u. and 4.106 cu. ft. per lb. and the relative entering velocity was 445 ft. per sec. compute, for a blade velocity coefficient of 0.87,

- (a) the relative exit velocity, (388.)
- (b) the exit enthalpy, (1214.4.)
- (c) the exit specific volume, (4.117.)
- (d) the ratio between the requisite cross-sectional areas at exit and entrance ends of the moving blade channels. (1.15/1.0.)

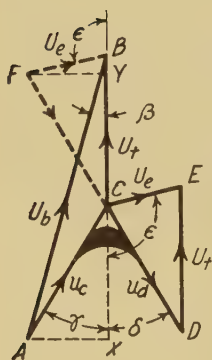


FIG. 91

147. Impulse Turbine Blading; Kinematic Analysis. — The foregoing analyses of impulse blading upon an energy basis are wholly essential. However, they do not serve to indicate those characteristics of blade formation which may afford conditions favorable for a shockless entrance of the jet into the blade channel, for minimization of the leaving loss in unutilized kinetic energy, et cetera. For those and other useful purposes a geometric and kinematic analysis is necessary. For that analysis vector diagrams such as those which appeared below the blading in Fig. 89 are valuable. Such a diagram, as it appears for impulse blading, is repeated for convenience and in a compacted form in Fig. 91.

It will be understood that in this diagram the jet velocity and direction U_b , represented by vector AB , are determined by the pressure difference existing across the nozzle and by the angular location of the nozzle axis with respect to the plane of travel of the blades.

The pressure difference is an item which will be established independently by the steam chest and exhaust pressures under which it is determined that the turbine shall operate, by the number of successive groups of nozzles which the designer may employ in the entire turbine for accomplishing the entire pressure drop and by the relative portion of this entire pressure drop which he may assign to any individual group of nozzles. These considerations are given some further attention in the following article. The methods for computing the jet velocity and for designing the nozzle for a given assigned pressure drop have been considered in Chapter XI.

The angular direction of the nozzle axis also is selected by the designer more or less independently and empirically, the selection being influenced by constructional considerations and by questions of manufacturing facility and of efficiency as determined by test, et cetera. In this connection it is to be remarked that the angular direction of the nozzle is not exactly the direction of the jet, due to a *jet deflection* which occurs with the beveled-end type of nozzles which are required in the steam turbine of usual type. In general it is desirable that the jet angle, angle β of Fig. 91, shall be relatively small. Angles of 10° to 30° are used, with more common values of about 12° to 25° .

Referring further to Fig. 91, the vector CB represents the linear tangential velocity U_t of a point on the blade. Concretely this velocity will equal $2 \pi R N$, where N is the number of revolutions per second at which the rotor is turning and R is the radial distance from the shaft axis to any point in question along a blade length. This radius is evidently somewhat variable and it becomes seriously so in the case of longer blades, with consequent real difficulty in the proper design of such blades. For present purposes we shall regard the blade length to be moderate and a radius to the center-line of the jet to be adequately representative. However, the blade speed is still dependent upon the rotative speed of the shaft, which may be highly variable in a variable speed machine such as a marine turbine. A purpose of this analysis will be the indication of the optimum relation between the blade velocity and the jet velocity. This velocity ratio is referred to so frequently that it may conveniently be assigned a symbol. It is commonly designated as the **blade speed ratio** and by the symbol ρ (rho). Thus,

$$\text{Blade speed ratio, } \rho, = \frac{U_{\text{blade}}}{U_{\text{jet}}} = \frac{U_t}{U_b}. \quad (7)$$

For a given blade velocity and jet velocity the relative velocity of the stream with respect to the rotor and blade is the resultant of the two vectors AB and CB , or the vector AC . This vector thus measures the relative velocity u_c at the entrance to the blade channel, and it also portrays its angle with the plane of the blade travel. This angle is the angle γ of Fig. 91. It seems rather evident that if a blade contour is employed such that at the entrance end of the channel the angle formed by the back of a blade shall equal this angle and the curve forming the face of a blade shall similarly be tangential to this angle, then conditions should be most favorable for a smooth entry of the jet stream into the channel with minimum shock and turbulence. It is upon this general principle that a selection is made for the blade contour at the entrance edge.

From the trigonometry of the entrance triangle, ABC ,

$$u_c \sin \gamma (= AX, \text{ Fig. 91}) = U_b \sin \beta$$

and

$$u_c \cos \gamma (= XC, \text{ Fig. 91}) = U_b \cos \beta - U_t.$$

Dividing the first of these relations by the second it develops that for minimum shock at entrance we should have

$$\tan \gamma = \frac{\sin \beta}{\cos \beta - U_t/U_b}, \text{ or}$$

$$\text{Blade entrance angle, } \gamma, = \tan^{-1} \frac{\sin \beta}{\cos \beta - \rho}. \quad (8)$$

The use of an actual blade entrance angle materially *less* than that as designated by equation 8 will evidently result in an impinging of the jet on the back of the blades, causing a fairly serious shock and disturbance to the stream. The use of a *greater* entrance angle will cause the jet to impinge more abruptly on the face of the blades but without so disastrous an influence on the stream formation, however, if the angular excess is moderate. This latter usage will cause an increase in the driving impulse. Also an analysis of the figure would show that for the operation of a constant-speed turbine with a decreased nozzle pressure difference and correspondingly decreased jet velocity, such as would accompany the operation of the turbine at reduced load with partial throttle opening, the greater blade entrance angle would be beneficial. For these and other reasons the use of entrance angles moderately in excess of the theoretically ideal is common.

In a like manner to that in which the relations between jet, blade, and relative entrance velocities are portrayed by the vector triangle ABC ,

so the relations between the relative exit, blade, and absolute exit velocities are portrayed by the triangle CDE (or the dotted triangle FCB). In that triangle the vector DE is established by the blade velocity U_t . Concerning the vector CD , however, which represents the relative exit velocity u_d , the magnitude of that vector is directly dependent on the relative entrance velocity u_c and the blade velocity coefficient, that is, $u_d = \psi u_c$. Also its direction is dependent on a partially rational and partially arbitrary selection of the blade exit angle δ which is formed by the back of the blading at the exit end. If symmetrical blade contour is desired the exit angle would equal the entrance angle, or $\delta = \gamma$. However, it was shown in the discussions immediately following equation 2 that in general it is advantageous for the absolute exit velocity U_e to be a minimum, and this condition may frequently not be served by symmetry. Thus there are a number of items which must be considered in the selection of the angle δ and it will be desirable to make a fairly comprehensive analysis of the blading as a whole prior to attempting to specify the proper magnitude of the angle.

An analysis will therefore be directed toward the development of a general expression for the proportion to which the kinetic energy of the jet may be utilized in the blading. This analysis will be based on the presumption of a proper selection of the entrance angle γ and a negligible shock loss at blade entrance and exit. For the analysis reference to the energy equations 2, 3 and 4 of Art. 145 will be required, together with some degree of trigonometric manipulation of the vector diagram of Fig. 91. The analysis follows. The several contributory relations associated with the several steps of the analysis are supplied by footnotes.

From equation 4 of Art. 145 and the diagram of Fig. 91,

Work output by blading, ft.-lb. per pound of fluid,

$$= \{[(U_b)^2 - (u_c)^2] + [(u_d)^2 - (U_e)^2]\}/64.34 \quad (4)$$

$$= \{[2 U_b U_t \cos \beta - (U_t)^2] + [(U_t)^2 + 2 U_e U_t \cos \epsilon]\}/64.34^2$$

$$= 2 U_t (U_b \cos \beta + U_e \cos \epsilon)/64.34, \text{ or also} \quad (9)$$

$$= 2 U_t (u_c \cos \gamma + u_d \cos \delta)/64.34^3$$

$$= 2 U_t u_c \cos \gamma \left[1 + \psi \frac{\cos \delta}{\cos \gamma} \right] / 64.34^4$$

² From triangle ABC , $(u_c)^2 = (U_b)^2 + (U_t)^2 - 2 U_b U_t \cos \beta$;

from triangle CDE , $(u_d)^2 = (U_e)^2 + (U_t)^2 - 2 U_e U_t \cos (180 - \epsilon)$.

³ From triangles ABC and FBC ,

$$U_b \cos \beta + U_e \cos \epsilon = XY = u_c \cos \gamma + u_d \cos \delta.$$

⁴ $u_d/u_c = \psi$, equation 6.

$$\begin{aligned}
 &= 2 U_t (U_b \cos \beta - U_t) \left[1 + \psi \frac{\cos \delta}{\cos \gamma} \right] / 64.34^5 \\
 &= 2 (U_t)^2 \left(\frac{\cos \beta}{\rho} - 1 \right) \left(1 + \psi \frac{\cos \delta}{\cos \gamma} \right) / 64.34^6 \quad (10)
 \end{aligned}$$

As the jet kinetic energy is $(U_b)^2/64.34$ and $U_t/U_b = \rho$, it follows that:
 Proportion of jet energy utilized in blading

$$= 2 (\rho)^2 \left(\frac{\cos \beta}{\rho} - 1 \right) \left(1 + \psi \frac{\cos \delta}{\cos \gamma} \right) \quad (11)$$

$$= 2 \rho (\cos \beta - \rho) \left[1 + \frac{\psi \cos \delta}{\cos \tan^{-1} \left(\frac{\sin \beta}{\cos \beta - \rho} \right)} \right]^7 \quad (11a)$$

For a single row of moving blading following a nozzle, such as that to which attention to this point has virtually been confined, the proportion expressed by the relation of equation 11 (or 11a) would evaluate the anticipated success of a blading design in utilizing the jet kinetic energy. Although it obviously would not necessarily measure the actual effectiveness of the real blading it still would serve as a valuable index in the development of the design. The performance so evolved by a diagram analysis is known as the **diagram efficiency** of the blading.

As they stand the expressions for the diagram efficiency are perhaps not immediately illuminating, except as the latter portion of equation 11 definitely indicates a betterment of efficiency if the blade exit angle δ may be reduced and the blade velocity coefficient ψ increased. However, joint consideration of the blade angle and the velocity coefficient is necessary as the former may markedly influence the latter. The equations may be more effectively interpreted if they are solved for a variety of jet angles, blade exit angles, and blade speed ratios. The results of such solutions, employing reasonable corresponding values of ψ , are shown graphically in Fig. 92. This figure indicates rather clearly the important phenomena that

(a) A reasonable variation of the jet angle does not greatly influence the diagram efficiency.

(b) Decrease of the blade exit angle does definitely increase the diagram efficiency.

(c) Efficiency is very materially influenced by the blade speed ratio.

⁶ From triangle ABC , $U_b \cos \beta = u_c \cos \gamma + U_t$.

⁶ $U_t/U_b = \rho$, equation 7.

⁷ $\tan \gamma = AX/XC = (U_b \sin \beta)/(U_b \cos \beta - U_t)$
 $= (\sin \beta)/(\cos \beta - \rho)$, equation 8.

Evidence thus secured shows that for *single-row impulse blading* maximum value of the diagram efficiency is attained when the value of the blade speed ratio ρ is closely equal to **0.50**, and further that the efficiency decreases at a rapidly increasing rate as the blading is designed for operation or is operated at greater or lesser values of the ratio. Adequate attention to this pronounced influence of the blade

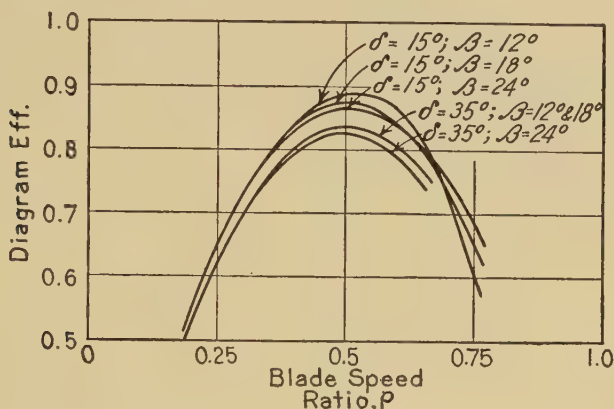


FIG. 92

speed ratio on efficiency in the design and operation of an impulse turbine is perhaps the most important of any individual feature.⁸ In practice values of ρ of 0.45 or less are frequently employed with single row blading such as that under discussion. The lower values have a definitely unfavorable effect on efficiency but are used because they permit the use of lower rotative speed or of smaller rotors and correspondingly smaller and less expensive machines.

The actual values employed for the blade exit angle δ are rather variable, ranging from about 15° for single blading used in dense high pressure fluid up to 35° or more for low pressure use. Consideration of the blade sections of Fig. 89A will show that a reduction of the angle acts to reduce the distance (perpendicular to the stream) between the face of a blade and the back of the adjoining blade and thus tends to reduce the channel area. To provide the necessary increased stream-section at exit (see equation 5 *et sequi*) with a reduced exit angle will evidently

⁸ In this connection it may be noted that, for a symmetrical blade and one which is ideal in the feature of 100 per cent velocity coefficient, equation 11 would give a diagram efficiency equal to $4(\rho \cos \beta - \rho^2)$. By the calculus this quantity would be a maximum when $\rho = \cos \beta / 2$ and the corresponding maximum blading efficiency would be $\cos^2 \beta$. At a jet angle of, say, 15° this would give an optimum blade speed ratio of 0.48 and a maximum diagram efficiency of 0.93.

require a marked excess in the blade length at the exit edge over that at the entrance edge. This may be permissible when dense steam is flowing through the blades, for which case short blades will provide ample sectional area for the flow. However, with attenuated low pressure vapor, blades which at best are of troublesome length may be required in order to accommodate the flow, and material increase of that length may be prohibitive. Therefore the larger exit angles are employed in the lower pressure portions of a turbine in order to avoid the excessive blade lengths.

For purposes of blading design with reference to strength it is essential that information should be available concerning the forces imposed on the blading by the jet. Referring to equation 9, it follows from it that for a given jet and a rate of flow of M' pounds per second,

Work output to blading, ft-lb. per second,

$$= M' \frac{U_t(U_b \cos \beta + U_e \cos \epsilon)}{32.17}.$$

However, the work output per second by the blading upon which the jet is impinging must also equal the product of the net average force F acting on the blades *in their direction of motion* times their velocity, or

Work output to blading, ft-lb. per second = FU_t .

Combining these two relations,

Force (F) on blades in direction of travel, lb.,

$$= \frac{M'}{32.17} (U_b \cos \beta + U_e \cos \epsilon). \quad (12)$$

Note that in this expression $U_b \cos \beta$ and $U_e \cos \epsilon$ are the components of the absolute jet and exit velocities parallel to the direction of the blade travel, whence the expression may alternatively be regarded as evaluating the *rate of change of momentum* of the jet with reference to the direction of blade travel. Frequently the expression is developed upon that basis, by consideration of the rate of change of momentum. In the same manner it may be developed that the axial force which will act to cause transverse or *end thrust* upon the blading and rotor will equal the rate of change of momentum in the axial direction, or

$$\text{Axial force on blades, lb.,} = \frac{M'}{32.17} (U_b \sin \beta - U_e \sin \epsilon). \quad (13)$$

Attention should perhaps be called to the feature that in all of the foregoing analyses and relations the angle ϵ is invariably to be interpreted as the angle ECX (Fig. 91), rather than the angle ECB , and that if the angle should exceed 90° its cosine should be taken in the proper negative sense.

Example 4. The following data shall be taken to apply to a *stage* of an impulse turbine, said stage consisting of a group of nozzles and one row of moving blades.

Nozzles: steam supply pressure, 20 lb. per sq. in. abs.; steam supply temperature, 280° F.; approach velocity, negligible; exit pressure, 10 lb. abs.; nozzle efficiency, 0.93; jet angle, 20°.

Moving blades: peripheral velocity, 700 ft. per sec.; velocity coefficient (ψ) = 0.86; blade exit angle, 30°.

Ascertain the following:

- Jet velocity and the ratio of blade velocity to jet velocity;
- Proper entrance angle for the moving blades, the relative velocities at entrance and exit ends of the blade channels, and the absolute velocity and direction of the stream leaving the blades;
- Enthalpy, specific volume and entropy of the steam at entrance to and at exit from the blades;
- Requisite ratio between the transverse area of the blade channels at exit and entrance ends;
- Work output per pound of steam, computed by equations 2, 4, 9, and 10, and the diagram efficiency of the blading, computed directly from the energy relations and checked by equations 11 or 11a;
- Total tangential and axial forces on blades when in path of jet, pounds per pound of steam passing per second.

Solution.

- $H_a = 1181.5$; $S_a = 1.7675$; $(H_a - H_b)_S = (1181.5 - 1130.0) = 51.5$;
 $(H_a - H_b)_{\text{actual}} = 51.5 \times 0.93 = 47.9$; $U_b = 223.7 \sqrt{47.9} = 1545$ ft. per sec.;
 $\rho = 700/1545 = 0.453$.

- By trigonometric analysis of vector triangle for blade entrance,
 $\gamma = 35.1^\circ$ and $u_c = 919$;
 $u_d = 0.86 \times 919 = 790$;

by trigonometric analysis of vector triangle for blade exit,
 $\epsilon = 92.3^\circ$ and $U_e = 396$.

- $H_b = 1181.5 - 47.9 = 1133.6$; $V_b = 38.08$; $S_b = 1.7729$;

$$H_e = 1133.6 + \frac{(919)^2 - (790)^2}{64.34 \times 778} = 1133.6 + 4.4 = 1138.0;$$

$$V_e = 38.26$$
; $S_e = 1.7797$.

- $\frac{A_d}{A_c} = \frac{38.26}{38.08} \times \frac{919}{790} = 1.17$.

- $W = (\text{Eq. 2}) \frac{(1545)^2 - (396)^2}{64.34} + 778 (1133.6 - 1138.0) = 31,300$ ft.-lb.,

$$\text{or} = (\text{Eq. 4}) \frac{(1545)^2 - (919)^2 + (790)^2 - (396)^2}{64.34} = 31,300 \text{ ft.-lb.,}$$

$$\text{or} = (\text{Eq. 9}) \frac{2 \times 700 (1545 \times 0.9397 - 396 \times 0.0401)}{64.34} = 31,300 \text{ ft.-lb.}$$

$$\text{or} = (\text{Eq. 10}) \frac{2 \times (700)^2 \left(\frac{.9397}{.453} - 1 \right) \left(1 + 0.86 \frac{0.8660}{0.8171} \right)}{64.34} = 31,300 \text{ ft.-lb.}$$

$$\text{Eff.} = \frac{31,300}{(1545)^2/64.34} = 0.842,$$

$$\text{or} = (\text{Eq. 11}) 2 \times (0.453)^2 \left(\frac{0.9397}{0.453} - 1 \right) \left(1 + 0.86 \frac{0.8660}{0.8171} \right) = 0.842.$$

$$(f) F_{\text{tang.}} = \frac{1545 \times .9397 - 396 \times 0.0401}{32.17} = 44.6 \text{ lb.};$$

$$F_{\text{axial}} = \frac{1545 \times 0.3420 - 396 \times 0.9992}{32.17} = 4.16 \text{ lb.}$$

Example 5. Employing such data and results from examples 1 and 3 as are given below, and taking the jet angle as 16° , the blade velocity as 400 ft. per sec., and the blade exit angle as 25° , compute the relative velocity and the suitable back-angle at the blade entrance, the absolute velocity and angular direction of the stream leaving the blading, and the diagram efficiency of the blading. Check the results by laying out the diagram to scale and compare your results with pertinent data and results of examples 1 and 3.

Data as adopted from examples 1 and 3; $U_b = 816$ ft. per sec., $\psi = 0.87$.

($\gamma = 30.3^\circ$; $u_c = 445$; $\epsilon = 106^\circ$; $U_e = 171$; eff. = 0.885; $W = 9160$ ft.-lb.)

Example 6. A 1000-hp. impulse turbine is supplied with steam at the nozzles at 156 lb. per sq. in. abs. and 420° F. and exhausts at 2 in. Hg. abs. The over-all engine efficiency of the turbine may be taken to be 0.763.

The first-stage nozzles are designed to expand to a stage pressure of 40 lb. per sq. in. abs. There is one row of moving blades in the first stage. The nozzle efficiency is estimated at 0.91; the jet angle is 18° ; the blade speed ratio, ρ , shall be 0.45; the blade velocity coefficient, ψ , may be taken as 0.86; the blade contours are symmetrical.

Compute:

(a) the steam rate and total steam required per hour at full load on the turbine; (10,000.)

(b) the total transverse area required at the throats and at the exits of the first stage group of nozzles;

$$\left(A_{\text{th.}} = 1.28; A_{\text{ex.}} = \frac{2.78 \times 10.05}{2237} \times 144 = 1.8 \text{ sq. in.} \right)$$

(c) the relative velocity and the suitable back-angle at the blade entrance and the total transverse area required for the stream at the entrance end of the blading channel (for the group of blades which at any instant are in the path of the jet); (1322, 31.5° , 3.04 sq. in.)

(d) the requisite ratio between the channel areas at blade exit and entrance;

$$\left(\frac{A_d}{A_e} = \frac{1322}{1137} \times \frac{10.15}{10.05} = 1.175. \right)$$

(e) the absolute velocity and direction of the stream leaving the blades; (600, 94° .)

(f) the work output per pound of steam, the diagram efficiency of the blading and the over-all efficiency of the stage (including the nozzle efficiency); (65,400 ft.-lb., 0.84, 0.76.)

(g) the enthalpy of the steam leaving the blading and the entropy increase across the entire first stage; (1138.2, 0.026.)

(h) the total tangential force acting on the group of blades at the instant they are in the path of the jet and the total axial thrust at the same instant; (181, 8.3.)

- (i) the required rotative speed of the rotor shaft if the radius to the nozzles is 24 in.; (4800 r.p.m.)
- (j) the diagram and stage efficiency if the blading were designed for operation at a blade speed ratio of 0.225 (vs. the above 0.45), symmetry of the blading contour still being maintained but the blade velocity coefficient being reduced to 0.85. (0.605, 0.55.)

148. Multi-staging; Impulse Turbine. — In the foregoing analyses attention has in effect been limited to the circumstance of a single group of nozzles followed by a single row of moving blades. Impulse turbines of that simple form, in which the full expansion of the steam from the supply state to the exhaust pressure was accomplished in one step and the resulting jet kinetic energy was more or less fully transformed to shaft-work in one row of blades, were characteristic of the earlier phases of turbine development. As they were developed commercially by the Swedish engineer deLaval they were commonly designated as the **deLaval** type of turbine.

In the then state of power plant evolution, when steam pressures of about 100 lb. and atmospheric exhaust pressures were usual, they were fairly adequate. However, even under such conditions, jet velocities of several thousand feet per second would be obtained and, if the optimum blade speed ratio of about 0.5 were to be employed, blade speeds of about a thousand feet per second would be necessary. Although blades speeds of such a range were very high and caused troublesome centrifugal stresses and rotor balancing difficulties they were actually handled more or less efficiently by the use of small and light rotors operating at rotative speeds as high as some 6000 r.p.m. As such rotative speeds are well above the range of suitable speeds for electric generators or for most centrifugal machinery, and are far above the range of suitable speeds for marine propulsion machinery, such turbines were always provided with high-ratio reduction gears.

With the progress of the art and the progressive increase of steam pressures and condenser vacua in power plant practice, which would make virtually impossible the use of such single-stage turbines, there were devised several methods by which lower rotative speed and, simultaneously, better efficiency may be secured in the impulse turbine. These methods are known as **pressure staging** and **velocity staging**. In the modern impulse turbine various combinations of these methods are also employed. The two basic ones are described in the following.

149. Pressure Staging; Impulse Turbine. — By pressure staging is meant the simple procedure of accomplishing the total expansion of the steam from the supply to the exhaust pressure in a succession or series of nozzle groups instead of in a single group, thereby reducing the pres-

sure drop through and the jet velocity from any individual nozzle. The utilization of the correspondingly moderate jet velocities is then effected by employing proper blading after each group of nozzles, with a material reduction of the blade and rotor speeds which will be required for efficient performance.

The general physical arrangement of a simple turbine employing pressure staging is indicated in Fig. 93, in which ten pressure stages are shown. A turbine of the type illustrated, in which a *single* row of moving blades appears after each group of nozzles, is frequently

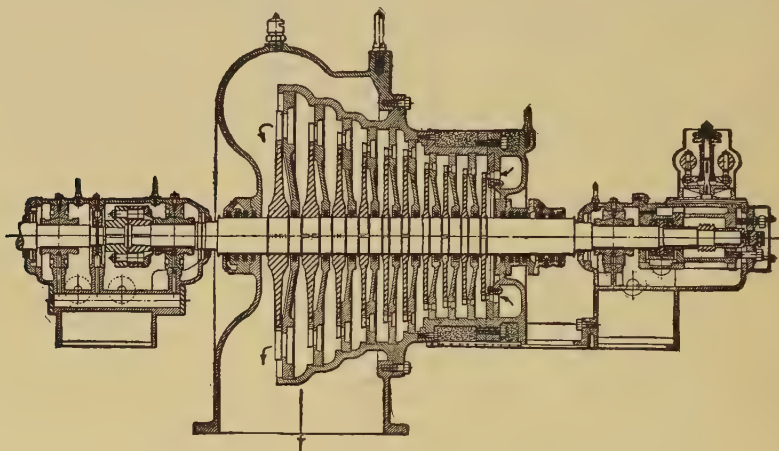


FIG. 93. Impulse turbine with pressure staging.

designated as a **Rateau** type turbine. In the actual commercial turbine as many as 20 to 25 pressure stages may be employed, the number depending upon the total pressure and enthalpy drop which is to be accommodated and on the blade and rotative speeds desired. In the design of such a turbine the pressure drop through each successive nozzle group may be so assigned and the nozzles so designed that an approximately equal enthalpy drop shall occur through each of the series. Thereby like jet velocities, like blade speeds, and like rotor radii may be employed throughout, with corresponding simplification of design and manufacture. However, a rather more conventional manner of pressure distribution is one in which a generally increasing enthalpy drop is assigned to succeeding stages.

It will be observed in the figure that each of the series of nozzles is located in a separate **diaphragm** which is secured to the casing and extends inward to the shaft. Each compartment formed by a diaphragm with nozzles and all blading which is used for the absorption of the jet

velocity would be known as a *pressure stage*. All blading which is associated with a given stage is mounted on a separate disk, which is in turn secured to the shaft. It is evident that by reason of the pressure difference between each successive compartment there would exist a tendency for steam leakage via the channel between the diaphragm and the shaft. Due to inability to lubricate, this channel may not be so constructed as to be wholly steam tight but the leakage is minimized by placing in that channel a **labyrinth packing** which imposes a tenuous and tortuous path for the steam. Obviously any steam which may so make its way from stage to stage has done so by a simple throttling process, without utilization of any portion of its inherently available energy.

Example 7. An impulse turbine with six pressure stages is supplied with steam at 175 lb. gage and 72.5° F. superheat and exhausts at 3 in. Hg. abs. Neglecting all losses and presuming a negligible useful velocity in the steam approaching any group of nozzles, ascertain the proper stage pressures if an equal enthalpy drop and work output were desired from each stage. (Note that, having ascertained the ideally constant entropy of the steam throughout the turbine and the enthalpy at the exit from each stage, the requisite pressure distribution may be readily obtained by use of the *H-S* diagram or by a pressure-entropy table if such is available.) Observe the markedly decreasing pressure drops associated with each successive stage.
(104, 51, 24, 10.5 and 4 lb., 3 in.)

Estimate the optimum blade speed, feet per second, for this turbine if a single row of moving blades were employed in each pressure stage and show that this speed is $1/\sqrt{N_P}$ times the corresponding optimum blade speed for a single-pressure-stage turbine operating through the same pressure range, N_P being the number of pressure stages.
(825.)

150. Velocity Staging; Impulse Turbine.—The second alternative in the reduction of the requisite blade and rotor speeds is that of absorbing the kinetic energy of a given jet in more than one row of moving blades. Such a procedure is known as **velocity staging**. The physical arrangement of a simple turbine employing such staging is illustrated in Fig. 94. In that figure appears the usual nozzle but followed in this case by an initial row of moving blades and then a row of stationary blades which serve to redirect the stream into a *second* row of moving blades. All of these blade rows are contained in the same compartment and so the same pressure will exist at the entrance and exit of any row of blades in the series, whereby the essential characteristic of impulse blading is complied with.

By reason of the presence of the two rows of moving blades and the consequent opportunity for absorbing the jet kinetic energy in two steps there would be said to be *two* velocity stages. In the modern turbine more than three such stages are rarely used after any nozzle

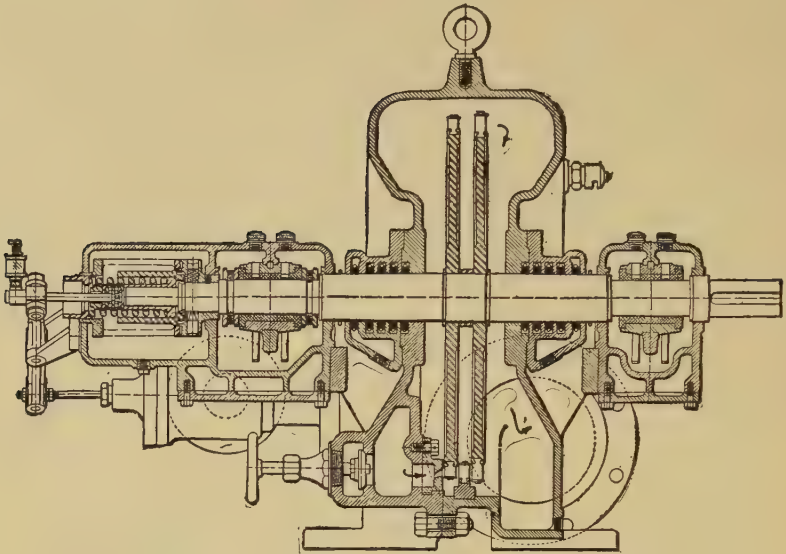


FIG. 94. Impulse turbine with velocity staging.

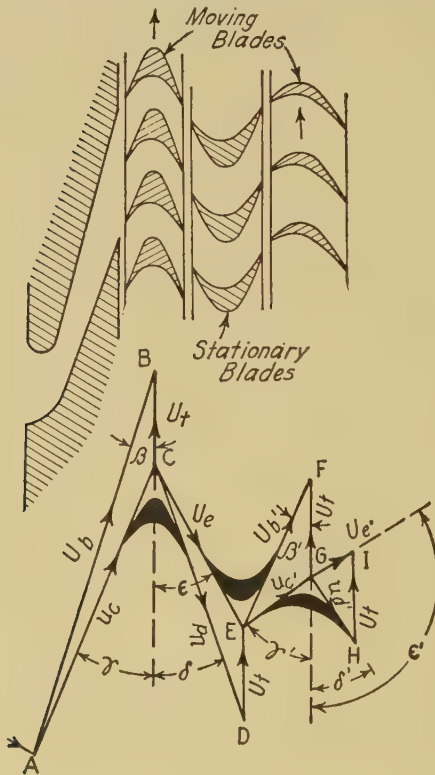


FIG. 95

group, and frequently only two. However, velocity staging may very frequently be repeated in successive compartments of a pressure-staged turbine. A turbine which contains more than one pressure stage but employs velocity staging in *each* of these pressure stages is commonly designated as a **Curtis** type turbine. Also, in any turbine, a pressure stage which employs velocity staging is commonly designated as a **Curtis stage**.

In Fig. 95 there appears the typical vector diagram which serves to represent and correlate the successive absolute and relative velocities throughout the several rows of moving and stationary blades in a two-velocity stage arrangement. The principles and methods employed in the development and analysis of such a diagram for multiple velocity staging are essentially parallel to those used in the analysis of the single-velocity stage diagram of Fig. 91 (Art. 147). However, a complete trigonometric analysis and formulation (such as that of equations 9, 10 and 11) for the rather more complex diagram of Fig. 95 will not be made here, nor can it commonly be made to advantage, due to the considerable number of variables which would be involved. As a result the selection of the blade contours for optimum results in multi-velocity stages is something of a cut-and-try procedure, carried out by scalar drawing of the vector diagram and assisted by experience and empirical rules. However, such procedures would again develop or verify the importance of the design and operation at a proper blade speed ratio, and from them would be drawn the rather natural conclusion that in general maximum diagram efficiency is secured if

Blade speed ratio, ρ , for optimum diagram efficiency

$$= (\text{closely}) \frac{0.5}{N_v}, \quad (14)$$

where N_v is the number of velocity stages in a given pressure stage.

The selection of the entrance angles of the moving blades is, as before, based on the angles of the vectors which give the stream velocity relative to those blades, that is, the angles γ and γ' of vectors AC and EG in Fig. 95. Similarly the entrance angle of the stationary reversing blades is based on the angle of the vector giving the absolute velocity leaving the first moving blades, that is, the angle ϵ of vector CE . The blades might be made symmetrical in contour or, as is the more usual and more efficient practice, the blade exit angles may be made somewhat less than the entrance angles. A characteristic of velocity staging is the progressive flattening of the blades which must evidently occur in successive rows.

It is to be noted that invariably the blading design which will give the

optimum utilization of the jet kinetic energy and the best efficiency is that which will give the minimum magnitude to the sum of (a) the absolute exit kinetic energy of the stream from the last row of moving blades and (b) the enthalpy increase through the series due to friction and turbulence. This is directly evident from the energy equation 2 of Art. 145, in which U_e and H_e may now be regarded as the exit velocity and exit enthalpy from the last of the series of blades. Joint attention to both of these features is necessary because a low exit velocity might be secured without due attention to and at the expense of an incommensurate enthalpy increase due to unfortunate blading design, with corresponding loss of over-all efficiency. In point of fact the guarding against excessive friction and turbulence losses may frequently be the item of major concern as in some instances a moderate leaving velocity may be utilized fairly effectively if the exit stream can pass without great disturbance directly into the nozzles of a succeeding stage.

By direct attention to the rate of change of the tangential component of the stream momentum, or alternatively by analogy with equation 12 of Art. 147, one may set down the expressions for the tangential driving forces exerted by the stream on the several rows of moving blades in a multi-velocity stage and so may ascertain the work distribution between them. Thus for the two-stage arrangement of Fig. 95,

$$\text{Driving force, first row,} = \frac{M'}{32.17} (U_b \cos \beta + U_e \cos \epsilon);$$

$$\text{Driving force, second row,} = \frac{M'}{32.17} (U_b' \cos \beta' + U_e' \cos \epsilon'). \quad (15)$$

From inspection of the figure in connection with these relations it is apparent the driving force decreases materially in the successive rows. It is this phenomenon, combined with the serious increase of enthalpy and entropy due to friction and turbulence as the number of velocity stages is increased, which makes it disadvantageous to employ more than two or three velocity stages in a pressure stage.

In certain types of small turbines interesting arrangements are employed in which speed reduction is obtained by velocity staging but with the use of only one row of moving blades or buckets. This is accomplished by so arranging the stationary reversing channels as to redirect the steam leaving the blades back to and through the same row of blades. Such turbines are known as the **re-entry** type of turbine and have been used considerably for auxiliary purposes when higher efficiency was not considered to be essential.

Example 8. The nozzles of a given pressure stage of an impulse turbine are supplied with steam at 156 lb. abs. and 420° F. and the stage pressure is 40 lb. abs. The velocity of approach to the nozzles is negligible. There are two velocity stages in the pressure stage. Adopt the following additional data:

Nozzle efficiency	0.91	Mean radius to nozzles and blades . .	24 in.
Jet angle	22°	Blade speed ratio, ρ	0.224
Exit angle,		Blade velocity coefficient, ψ ,	
first moving blades	26°	first moving blades	0.82
stationary blades	30°	stationary blades	0.86
second moving blades	34°	second moving blades	0.88

(Note that the nozzle data are the same as those of example 5, except for the jet angle.)

Estimate by velocity diagram analysis and by attention to the energy equation:

- the successive absolute and relative velocities throughout the stage;
(2237; 1783; 1462; 1036; 891; 522; 459; 282.)
- suitable blade entrance angles throughout; (28.0; 38.2; 58.7.)
- the increase of enthalpy in each successive blade channel and the enthalpy of the steam leaving the nozzles and each successive blade row;
($H = 1149.8$; 1155.4; 1156.6.)
- the requisite ratio between the channel area at exit from last row and at entrance to first row of blades; (4.0.)
- the tangential driving force on the several rows of moving blades, lb. per pound of steam per second; (89.8; 20.3.)
- the work output by the blading, foot-pounds per pound of steam per second, computed (a) by the results of paragraph e for the several driving forces and (b) by the results of paragraph c for the enthalpies and the energy equation for the stage (Eq. 2); (55,400.)
- the proportion of the jet kinetic energy which is utilized by the blading, or the diagram efficiency. (0.71.)
- r.p.m. for operating at the specified value of ρ .
- Consider results of item (g) in connection with those of items (f) and (j) of example 6.

151. Reaction Turbine; Energy Analysis. — It was indicated in Art. 144 and thereafter that the fundamentally distinctive characteristic of the *impulse* turbine is the provision whereby *pressure equality* may and shall exist at the entrance and at the exit of any blade channel, whether the blade row is the single one of a deLaval stage or any one of the series of a Curtis stage. Similarly it was noted that the characterizing feature of the *reaction* turbine is the opposite one of a blade channel so formed as to require and provide for a *pressure drop* from entrance to exit of any moving blade. For conformity with this requirement of reaction blading a progressive pressure drop must occur throughout the entire turbine, whereby it will be found to take the physical form of a virtually continuous series of alternate rows of stationary nozzles and moving blades, without the characteristic grouping into individual stages

which is typical of the impulse turbine. It is convenient, however, for purposes of analysis to regard any pair made up of one stationary row and the following moving row as in effect a unit. Such a pair is known as an **element** of the turbine, or sometimes it is in fact likewise designated as a *stage*.

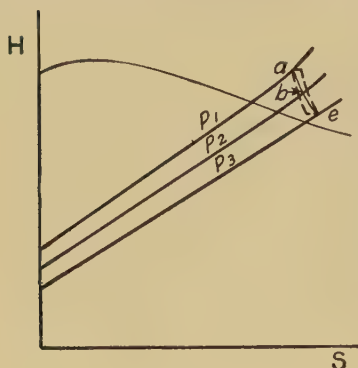


FIG. 96

Giving attention to such an element, let the states *a* and *e* on the *H-S* diagram of Fig. 96 represent the condition of the stream at respectively the entrance and the exit of the element. The total pressure drop, from P_1 to P_3 , which takes place across the element is determined by the operating conditions and by the number of elements which the designer may have pleased to provide in the turbine. The increase of entropy between entrance and exit is as usual occasioned by the inevitable friction

and turbulence accompanying the flow.

As regards the *distribution* of the pressure and enthalpy drop between the stationary and moving components of the element, the designer would in principle be at liberty to select and design for any distribution between the one extreme of a slight pressure drop but no net enthalpy change in the stationary portion and the full enthalpy drop in the moving portion of the element, and the other extreme of the full enthalpy drop in the stationary channels and a slight pressure drop but no enthalpy change in the moving channels. These two extremes are indicated by the dotted lines in the figure. In the first schedule the stationary nozzles would do no more than direct the stream into the moving channels, in which latter nearly the full expansion would occur. Such an element would be said to be one with **100 per cent reaction**. In the second schedule the moving blades would act in a manner almost parallel to those of an impulse turbine and the element would be said to be one with **0 per cent reaction**. Actually the conventional basis of design is one in which the enthalpy drop across the element is distributed about equally as between the stationary and moving components. With an exactly equal distribution the element would be said to be one with **50 per cent reaction**. The location of state point *b* on pressure line P_2 in Fig. 96, which point denotes the state at the exit from the stationary element and the entrance to the moving component, is representative of the latter 50 per cent distribution.

The occurrence of both pressure and enthalpy drop through the mov-

ing blades finds an essential parallel in the expansion of a fluid through a nozzle. It should not seem unreasonable therefore to conclude and state that *in the reaction turbine the moving blade channels may be regarded virtually as moving nozzles*, and such in fact they are. As such their function is the provision for an expansion and relative acceleration of the fluid stream in the after portion of the blade channel, this expansion taking place in a direction more or less opposite to that in which the blades are moving and acting to produce a reactive force or *recoil* which assists in driving the blades. It is this feature which justifies the application of the term **reaction** to the turbine type. By reason of their curvature the blades serve the further function of converting into work the kinetic energy of the jet from the preceding stationary nozzles, this by an impulse action virtually identical with that employed with impulse blading.

By reason of the somewhat wide latitude permissible in the selection of the percentage of reaction to be employed and, as we shall also see, in the blade speed ratio which may be used to advantage, the kinematic and energy analysis of reaction blading does not lead to as concrete conclusions as were evolved for impulse blading. The analysis will serve, however, to assist in justifying certain physical features which are found to be typical of the reaction turbine and reaction blading.

The conventional vector diagram as applied to the reaction element appears in Fig. 97. Subscripts *a* and *b* again refer to respectively the entrance and exit of the stationary nozzles, *c* and *d* to respectively the entrance and exit of the moving nozzles, *e* to the stream after leaving the moving blades, and *t* to the tangential velocity of the blade itself. Velocities designated by *U* are again absolute velocities and those designated by *u* are velocities relative to a point on the moving blade. The figure as presented is one representative of 50 per cent reaction. For that condition the vector triangles *ABC* and *CDE* will be identical.

In the figure there is shown the velocity U_a with which the stream approaches the stationary channels. Its inclusion is based on the presumption that by proper design and operation the kinetic energy of the fluid leaving the preceding element may be preserved and usefully delivered to the stationary nozzles. According to the same presump-

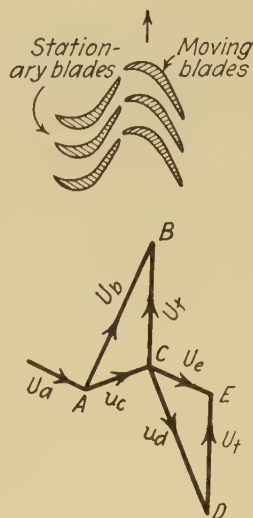


FIG. 97

tion the stream leaving the moving blades of the element under consideration might pass without material shock or turbulence and at velocity U_e into the following stationary channel. These presumptions may be realized to some degree in a well designed group of intimately conjoined elements in a reaction turbine.

The energy relations of the element are again the general ones of equations 1, 2 and 3 of Art. 145, or:

(a) For the stationary nozzles;

$$\frac{(U_a)^2}{64.34} + JH_a = \frac{(U_b)^2}{64.34} + JH_b, \text{ ft-lb. per lb.}; \quad (1)$$

(b) For the moving blades;

$$\text{Work output, ft-lb., per lb.,} = \frac{(U_b)^2 - (U_e)^2}{64.34} + J(H_b - H_e), \quad (2)$$

and also, presuming negligible shock and turbulence at blade entrance and exit,

$$J(H_b - H_e) = J(H_c - H_d) = \frac{(u_d)^2 - (u_c)^2}{64.34}. \quad (3, 3a)$$

Also, by combining equations 1 and 2 to eliminate U_b and H_b or by direct inspection,

(c) For the entire element,

$$\text{Work output, ft-lb., per lb.,} = \frac{(U_a)^2 - (U_e)^2}{64.34} + J(H_a - H_e). \quad (16)$$

This last relation is perhaps of outstanding significance in that it indicates that for a group of elements with 50 per cent reaction, in which $U_a = U_e$, the work output of any element would depend only on the enthalpy change across the element. However, inability or failure to receive the kinetic energy of the fluid approaching any element ($U_a^2/64.34$), through dissipation by shock and turbulence, is reflected directly in an increased value of S_e and H_e , with corresponding decrease of efficiency. Nevertheless it is significant that if the absolute exit kinetic energy from any element may be carried over effectively into the following element the importance of a minimum leaving velocity is correspondingly reduced. This is in distinction to the more positive importance in the impulse turbine of a leaving velocity which shall be the minimum which is compatible with efficient flow through the blading. As a result of this circumstance the diagram efficiency of a reaction element is markedly less dependent upon the blade speed ratio employed, whence the designer has considerably greater latitude in its selection than in the case of the impulse turbine. For the case of zero

carry-over of kinetic energy to an element with 50 per cent reaction the optimum blade speed ratio ρ becomes equal to cosine β . This will closely approach unity.

152. Reaction Turbine; Physical Characteristics. — By attention to various of the foregoing analytical aspects of the reaction turbine one may justify certain of the characteristic features of the physical form of the turbine. Those are considered briefly in the following paragraphs.

(a) **Number, Form and Arrangement of the Elements.** By reason of the pressure difference between the entrance and exit sides of the moving blades there is a distinct tendency toward wasteful leakage of steam around the outer ends of the blades. This will tend to become much more serious in amount than the leakage between the shaft and diaphragm of the impulse turbine, due to the much greater radius and consequent circumference of the ring through which it may occur. It is minimized by the provision of close mechanical clearances between the blade ends and the turbine casing, with close *radial* clearance in the conventional older turbine and *axial* clearance (or "end-tightening") in certain more recent ones. However, some leakage must persist and in order that it may not be excessive it is necessary that a relatively small pressure drop per moving blade or per element shall be employed and a correspondingly large number of elements. Forty and more elements are sometimes provided.

Because of this large number of elements it is not mechanically advantageous to place the moving blades on separate disks or to locate the stationary nozzle passages in distinct diaphragms, as was done in the impulse turbine. Therefore it is the practice in reaction turbine construction to place the moving blade rows on a continuous cylinder or **drum**, and also to form the stationary nozzles by the use of blades which are attached to and project radially inward from the casing and which in general form and contour are quite like to the moving blades. The turbine thus becomes virtually a series of alternate rows of stationary and moving blades, each pair of rows forming the element and the channels in each row forming nozzles. To accommodate the increasingly greater steam volume resulting from the progressive expansion of the steam through the turbine it is the practice to increase progressively the blade lengths and also, at less frequent intervals, to increase the diameter of the drum. Fig. 98 illustrates these features.

Due to the necessary avoidance of large pressure drops per blade row it is virtually the rule that the exit pressure from any row will exceed the critical pressure corresponding to the entrance pressure, whence it may be anticipated from the discussions of Art. 114 concerning nozzles that all of the blading channels will be convergent in so far as the exit

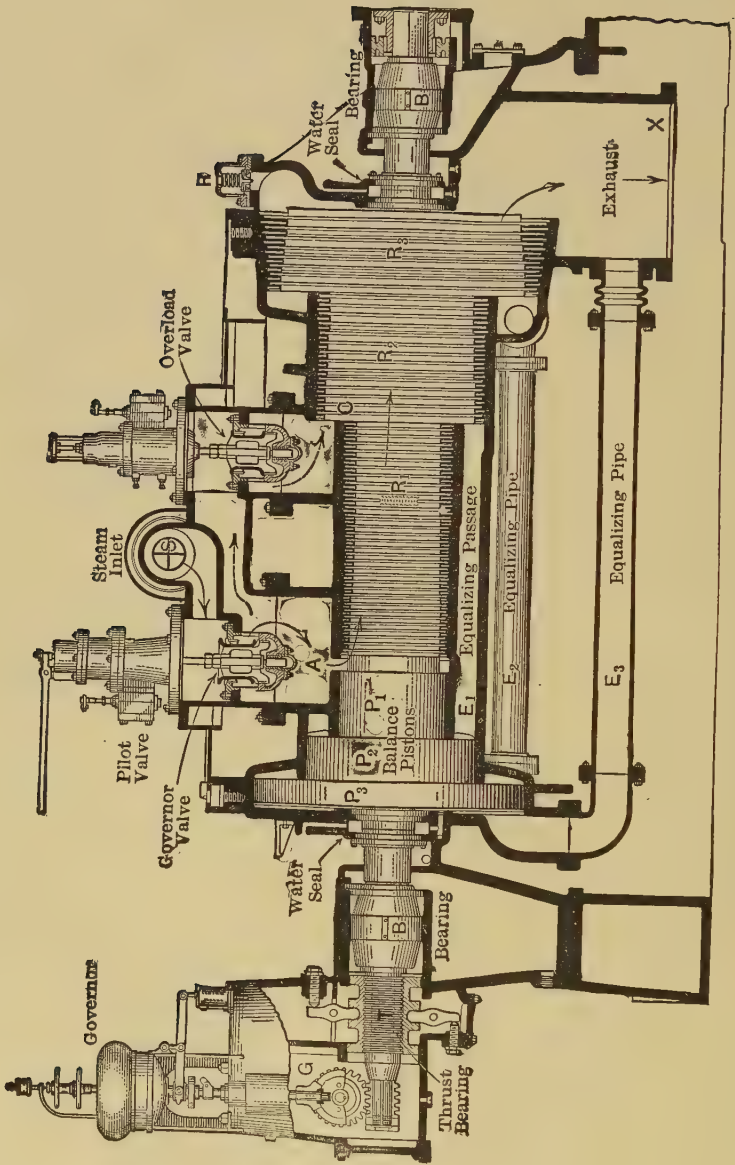


FIG. 98. Reaction turbine, single-flow type with balance pistons.

and entrance areas are concerned. This is in distinction to the slight net divergence which was seen to be essential for the moving blade channels of the impulse turbine. The necessary relation between those sectional areas will be, according to the Continuity Equation,

$$\frac{A_b}{A_a} = \frac{V_b}{V_a} \frac{U_a}{U_b}; \quad \frac{A_d}{A_c} = \frac{V_d}{V_c} \frac{u_c}{u_d}.$$

The necessary channel convergence may be conveniently obtained by use of non-symmetrical blading such as is illustrated in Fig. 97. Comparing further the requisite channel areas at, say, the exits from the successive stationary and moving blades of an element,

$$\frac{A_d}{A_b} = \frac{V_d}{V_b} \frac{U_b}{u_d} = \frac{V_d}{V_b},$$

if 50 per cent reaction is employed. The necessary increase of area as between the two, due to the progressively increasing specific volume, is commonly provided by a progressive increase of blade length (and also by decrease in the number of blades in successive rows). A result is that by proper tapering of individual blade lengths from entrance to exit edge the contours of both sets of blades in a given element or even of all blades of a limited group of elements may be made identical, with considerable advantage in manufacturing.

(b) **Full (vs. partial) Admission of Steam.** By reason again of the pressure difference between the entrance and exit sides of both the stationary blades and the moving blades it is necessary that in all portions of the turbine the steam shall be in passage throughout the entire periphery of any blade row. The reaction turbine is thus necessarily a **full admission** one, in that steam admission to the first (and all) stationary nozzle rows takes place entirely around their periphery. This is in distinction to the impulse turbine, in which only sufficient nozzles need be supplied in any pressure stage as is necessary to accommodate the flow and in which steam admission needs take place through only a portion of the wheel periphery in the higher pressure stages of the turbine, where the specific volume of the steam is small and a small number of nozzles will readily accommodate the flow.

(c) **Forces on Blading and on Drum.** The tangential force on any row of blades is again expressed by the relation of equation 12 (Art. 147). The total axial force is that due to the change of axial momentum of the fluid stream, as expressed by equation 13, plus the axial force due to the pressure difference between the two sides of the annular area (A_b) covered by the blades, or (for a moving row) —

$$\text{Axial force, lb.} = \frac{M'}{32.17} (U_b \sin \beta - U_c \sin \epsilon) + (P_b - P_c)A_b. \quad (13a)$$

Due to the wider sharing of the total tangential force in the full admission reaction turbine and the generally lower velocities the load imposed on any individual blade is in general much less than in the impulse turbine, whence reaction blading is materially lighter. Also the impulse blade is more subject to fatigue failure, due to the intermittent character of its load, in those sections in which there is partial admission.

Due to the necessary mounting of the blades on a continuous closed drum and the exposure of one end of the drum to the higher steam supply pressure and the other end to the condenser pressure, a large end-thrust exists on the drum itself. In practice this is taken care of either by *balancing rings* upon which an opposite steam pressure thrust is caused to be imposed or by constructing the turbine to give *double-flow* of the steam, that is, flow from the center to each end, whereby condenser pressure exists on both ends of the drum. This latter method offers the additional advantage of a reduction of the blade lengths required in the low pressure ends of the turbine, in which the specific volume of the steam is difficultly large. In marine reaction turbines the end thrust on the drum may be utilized to carry a portion of the propellor thrust and thereby partially relieve the load on the propellor thrust bearings.

It should be noted in closing this discussion that, due to the lower enthalpy drops per element or stage and the consequent lower velocities involved, the reaction turbine lends itself more readily to design for operation at lower rotative speed than does the impulse turbine. For that reason, and also due to the less harmful influence on efficiency which results from operation of a reaction turbine at blade speed ratios which depart from the design ratio, it was for a number of years considered to be outstandingly suitable for marine installations, with direct drive to the propellor. That usage was developed, as was the turbine type itself, by the British engineer Parsons, whence the reaction turbine is frequently designated as the **Parsons** type.

With the present practice of using reduction gears of various types in the marine plant and with the conventional shore type of alternating-current turbo-generator plant the lower inherent speed of the Parsons turbine is not an advantage and may become a disadvantage due to the added weight and space which it involves. As a result, turbines made up wholly of reaction elements are not much used at this writing. Instead the large modern turbine either is entirely of the impulse type or it uses one or two Curtis stages at the high pressure end of the turbine, through which perhaps a half or more of the total steam pressure drop is caused

to take place, followed by reaction blading for the remainder of the expansion. Reaction blading suffers rather distinct disadvantages in the high pressure region due to the facts that in that region (a) a given enthalpy drop through adiabatic expansion requires a markedly greater pressure difference than is required in the low pressure region, and (b) with full admission the passage of the dense steam requires only very short blades and the necessary area of the annular clearance ring at the tips of the blades therefore bears a quite appreciable proportion to the total area through the blade channels. Both of these conditions tend to aggravate the leakage seriously and to make it very difficult to secure suitable efficiencies in the higher pressure region of the turbine when reaction blading is employed.

Example 9. In a successive pair of elements of a reaction turbine using 50 per cent reaction, steam is supplied at 56 lb. abs. and 300° F. and is presumed to expand to 51, 46.3, 42 and 38.1 lb. successively in the several blades, doing so with 67 per cent nozzle efficiency in each. Compute the enthalpy and entropy at the exit from each blade.

(1177.3, 1172.3, 1167.2, 1162.2; 1.6610, 1.6645, 1.6679, 1.6713.)

Presuming an effective carry-over of 200 ft. per sec. to and from each element, compute the jet velocity from the stationary blade and the ratio between the requisite transverse area of the first stationary blade channels at entrance and exit ends.

(540, 2.5/1.)

At a blade speed of 430 ft. per sec. and with an exit angle of 20° for both blades of the element, determine the several velocities and angles associated with both. Show vector diagram to approximate scale.

153. The Condition Curve. — One of the primary steps taken by the designer in the initial development of the thermodynamic design of a turbine is that of putting down on H - S coördinates a curve showing the anticipated progressive state change of the steam en route through the entire turbine. Such a curve is known as the **condition curve**. One applicable to an impulse turbine is illustrated in Fig. 99. The curve is recognized to be simply a full extension of the line ae of Fig. 90. It should also be recognized that for the impulse turbine the entire line actually takes the form of a sequence of more or less irregular steps or *notches* rather than that of a smooth curve.

In the evolution of the curve, after selection of the initial supply state of the steam and the exhaust pressure and the determination of the corresponding isentropic enthalpy drop which would ideally be available for transformation to work, the designer must then arrive at a decision concerning the number and character of stages to be employed and from his experience or from other available information will needs make an estimate of the individual or average efficiencies obtainable in the several stages. He may thereby predict the engine efficiency

of the entire turbine and thus arrive at an estimate of the enthalpy of the steam leaving the turbine and so of the end point of the condition curve. Recall in this connection that from the energy equation for a stage or for the entire turbine,

$$W_{\text{out, actual}} = J(H_1 - H_2) + \frac{(U_1)^2 - (U_2)^2}{64.34};$$

$$W_{\text{out, ideal}} = J(H_1 - H_2)_S;$$

$$\text{Efficiency, } e, = \frac{J(H_1 - H_2) + [(U_1)^2 - (U_2)^2]/64.34}{J(H_1 - H_2)_S};$$

$$H_{2, \text{actual}} = H_1 - e(H_1 - H_2)_S + [(U_1)^2 - (U_2)^2]/64.34J;$$

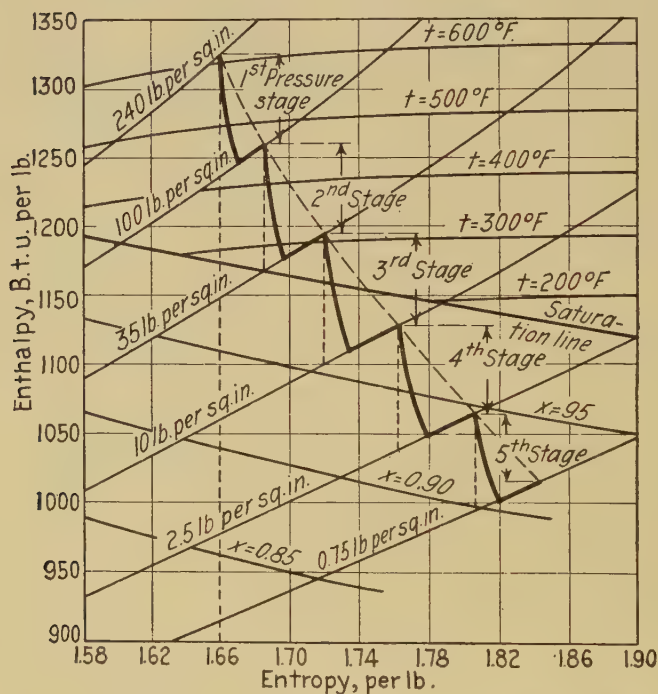


FIG. 99. Condition curve, impulse turbine with pressure staging.

where the subscripts 1 and 2 refer to respectively the entrance and exit of any stage or of the turbine and the subscript S designates the ideal isentropic state change between the two sections. The kinetic energy items may be relatively of sufficiently minor magnitude that they may be neglected in the initial projection of the condition curve.

In predicting the over-all engine efficiency of the turbine from estimates of the individual or average efficiencies of the several stages atten-

tion needs to be given to a certain interesting phenomenon. Thus, considering that portion of the ideally available energy which is prevented from transformation to kinetic energy or work by reason of friction, shock or turbulence in any given stage and is retained in the fluid, it is obvious that a certain portion of that retained energy is still available for transformation to work in the expansion through subsequent stages. The unique but natural result is that the over-all efficiency of the turbine will exceed the average efficiency of the individual stages. This result may be justified alternatively by referring to Fig. 99 and observing that, due to the lack of parallelism in the various pressure lines, *the sum of the energies which are available by the isentropic expansions in the several individual stages must exceed the available energy corresponding to a direct isentropic expansion from the initial state to the final pressure.* This sum is known as the **cumulative enthalpy drop** (or cumulative "heat" drop). Noting that the total work output of the turbine will equal the average stage efficiency times the cumulative enthalpy drop and by definition is also the product of the turbine efficiency times the enthalpy drop for a direct isentropic expansion, it follows that the turbine efficiency must exceed the average stage efficiency.

The ratio between the cumulative enthalpy drop and the direct enthalpy drop, or alternatively the ratio between the turbine and stage efficiencies is commonly known as the **reheat factor**. The name derives from a viewpoint sometimes taken to the effect that the retention of energy in the fluid by reason of friction and turbulence is in a sense equivalent to the supplying of energy to the fluid as heat. The factor may attain values of about 1.04, more or less.

Having arrived at an estimate of the final state of the steam leaving the turbine, the designer must fill in the intervening portion of the condition curve between the initial and the final state. This is done largely by experience and judgment, assisted by various empirical rules. Having done so and having selected the pressure distribution for the series of stages he then has available a convenient graph showing in general the sequence of states of the steam as it passes through the turbine, whereby he may proceed with some degree of exactness to a detailed design and analysis of the nozzles and blading in the individual stages. Eventually it will probably be necessary to revise the condition curve in the light of such information concerning stage efficiencies et cetera as may develop from the detailed design, but after having done so the designer should be in a position to predict with considerable accuracy the steam rate and efficiency which may be expected from the completed turbine.

It is to be remarked that in accurate design attention must be given

to the exact influence of various items which it has not been thought practicable to consider in any detail or even to mention in the foregoing general analyses. Such items are, for example, the actual magnitude of the losses due to shock, turbulence, friction, leakage et cetera, the losses occasioned by fluid friction between the drum or disks and the steam, the mechanical friction losses. Also in modern design an effort is made by some engineers to account accurately for the effect of the super-saturation phenomenon which was discussed briefly in Art. 115-*b*. That has not been considered here due to the present uncertainty as to the extent to which it persists in the turbine. Its omission is in no way intended to imply that it may not be an item of very real significance.

The condition curve is essentially the designer's tool. A simpler but valuable curve for the convenience of the operator is one on which the stage pressures are plotted as ordinates against the number of the corresponding stage as the abscissa, with a separate curve for each of a number of loadings on the turbine. Such a curve plotted when the turbine is new and in good internal condition and when it is operating with the proper steam and exhaust pressures offers useful reference data. Any progressive departure from the original pressure distribution at a given loading may be taken as valid evidence of a deteriorating internal condition in the turbine.

154. Summary. — The steam turbine is a relatively light, compact, and high speed engine which operates through the formation of jets by expansion of the fluid through nozzles and by the direction of those jets onto blades which are mounted upon the rotating element and also, in certain types of turbine, by a reactive recoil resulting from expansion occurring in the channels between the blades. Those turbines which operate solely by diverting the fluid stream and without expansion in the moving blade channels are designated as **impulse** turbines, whereas those in which advantage is also taken of the reactive recoil accompanying expansion are known as **reaction** turbines.

The conventional steady-flow energy equation is invariably applicable in the analysis of the turbine, both as regards the turbine as a whole and as regards any component part. In applying it to turbine processes those may properly be taken to be virtually adiabatic ones, and the ideal processes will as usual be ones without dissipation of available energy or increase of unavailable energy and thus isentropic processes. According to the energy equation the work output per pound of fluid passing will be

$$\text{Work output, ft-lb. per lb.} = \frac{(U_1)^2 - (U_2)^2}{64.34} + J(H_1 - H_2), \text{ where}$$

the subscripts 1 and 2 refer respectively to the entrance and exit sections of the turbine or of any component part.

Applying this relation to the blading, a characteristic of the impulse turbine is that there is a net increase of enthalpy between the entrance and exit of the blading due to the effect of friction and turbulence and the consequent loss of availability and increase of entropy, the state change being prescribed to be one with equal initial and final pressure. In distinction there is an increase of entropy but net decrease of enthalpy in the moving blades of the reaction turbine, due to the net drop of pressure between entrance and exit. In impulse blading, due to the increase of entropy and enthalpy at the given stage pressure, the specific volume must show a net increase and the relative velocity a net decrease across the blades. The blade channels must thus provide a slightly increasing area as between entrance and exit in order that pressure equality may be secured.

A kinematic analysis of the velocity relations in impulse blading by means of the vector diagram shows that there is an optimum relation between the tangential velocity of the blades and the jet velocity. For a single row of blading after a nozzle a design and operation at a ratio of about 0.5/1.0 between blade and jet velocities is required for most efficient performance. Designing for a materially different ratio or operation at a ratio materially different from that for which the design was developed has a markedly deleterious effect on the efficiency of the impulse type of turbine. Analysis by the vector diagram also serves to ascertain the proper entrance and exit angles and the blading contour which should be employed in manufacture and the tangential and axial forces which the blading must be designed to withstand.

An impulse turbine which would endeavor to accomplish the complete expansion of the steam from the modern higher boiler pressure to the condenser pressure in a single nozzle or group of parallel nozzles, and which would further attempt to absorb the resulting jet kinetic energy in a single row of moving blades, would need to operate at an excessively and impossibly high blade speed in order to obtain reasonable efficiency and even then the efficiency secured would be unsatisfactory. To better this condition it is the practice to effect the expansion through a series of nozzle groups, with moving blades after each group of nozzles, and also to absorb the kinetic energy of any individual group of jets in more than one row of moving blades, with stationary blades located between the moving blade rows in order to redirect the stream at a suitable angle for entrance into the following row of moving blades. The first procedure is called **pressure staging** and the second, **velocity staging**. Pressure staging reduces the requisite blade speed in

the ratio of, roughly, $1/\sqrt{N_P}$, where N_P is the number of pressure stages employed. Velocity staging reduces the requisite blade speed in the ratio of about $1/N_V$, where N_V is the number of velocity stages in the pressure stage. A score or more of pressure stages may be employed effectively but more than two or three velocity stages in any pressure stage is not advantageous.

By reason of the pressure drop which is caused to occur in both the stationary and the moving channels in the reaction turbine that type of turbine becomes in effect a series of successive stationary and moving nozzles. A pair of these is known as an **element**. The pressure and enthalpy drop per element is established by the number of elements employed and the total pressure range to be accommodated. An element may be designed for any desired distribution of the enthalpy drop as between the stationary and moving components but an approximately equal distribution is customary. An element designed for such a distribution is said to have 50 per cent reaction. For a 50 per cent reaction element the vector diagram becomes essentially two equal triangles. The steady-flow energy equation as usual is valuable for the analysis and interpretation of the energy relations of the reaction element or of any groups of elements.

Because of limitations resulting from the application of the reaction principle the reaction turbine takes a physical form which differs more or less distinctly from the impulse turbine. Certain of the characteristic features are:

(a) There is need for close mechanical clearance between the tips of the blades and the adjacent casing (or rotor), in order to minimize an otherwise excessive leakage.

(b) A relatively large number of elements is necessary, again to minimize leakage, for which reason it is mechanically advantageous to mount the moving blades on a continuous drum and to employ blades for formation of the stationary nozzle channels, this in distinction to individually formed blocks of nozzles which are characteristic of the impulse turbine.

(c) All blade contours are such as to give more or less convergent channels, with the result that the blade contours are typically non-symmetrical, in distinction to the more nearly symmetrical sections of impulse blading.

(d) Full admission of steam must exist throughout the entire periphery of the turbine.

(e) The tangential forces on the blading are much smaller than they are in the impulse turbine and they are effectively steady, whence the reaction blading may be structurally lighter.

(f) The axial forces on the blades are very small but in a *single-flow* reaction turbine a large end thrust will exist on the drum due to the action of the high pressure on the one end and the low condenser pressure on the other end of the closed drum.

(g) The reaction principle is not well adapted for use in the high pressure end of a turbine, therefore it is more commonly employed only in the medium and lower pressure portions.

The condition curve is a graphic representation, usually on H - S coördinates, of the entire sequence of state changes occurring during the passage of the steam through a turbine. By careful predetermination of the form of the curve the designer may proceed intelligently with the detailed thermodynamic design of all parts of the turbine and may accurately predict the operating efficiencies which may be secured from the completed machine.

155. Review Questions and Topics. —

1. (a) Of the reciprocating steam engine and the steam turbine which is the older in point of original conception but which enjoys the greater popularity at present, and why?

(b) Describe briefly the primary components of the steam turbine.

(c) What are the two primary classes of turbine and what is the fundamental basis upon which the classification is made?

2. (a) Show by sketch the typical arrangement of a nozzle and row of moving blades and the absolute path of the steam through the blades; and portray by vector diagram the several absolute and relative velocities at entrance and exit of the blade channels.

(b) Write the energy equation for the nozzle, express it in a form solved for the jet velocity, and depict the state change through the nozzles on H - S coördinates.

(c) Write the energy equation between a section in the approaching jet and one in the stream leaving the moving blades, express it in a form solved for the work output per pound, and discuss the evidence to be deduced from the last equation.

(d) Write the energy equation between a section at the entrance of the moving blade channel, a mid-point in the channel, and the exit from the channel; and discuss.

3. (a) Depict on H - S coördinates the state change ideally occurring in the moving blade channel of an impulse turbine and that actually occurring. Correlate the diagram and equation 2 of Art. 145.

(b) From the H - S diagram and equation 3 of Art. 145 draw conclusions as to the requisite relation between the channel area at entrance and exit of impulse blading, both for ideal and for actual conditions of flow.

(c) What is the term designating the ratio between the relative velocities at exit and at entrance of the blade channels of the impulse turbine?

4. (a) What are the purposes of the vector diagram and the kinematic analysis of the flow through impulse blading?

(b) Discuss briefly the considerations which control the pressure drop for which a given nozzle shall be designed, the basis of selection of the nozzle and jet angles, and the items involved in proper selection of the blade velocity. What is the term designating the ratio between the tangential velocity of the blades and the blade velocity?

(c) Draw an illustrative vector diagram employing a reasonable jet angle, a favorable blade-speed ratio, and a suitable blade exit angle; and then draw a similar diagram to like scale but with an unfavorable blade-speed ratio, and indicate the condition which makes the latter arrangement an unfavorable one. Show by the first diagram the effect of reduction of the blade exit angle.

(d) State the conclusion to be drawn from the vector diagram as regards the proper angle of the blades at entrance.

(e) Approximately what blade-speed ratio is found to be the optimum one upon which to base the design of a single row of moving blades? Show by a representative curve the effect of departure from the optimum ratio in the design. What would be the general effect of the operation of an impulse turbine at a blade-speed ratio materially different from that for which it was designed?

(f) Quote the relations expressing the tangential force and the axial force acting upon impulse blading when in the path of the jet.

5. (a) What condition makes impracticable the construction and operation of an impulse turbine with a single group of nozzles and a single row of moving blades?

(b) What means are employed for reduction of the individual jet velocities to be handled in the impulse turbine? Discuss briefly.

(c) What means are employed for reduction of the suitable blade speed when handling the jets from an individual nozzle or group of nozzles. Discuss briefly and show an illustrative vector diagram for the jet and blading when the procedure in question is employed.

(d) State the distinguishing characteristics of the deLaval type and the Rateau type of turbine and of the Curtis stage.

(e) Compare the relative effectiveness of a given number of pressure stages and the same number of velocity stages as regards their influence on the proper blade speed.

(f) Describe the general mechanical form taken by an impulse turbine.

6. (a) Define an element of a reaction turbine and represent on H - S coordinates the state change taking place from entrance to exit of a given element. Indicate the meaning of 50 per cent reaction in the element.

(b) Draw an illustrative vector diagram for an element with 50 per cent reaction, discuss briefly and correlate with the energy equation 16 of Art. 151.

(c) Indicate briefly the characteristic and distinguishing physical features of a reaction turbine as regards the mechanical clearance of the blade tips, the number of elements in the turbine and the resultant advantageous form of the rotating member, the typical contour and the symmetry or dissymmetry of the blading, the degree to which steam admission and passage occupy the full periphery of the turbine, the forces which load the blades and the consequent structural size and strength of the blades, the axial forces on the blading and on the rotating element as a whole.

(d) Discuss briefly the relative importance of the employment of a closely defined blade-speed ratio in the design of a reaction turbine.

(e) Discuss briefly the advantages or disadvantages of reaction blading in the high pressure end of a turbine.

7. (a) Describe the condition curve and the general method by which it is evolved and its utility.

(b) Define the cumulative enthalpy drop and the reheat factor.

(c) What is the utility of a graphic record of the stage pressures in turbine operation?

Abbreviations and Symbols, Chapter XIV

- A = cross-sectional area of a channel, square feet.
 a = subscript referring to the entrance of a stationary nozzle.
 b = subscript referring to the exit from a stationary nozzle.
 c = subscript referring to the entrance end of a moving blade channel.
 d = subscript referring to the exit end of a moving blade channel.
 e = subscript referring to the stream after leaving the moving blade, and
 = efficiency.
 F = force, pounds.
 H = enthalpy ($E + PV/J$), B.t.u. per pound.
 J = Joule's equivalent, foot-pounds per B.t.u. (= 778).
 M' = mass-rate of flow, pounds per second.
 N_P = number of pressure stages.
 N_V = number of velocity stages.
 S = entropy and, as subscript, designating constancy of entropy.
 t = subscript referring to the tangential velocity of a moving blade.
 U = absolute velocity, feet per second.
 u = relative velocity, feet per second.
 V = specific volume, cubic feet per pound.
 W = work, foot-pounds of fluid.
 α = angle taken by stream entering a stationary nozzle or blade, the angle being
 that with respect to the plane of motion of the moving blades.
 β = angle taken by the stream leaving the stationary nozzle.
 γ = angle of the vector showing the relative velocity of the stream entering a
 moving blade channel.
 δ = angle of the vector showing the relative velocity of the stream leaving the
 moving blade channel, and established by the blade exit angle.
 ϵ = angle taken by the stream after exit from the moving blade channel.
 ρ = ratio of the tangential velocity of a moving blade to the jet velocity.
 ψ = ratio between the relative exit velocity and relative entrance velocity
 within the blading channel of an impulse turbine.

CHAPTER XV

COMBUSTION

At this point let us again review very briefly the outstandingly significant aspects of man's efforts toward procuring and directing to his purposes the stores of energy which he finds in nature. Aside from several current supplies of mechanical energy in the winds and in impounded streams, which he utilizes in the wind mill and the sail ship and in the hydraulic power plant, virtually the only other supply of natural energy which man has discovered and learned to release, control, and utilize in large amounts is the internal chemical energy that is stored in certain substances which he designates as **fuels** and that is released by the process of oxidation known as **combustion**.

The process of energy release by the combustion of a fuel has its intense significance to mankind for two general reasons. The one is that the *oxygen* which constitutes the second vital element for effecting the combustion process is so abundantly and accessibly provided in our atmosphere. The second is that the combustion reaction effects a transformation of the chemical energy of the fuel into molecular energy in the physical products of the combustion, thereby causing the temperature of those products to become markedly higher than that of the atmosphere — and it has been indicated repeatedly and emphatically that an elevated temperature is the *sine qua non* of all of the processes which may be devised for the conversion of molecular energy into work. Perhaps we may eventually find means of securing natural energy from other distinctly different sources of supply and may perchance learn to transform that stored energy directly into electric current and thus into work, without the interposition of the inherently inefficient and unfortunate temperature phase, but until such time we shall need to continue our dependence on the combustion process.

In the several immediately preceding chapters we have considered the means that have been developed whereby, given a source of energy at high temperature, that energy may be transferred to a vapor and a greater or less portion of the energy may then be transformed into work by a cyclic series of processes properly carried out on the vapor by means of certain types of engines, together with other associated apparatus. In a following chapter we shall consider those devices in which the

energy release is effected and the work delivery is accomplished jointly and directly, that is, the internal-combustion engine. In the present chapter attention will be given to certain of the primary principles of the combustion process itself.

156. Combustion; Quantitative Chemistry of Combustion. — As indicated above, by the term **combustion** is meant the more or less rapid reaction between the element oxygen, which makes up about one-fourth of our atmosphere as it exists at the earth's surface, and certain simple substances or certain constituents of more complex substances which we know as **fuels**.

The particular forms taken by the various natural and fabricated fuels are very numerous indeed. The incomplete list of such fuels which appears in the accompanying table will serve to indicate their variety.

TABLE XI

Solid Fuels	Liquid Fuels	Gas Fuels
Anthracite coal. Bituminous coal. Lignite. Peat. Wood, cane, etc. Coke and other residues from distillation of solid and liquid fuels.	Petroleum and its distil- lates and fractionates. Various by-products of bituminous coal dis- tillation. Alcohols.	Natural gas. Artificial gases produced by distillation or in- complete combustion of coal or oil.

For the purposes of engineering thermodynamics we do not need to have a complete picture of the internal mechanism of the combustion process, but it is essential that we have available a method for evaluating or predicting quantitatively the amount of energy transformed during a given reaction, as well as a basis for determining the relative masses of fuel and air which need be brought together for proper carrying out of the reaction. In subsequent articles some general consideration will be given to the energy relations during combustion. For determining the proper mass relations some use of the formulations of quantitative chemistry is necessary. They are considered briefly in the following article. The reader is presumably familiar with the conventional symbols and methods of the chemistry.

A characteristic which will be found to be common to all fuels is that they contain only two combustible elements of primary importance, carbon (C) and hydrogen (H_2), with small proportions of an addi-

tional combustible, sulphur (S).¹ More often however the latter is an undesirable impurity rather than a beneficial constituent.

The carbon constituent may occur almost uncombined, as in anthracite coal or in coke; in a great variety of combinations with hydrogen or with hydrogen and oxygen (in the form of hydrocarbons, C_xH_y , or carbohydrates, $C_xH_yO_z$) in the *volatile matter* of solid fuels and in the liquid and gas fuels; or as the monoxide (CO) or dioxide (CO_2) in certain gas fuels. The hydrogen constituent similarly may be uncombined in gas fuels, combined with carbon or carbon and oxygen in solid and liquid fuels, or combined with oxygen as water (that is, as the *moisture* in solid fuels). Not much is known concerning the forms in which sulphur occurs.

Aside from the combined oxygen already mentioned the solid (and liquid) fuels also contain certain non-combustible materials. These include small proportions of nitrogen and greater or less amounts of various inorganic compounds of silica, aluminum, iron, calcium, et cetera. The latter form the **ash** of the solid fuels. The nitrogen may also appear uncombined in relatively large quantity in artificial gas fuels. The mechanical engineering hand books offer convenient tables which show typical chemical analyses of the various common fuels.

The combustion of the carbon in a fuel may proceed in several distinct manners. Thus it may be burned incompletely to form carbon monoxide, with partial conversion of the stored chemical energy to molecular energy; or the monoxide may combine with additional oxygen to form the dioxide, thereby completing the energy transformation; or finally the original carbon may be burned completely to CO_2 in a single reaction or in the two reactions taking place in immediate succession. Hydrogen seems to combine with oxygen more avidly than does carbon. It may in general be taken to burn completely to water vapor, H_2O .

The composition of atmospheric air at or moderately above ground level, from which the oxygen for the usual combustion process is obtained, is closely as follows:

Percentage Composition of Air

Oxygen	20.9, by volume: 23.1, by weight.
Nitrogen, including about 1 per cent of argon and traces of CO_2 , H_2O , He, etc.	79.1, by volume: 76.9, by weight.

¹ It is evident that Table XI contains none of the fabricated explosives such as gunpowder, et cetera. In general those substances differ from the fuels in that they do not depend on the atmosphere for oxygen but are physical mixtures or chemical compounds carrying sufficient oxygen in intimate relation to the combustible material.

The equivalent molecular weight of air is, closely, 29.0. It will be convenient to note that, in accordance with the above composition,

$$\text{Volume of N}_2 \text{ per cu. ft. of O}_2 = 0.791/0.209 = 3.78 \text{ cu. ft.} \quad (1)$$

$$\text{Volume of air per cu. ft. of O}_2 = 1.00/0.209 = 4.78 \text{ cu. ft.} \quad (2)$$

$$\text{Mass of N}_2 \text{ per lb. of O}_2 = 0.769/0.231 = 3.32 \text{ lb.} \quad (3)$$

$$\text{Mass of air per lb. of O}_2 = 1.00/0.231 = 4.32 \text{ lb.} \quad (4)$$

To determine the relative masses and volumes of fuel, of air and of the gaseous products of combustion involved in a combustion process recourse is had to the conventional equations used by the chemist for portraying the reactions. These equations simply represent symbolically his accumulated experience concerning the manner and relative proportions in which the elements concerned will combine. For an ideal combustion, in which only the theoretically requisite amount of atmospheric oxygen is supplied but combustion is complete, the following equations apply:

(a) For carbon burning completely to CO₂,



(b) For hydrogen burning to H₂O,



(c) For sulphur burning to SO₂,



It is recognized that in such expressions the subscripts represent the number of atoms of any element in a molecule of the element or in a molecule of a chemical compound containing the element. Also the coefficient of any element or molecule represents the relative number of molecules of that element or compound going to make up the original mixture of combustible and air or resulting from the combustion. For convenience, however, the subscript or the coefficient *unity* is never written but it is indicated by its very absence. Note further in this connection that, in accordance with *Avogadro's law*,² it may be taken as an approximation of sufficient exactness that at a given temperature and pressure the number of molecules per unit volume is the same for all *gases*. It follows that, *for the gaseous elements or compounds*, the coefficients likewise represent the relative volumes occupied at a given pressure and temperature by the individual gas in the mixture or in the products of combustion. Thus, interpreting equation 5, it states that

² See footnote 5, Art. 81.

one atom of carbon combines with one diatomic molecule of oxygen to form one molecule of carbon-dioxide, and also that if the oxygen is atmospheric oxygen each molecule or volume of oxygen carries with it in physical mixture the equivalent of 3.78 molecules or 3.78 volumes of nitrogen. The nitrogen is extremely inert, is unaffected by the combustion, and remains unchanged in form and amount after the combustion.

Giving further attention to the interpretation of the combustion equations, observe that the relative mass of the atom of any element is expressed and measured by its *atomic weight*, and correspondingly the relative mass of a molecule, or its *molecular weight*, is determined by summing the products of the number of atoms per molecule times its atomic weight for each element occurring in the molecule. Finally, therefore, the relative mass of any of the several constituents of a fuel-air mixture or of the products of combustion will be represented by the product of the coefficient of that constituent times its molecular weight.³ The atomic weights of the several elements which will be involved in the consideration of combustion are, closely,

Carbon	— 12.0;	Hydrogen	— 1.01;
Sulphur	— 32.1;	Oxygen	— 16.0;
Atmospheric nitrogen (including argon, etc.) — 14.08.			

The molecular weights of various of the gaseous elements and compounds are, closely,

Hydrogen, H_2	— 2.02;	Carbon monoxide, CO	— 28.0;
Oxygen, O_2	— 32.0;	Carbon dioxide, CO_2	— 44.0;
Water vapor (H_2O)	— 18.02;	Sulphur dioxide, SO_2	— 64.1;
Atmospheric nitrogen — 28.15.			

On the basis of the above the reaction equations 5, 6, and 7 may be more fully interpreted in the relations given in tabulated form here. The constants developed in this table will be frequently employed from this point on. The practical application of the principles tabulated is illustrated by examples.

³ The *mol* conception of Art. 81 may frequently be employed with advantage. Recall that the mol is defined as a mass of m lb. of a substance, where m is its molecular weight. Since, with permissible approximation for gases, $R = 1545/m$ and $P(MV) = MRT$, the volume of m lb. (1 mol) of any gas = $mRT/P = 1545 T/P$ and thus is the same for all gases at a given temperature and pressure. At 60° F. and 14.7 lb. the mol volume of the gases is 380 cu. ft. (per m . lb.).

The coefficients in the combustion equation may thus be taken to indicate the relative number of mols of any gaseous constituent of the mixture or products.

For carbon	C	+ O ₂	+ 3.78 N ₂	= CO ₂	+ 3.78 N ₂
Relative volumes...	(solid)	+ 1	+ 3.78	= 1	+ 3.78
	(solid)	+ 4.78 (air)		= 4.78 (products)	
Relative masses....	12	+ 32	+ 106.4	= 44	+ 106.4
	1	+ 2.67	+ 8.86	= 3.67	+ 8.86
	1	+ 11.53 (air)		= 12.53 (products)	

For hydrogen	H ₂	+ 0.5 O ₂	+ 1.89 N ₂	= H ₂ O	+ 1.89 N ₂
Relative volumes...	1	+ 0.5	+ 1.89	= 1 (if vapor)	+ 1.89
	1	+ 2.39 (air)		= 2.89 (products)	
Relative masses....	2.02	+ 16	+ 53.2	= 18.02	+ 53.2
	1	+ 7.94	+ 26.4	= 8.94	+ 26.4
	1	+ 34.3 (air)		= 35.3 (products)	

For sulphur	S	+ O ₂	+ 3.78 N ₂	= SO ₂	+ 3.78 N ₂
Relative volumes...	(solid)	+ 1	+ 3.78	= 1	+ 3.78
	(solid)	+ 4.78 (air)		= 4.78 (products)	
Relative masses....	32.1	+ 32	+ 106.4	= 64.1	+ 106.4
	1	+ 1	+ 3.33	= 2	+ 3.33
	1	+ 4.33 (air)		= 5.33 (products)	

Example 1. Find the mass and volume of air theoretically required and the mass, volume, and volumetric analysis of the products of the combustion of 1 lb. of a fuel made up of

Carbon, 84 per cent by weight
Hydrogen, 16 per cent by weight

Determine the volumes for 14.7 lb. abs. and 70° F. and also the volume of the products at 1000° F., giving due attention to the degree of vaporization or condensation of the H₂O in the products.

Solution.

Mass of air required, pounds:

$$\text{For 0.84 lb. of carbon,} \quad = 0.84 \times 11.53 \quad = 9.68$$

$$\text{For 0.16 lb. of hydrogen,} \quad = 0.16 \times 34.3 \quad = 5.49$$

$$\text{Total air required, pounds per pound of fuel} = 15.17$$

Volume of air required, at 70° and 14.7 lb., cubic feet per pound of fuel:

$$= 15.17 \times \frac{53.3 \times 530}{14.7 \times 144} = 15.17/0.075 \quad = 202$$

Mass of combustion products, pounds per pound of fuel:

$$\text{CO}_2 - 0.84 \times 3.67 \quad = 3.08$$

$$\text{H}_2\text{O} - 0.16 \times 8.94 \quad = 1.43$$

$$\text{N}_2 - 15.17 \text{ (lb. of air)} \times 0.769 \quad = 11.67$$

$$\text{Total mass, pounds per pound of fuel} = 16.18$$

Volume of combustion products at 14.7 lb. and 1000° F., cubic feet per pound of fuel:

$$\text{CO}_2 - 3.08 \times \frac{1545 \times (460 + 1000)}{44 \times 14.7 \times 144} = 3.08 \times \frac{1066}{44} = 74.5$$

$$\text{H}_2\text{O} - 1.43 \times \frac{1066}{18.02} = 84.6$$

$$\text{N}_2 - 11.67 \times \frac{1066}{28.15} = 442$$

$$\text{Total volume, cubic feet per pound of fuel} = 601$$

Volume of combustion products at 14.7 lb. and 70° F. At this temperature the water vapor will have condensed out to a large degree, but that which has not condensed must exist as saturated vapor at the pressure corresponding to 70° F., which pressure is 0.363 lb. per sq. in. The partial pressure of the remaining constituents is thus $(14.7 - 0.363 =) 14.337$ lb. To ascertain their volume at this partial pressure, and thus the volume of the mixture, note that the gas constant for the residual mixture of CO_2 and N_2 is, according to equation 4 of Art. 95,

$$R_m = \frac{1545}{3.08 + 11.67} \left(\frac{3.08}{44} + \frac{11.67}{28.15} \right) = 50.7.$$

Thus the volume is that of $(3.08 + 11.67 =) 14.75$ lb. of CO_2 and N_2 at 70° F. and 14.337 lb., or

$$\text{Total volume} = 14.75 \times \frac{50.7 \times 530}{14.337 \times 144} = 192 \text{ cu. ft}$$

Note from the results of the example that for the fuel in question the volume of the combustion products *after recooling to the original temperature of the air* is slightly less than that of the air. This condition will hold for any hydrogenous fuel. Also note that if a volumetric analysis of the products of combustion is made in or over water, as is the condition in the usual gas analysis apparatus of **Orsat** type, no record of the water vapor in the products is obtained. Such an analysis would show only the relative volume of the gaseous components of the products; or, referring to the data of the example, the analysis would result as follows:

$$\text{CO}_2 - 74.5 / (74.5 + 442) = 0.144, \text{ or } 14.4 \text{ per cent } \text{CO}_2 \text{ by volume;}$$

$$\text{N}_2 - 442 / (74.5 + 442) = 0.856, \text{ or } 85.6 \text{ per cent } \text{N}_2 \text{ by volume.}$$

Example 2. For the same fuel as that of example 1, but supplied with air for combustion in an amount 20 per cent in excess of the ideal requirements, find the mass and volume of air supplied and the mass and volumes of the combustion products.

Example 3. Given an oil fuel of the following analysis, in per cent by weight

Carbon	— 86.36;	Hydrogen	— 11.27;
Nitrogen	— 0.74;	Oxygen	— 0.74;
Sulphur — 0.89.			

Find for this fuel

- the mass and volume of air, at 62° F. and 14.7 lb., ideally required for complete combustion of each pound of fuel; (13.8; 182.)
- the mass and volume of air at 15 per cent in excess of ideal requirements;
- the mass and volume of the combustion products for case (b), at 14.7 lb. and 1000° F. and at 14.7 lb. and 62° F.; (616; 202.)
- the volumetric composition of the gaseous components of the combustion products, for both case (a) and case (b). (Note. — In the gas analysis SO₂ will appear as CO₂. (CO₂ — 16.0 and 13.8 per cent.) (O₂ — 0.0 and 2.9 per cent.)

Observe in the results of example 3*d* that the provision of air in excess of the ideal requirements will act to reduce the volumetric percentage of CO₂ in the gaseous components of the combustion products and will cause the appearance of free O₂. Further attention to this phenomenon is given in the following article. The percentage of CO₂ which may ideally be secured is also influenced by the amount of hydrogen which exists in the original fuel, this due to the H₂O which disappears in the analysis of the combustion products and the N₂ which entered with the oxygen for the hydrogen and persisted as nitrogen in the products. The following example illustrates.

Example 4. Compute the volumetric percentage of CO₂ which should appear in the gaseous constituents of the products of complete combustion with the ideal air requirements, doing this for fuels with combustible made up (per cent by weight) of:

	(a)	(b)	(c)	(d)
Carbon	70	80	90	100
Hydrogen	30	20	10	zero
				(10.3; 13.2; 16.6; 20.9)

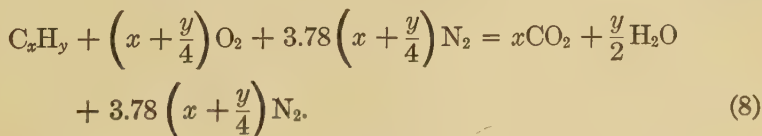
Solution, at 70 per cent carbon.

$$\text{Relative volume of CO}_2 = \frac{0.7 \times 3.67}{44} = 0.0583$$

$$\text{Relative volume of N}_2 = \frac{(0.7 \times 8.86 + 0.3 \times 26.4)}{28.15} = 0.5015$$

$$\text{Per cent CO}_2 = 0.0583 / 0.5598 = 0.103 = 10.3 \text{ per cent.}$$

For a simple hydrocarbon fuel of the form C_xH_y a composite reaction equation may be formulated as follows. The basis of selection of the various coefficients should be evident from the foregoing.



Similarly for a simple carbohydrate of the chemical form $C_xH_yO_z$,

$$C_xH_yO_z + \left(x + \frac{y}{4} - \frac{z}{2}\right)O_2 + 3.78 \left(x + \frac{y}{4} - \frac{z}{2}\right)N_2 = xCO_2 + \frac{y}{2}H_2O \\ + 3.78 \left(x + \frac{y}{4} - \frac{z}{2}\right)N_2. \quad (9)$$

Example 5. Validate the various coefficients in equations 8 and 9. Note that for a carbohydrate the oxygen appearing in the fuel acts to reduce correspondingly the amount of additional oxygen which may be absorbed or required.

Example 6. Find the mass and the volume of the air, at 14.7 lb. and 70° F., which is ideally required for the complete combustion of one pound of (a) hexane, C_6H_{14} , and (b) ethyl alcohol, C_2H_6O . (204; 120.)

These analyses and problems have considered only the complete combustion of the fuel with the ideal amount of air or with air in excess of that amount. In the event of a deficit of air with respect to the ideal requirements, the hydrogen will tend to acquire its full quota of oxygen but some portion of the carbon will necessarily burn incompletely to CO or pass out entirely unburned. In point of fact, for most fuels and most physical conditions under which practical combustion processes are carried out some excess of air is required to avoid some incomplete combustion of the carbon. Due to insufficient mixing of the air with the combustible or to a chilling of the mixture from various causes, unburned carbon and carbon monoxide may frequently be detected even when free oxygen is also present in the combustion products, and this condition is much aggravated when insufficient oxygen has been provided. In view of the variety of conditions which may exist when a deficit of air occurs, no attempt may well be made to set up definite reaction equations for that condition.⁴

157. Interpretation of the Analysis of Combustion Products.—It should be evident that the incomplete combustion resulting from a deficit of air supply will result in only a partial energy delivery, and that usually such a deficit is undesirable both in furnace or internal-combustion engine operation. As regards excess air, although either in furnace or engine operation some excess may be necessary or advantageous, too great an amount will be fatal to efficiency or to satisfactory perform-

⁴ If it is assumed that, *after* the satisfaction of the hydrogen in a fuel, all carbon does combine as completely as possible to either CO or CO₂ then a reaction equation for the carbon may be set up. It is as follows:

$$C + xO_2 + 3.78 xN_2 = (2x - 1)CO_2 + 2(1 - x)CO + 3.78 xN_2,$$

where x represents the proportion between the air actually available for combination with the carbon and the amount of air ideally required for its combustion.

ance. It appears therefore that in all practical combustion processes it is essential that effective means be provided for air control and that convenient facilities be available for judging the relative degree of deficit or excess in the amount of air supply.

This information might be secured by actual metering of the air and fuel supply, but in practice air metering may not be feasible. In lieu of it, recourse is frequently had to chemical analysis of the products of combustion — that is, the flue gas from a furnace or the exhaust gas of an engine. In the operating fire-room the extent of the analysis usually is only that for the percentage of carbon dioxide, as made by the automatic, indicating or recording **CO₂ Meter**. For test purposes a more complete analysis for CO₂, O₂, and CO is frequently made, accompanied at times by further determination of H₂ and of such hydrocarbons as CH₄, et cetera. The residue after determination of these is taken to be N₂. The apparatus employed is of such character that there is secured a volumetric analysis only of the gaseous portion of the products of combustion, or exclusive of the water vapor.

Two general types of expression are available for estimating from the gas analysis the amount of air supplied per pound of fuel burned. The first is limited in its application to the case where the relative proportion of carbon in the fuel is known from a chemical analysis. The second is more involved but is suitable particularly for a hydrocarbon fuel (such as oil). It has the advantage that it is available whether the composition of the fuel is known or not.

It is convenient in the following to employ the symbols CO₂, CO, O₂, N₂ (and H₂ or CH₄) to represent the relative or percentage volumes of those gases in the gaseous portion of the products of combustion, that is, as secured by the gas analysis. Let us also recall that the relative mass of a constituent may be computed from its relative volume by multiplying by its molecular weight. Finally note that for a solid or liquid fuel the nitrogen in the products may without material error be taken to represent nitrogen coming solely from the air supplied, and that the carbon in the products comes solely from the carbon which actually is burned. The development of the several expressions is then as follows:

Expression A.

(a) Relative mass of air represented by the nitrogen in any unit volume of combustion products

$$= N_2 \times 28.15 \times \frac{1}{0.769}.$$

(b) Relative mass of carbon represented in the volumes of CO_2 , CO , and CH_4 in the same volume of the gas

$$= (\text{CO}_2 \times 44) \times \frac{1}{4} + (\text{CO} \times 28) \times \frac{1}{2} + (\text{CH}_4 \times 16) \times \frac{1}{8} \\ = 12 (\text{CO}_2 + \text{CO} + \text{CH}_4).$$

Combining these relations,

(c) Mass of air supplied, pounds per pound of carbon *in the gas*,

$$= \frac{\text{N}_2 \times 28.15 \times \frac{1}{0.769}}{12 (\text{CO}_2 + \text{CO} + \text{CH}_4)} = 3.05 \frac{\text{N}_2}{\text{CO}_2 + \text{CO} + \text{CH}_4};$$

(d) Mass of air supplied, pounds per pound of fuel

$$= 3.05 \frac{\text{N}_2}{\text{CO}_2 + \text{CO} + \text{CH}_4} \times \text{carbon supplied and burned per pound of fuel}, \quad (10)$$

where the mass of carbon supplied and burned per pound of fuel is that as determined by chemical analysis of the fuel and corrected as closely as ascertainable by measurement of the relative amount of unburned carbon passing out as soot or to the ashpit of a coal-burning installation.

Expression B. In the second method we may again consider that (a) the relative mass of air represented by the nitrogen in the gas

$$= \text{N}_2 \times 28.15 \times \frac{1}{0.769}. \text{ Likewise (b) the relative mass of carbon in}$$

the gas is again taken to be $12 (\text{CO}_2 + \text{CO} + \text{CH}_4)$. Presuming the chemical analysis of the fuel to be unknown, but with the information that for hydrocarbon fuel the only constituent aside from carbon is hydrogen (minor impurities excepted), we may estimate the relative mass of hydrogen in the total products as follows:

(e) Relative mass of hydrogen represented in the volumes of H_2 and CH_4

$$= (\text{H}_2 \times 2.02) + (\text{CH}_4 \times 16) \times \frac{4.04}{16} = 2.02 \text{ H}_2 + 4.04 \text{ CH}_4.$$

(f) Total relative mass of oxygen, as represented by the total volume of nitrogen in the products

$$= \text{N}_2 \times 28.15 \times \frac{.231}{.769} (= Y).$$

(g) Relative mass of oxygen represented in the volumes of CO_2 , O_2 and CO

$$= (\text{CO}_2 \times 44) \times \frac{3}{4} + (\text{O}_2 \times 32) + (\text{CO} \times 28) \times \frac{1}{2} \\ = 32 (\text{CO}_2 + \text{O}_2 + \text{CO}/2) (= Z).$$

(h) Remaining relative mass of oxygen, which must have combined with hydrogen to form H_2O , $= Y - Z$.

(i) Relative mass of hydrogen thus passing out in H_2O

$$= \frac{2.02}{16} [Y - Z].$$

$$= \frac{2.02}{16} \left[\text{N}_2 \times 28.15 \times \frac{.231}{.769} - 32 \left(\text{CO}_2 + \text{O}_2 + \frac{\text{CO}}{2} \right) \right].$$

Noting again that item *a* evaluates the relative mass of air represented by a given volume of the combustion products, that item *b* represents the relative mass of carbon in the same volume, and that *e plus i* represents the total hydrogen, again in the same volume of products, it appears that these items may be combined as follows to give the mass of air per unit mass of fuel. Thus,

Mass of air supplied, pounds per pound of fuel,

$$= \frac{a}{b + e + i}$$

$$= \frac{\text{N}_2 \times 28.15 \times \frac{1}{0.769}}{12 (\text{CO}_2 + \text{CO} + \text{CH}_4) + (2.02 \text{H}_2 + 4.04 \text{CH}_4) + \frac{2.02}{16} \left[\text{N}_2 \times 28.15 \times \frac{.231}{.769} - 32 \left(\text{CO}_2 + \text{O}_2 + \frac{\text{CO}}{2} \right) \right]}$$

$$= \frac{9.2 \text{N}_2}{2 \text{CO}_2 - \text{O}_2 + 2.5 \text{CO} + 0.268 \text{N}_2 + 0.5 \text{H}_2 + 4 \text{CH}_4} \text{ (closely), (11)}$$

in which the chemical symbols represent the relative *volumes* of each constituent.

Although these expressions provide valuable evidence of the air-fuel relations they should not be depended upon too implicitly, both because of the uncertainty in securing a truly representative gas sample for the analysis and the reasonable surety that some portion of the fuel goes out unburned or in forms not adequately determined in the analysis.

Example 7. With the fuel of example 1 compute by both of the above methods the ratio of air to fuel if the analysis of the gaseous products of combustion were, by volume, in percentages,

$\text{CO}_2 - 13.3;$	$\text{H}_2 - 0.18$
$\text{O}_2 - 0.5;$	$\text{CH}_4 - 0.06$
$\text{CO} - 0.5;$	$\text{N}_2 - 85.46 \text{ (by difference).}$

158. Heating Value of Fuels. — The terms **heating value** or **calorific value** are those used to designate the quantity of energy emitted as heat from a **calorimeter**, per unit quantity of a fuel, as the result of a combustion reaction. This is approximately the amount of chemical energy released by the reaction.

In laboratory practice there are two general methods and types of

calorimeters used for ascertaining calorific value. In one method, used for solid, liquid, and gaseous fuels, a known quantity of the fuel is ignited electrically in a closed **bomb** in an atmosphere of oxygen or in a mixture with some other active oxidizing agent, frequently under considerable pressure, and thus is caused to burn *at constant volume*. The energy emitted as a result of the reaction is delivered from the hot products of combustion through the walls of the bomb as heat to a surrounding water-bath. The amount of energy so delivered is evaluated from data on the mass and temperature change of the water and bomb. The products are recooled to practically the initial temperature of the fuel.

Because the process is a state change without flow, without volume change and thus with no energy delivery other than as heat, the energy equation is

$$\begin{aligned} C + ME_m &= Q_V + ME_p, \quad \text{or} \\ Q_V &= C + M(E_m - E_p) \end{aligned} \quad (12)$$

where C = the chemical energy released as a result of the reaction, B.t.u. per pound of fuel;

M = mass of mixture or products, pounds per pound of fuel;

E_m = the molecular energy of the *original mixture* of fuel and oxygen, B.t.u. per pound;

E_p = the molecular energy of the *recooled products* of combustion, B.t.u. per pound;

Q_V = the energy emitted as heat from calorimeter, B.t.u. per pound of fuel, known as the **heating value at constant volume**.

In the second general type of calorimeter, used particularly with gaseous and liquid fuels, the reaction is caused to take place in a suitably designed burner to which fuel and atmospheric air are delivered continuously, the fuel at a measured rate. The process thus simulates the combustion in the ordinary furnace. The hot products of combustion pass from the burner through a system of water-jacketed tubes and are there recooled to practically the initial temperature. The rate of energy transmission as heat through the tubes to the cooling water is evaluated from data on the temperature rise and rate of flow of the water.

The process is a state change with steady-flow *at constant (atmospheric) pressure*, with or without volume change as the volume of the products may or may not differ from the volume of the fuel mixture. The energy equation becomes

$$\begin{aligned} C + M(E_m + PV_m/J) &= Q_P + M(E_p + PV_p/J), \\ \text{or} \quad Q_P &= C + M[(E_m - E_p) + P(V_m - V_p)/J] \\ &= C + M(H_m - H_p) \end{aligned} \quad (13)$$

where C , M , E_m and E_p are as before, and

- P = constant total pressure in calorimeter, pounds per square foot;
 V_m = specific volume of mixture, cubic feet per pound;
 V_p = specific volume of recooled products, cubic feet per pound;
 Q_P = energy emitted as heat from calorimeter, B.t.u. per pound of fuel, known as the **heating value at constant pressure**;
 H_m = the enthalpy ($E_m + PV_m/J$) of the fuel mixture, B.t.u. per pound;
 H_p = the enthalpy ($E_p + PV_p/J$) of the products, B.t.u. per pound.

It might appear desirable to ascertain and ascribe to a fuel as a definite characteristic the amount of chemical energy C which is released by the combustion. That we are unable to do with exactness by reason of our inability to evaluate the absolute magnitude of the molecular energy or the enthalpy assignable to the two distinct fluids which comprise on the one hand the original mixture and on the other hand the final products of combustion. However for certain purposes it is convenient to assign a hypothetical or equivalent value to the chemical energy term (see footnote 7, Art. 159).

From the foregoing consideration of the two types of calorimeter it would appear that determination of the calorific value of a given fuel by the use of each could give identical results only if the initial and final temperatures were virtually the same in each and if the fuel were one the combustion of which incurred no volume change. Strictly that is true but for most commercial fuels any difference in the two heating values *due to the volume change* will not exceed about $\frac{1}{3}$ of 1 per cent. As that is within the range of experimental uncertainty in calorimetric determinations it may be taken that the constant volume and constant pressure calorific values, Q_V and Q_P , differ by an effectively negligible amount, *except as the presence of hydrogen in the fuel and water vapor in the products of combustion* may incur independently an appreciable variation in the calorimetric determinations. This feature is considered in the following.

For fuels which contain hydrogen — and most fuels do — there may, however, be a much more appreciable variation in calorimetric determinations when using the two types of calorimeter, or even with the same type, due to possible variation in the final state of the water vapor which is formed by the combustion of the hydrogen. More specifically, the variation would be due to difference in the degree to which the water vapor may be condensed in the recooling of the combustion products, with consequent difference in the heat emission accompanying

the condensation. The degree of condensation is in turn influenced by a variety of conditions, the outstanding ones of which are (1) the variation of the *dewpoint* of the combustion products (see Art. 99) as affected by the amount of hydrogen in the fuel and by the resulting amount of water vapor in the products, and also by the amount and original humidity of the oxygen or air provided for support of the combustion, and (2) the final temperature to which the combustion products are recooled. The influence of these conditions is illustrated in the accompanying table, which is based upon the combustion of the typical liquid (hydrocarbon) fuel of example 1. The basis of arriving at the indicated results is shown by the illustrative example which follows:

TABLE XII

EFFECT OF CALORIMETRIC CONDITIONS ON THE HIGHER HEATING VALUE OF A HYDROGENOUS FUEL (C_7H_{16})

Conditions	Dew-point of products, °F.	Per cent condensation	B.t.u. emission due to condensation;	
			per pound of H_2O from H_2 in fuel	per pound of fuel
(a) Constant volume bomb calorimeter, with oxygen at 200 lb. per sq. in. and 20 per cent in excess of ideal requirements and with final products temperature of 70° F.	378	99.7	1072	1533
(b) Constant (atmospheric) pressure calorimeter, with dry air supplied in ideal amount and with final products temperature of 70° F. . . .	127	84.5	915	1310
(c) Constant pressure calorimeter, with final products temperature of 70° F. but with air supply at 65 per cent humidity and 20 per cent in excess	125	83.3	1020	1460
(d) Constant pressure calorimeter, with dry air supplied in ideal amount but with final products temperature of 62° F.	127	88.4	965	1390

An item in the table which is of some considerable significance in the dewpoint found in connection with the constant pressure calorimeter. The conditions of the combustion products from this calorimeter are not unrepresentative of the conditions of the stack gases from a furnace and boiler or the exhaust from an internal-combustion engine. The dewpoint seems to lie in the vicinity of 125° F. It will be recalled that the dewpoint measures the temperature below which the gases must be cooled at constant pressure before condensation may begin.

Example 8. Check the results of Table XII for conditions (a) and (b).

Solution. —

Condition (a)

Oxygen required (by Eq. 8) = $\frac{(7 + 16/4) \times 32}{(7 \times 12) + (16 \times 1.01)} = 3.51$ lb. per pound of fuel.

Oxygen supplied = $1.20 \times 3.51 = 4.21$ lb. per pound of fuel, and

$= 4.21 \times \frac{1545 \times 530}{32 \times 200 \times 144} = 3.74$ cu. ft. (at 70° F.) per pound of fuel.

As the combustion is at constant volume this volume must also be the volume of the products of combustion.

Products; $\text{CO}_2 = \frac{7 \times (12 + 32)}{(7 \times 12) + (16 \times 1.01)} = 3.08$ lb. per pound of fuel;

$\text{O}_2 = 4.21 - 3.51 = 0.70$ lb. per pound of fuel;

$\text{H}_2\text{O} = \frac{16}{2} \times \frac{18.02}{(7 \times 12) + (16 \times 1.01)} = 1.43$ lb. per pound of fuel;

Total..... = 5.21 lb. per pound of fuel.

Gas constant R_m of the products at or above the dewpoint (Art. 95, Eq. 4)

$$= 1545 \left(\frac{3.08}{5.21} \times \frac{1}{44} + \frac{0.70}{5.21} \times \frac{1}{32} + \frac{1.43}{5.21} \times \frac{1}{18} \right) = 50.8.$$

Total pressure, P_{total} , of products at dewpoint temperature T_d

$$= \frac{R_m T_d}{V} = \frac{50.8 \times T_d}{3.74/5.21} = 70.7 T_d.$$

Partial pressure, P_v , of H_2O component at dewpoint (from Art. 94, Eq. 3)

$$= P_{\text{total}} \times \frac{1.43/18.02}{(1.43/18.02) + (3.08/44) + (0.70/32)} = 0.465 P_{\text{total}};$$

and thus = $0.465 \times 70.7 \times T_d = 32.9 T_d$, or

$p_v/T_d = 0.228$, where p_v is the vapor pressure in pounds per square inch.

The vapor pressure p_v and the temperature T_d at the dewpoint are the necessarily corresponding saturation pressure and temperature, for which values are listed in the Steam Tables. By search of the tables it may be found that at 191 lb. per sq. in. abs. and 378° F. the requisite p_v/T_d relation is satisfied [$191 = 0.228 \times (378 + 460)$] and the dewpoint of the products is therefore 378° F.

At the specified final products temperature of 70° F. V_g for water vapor is, by the steam tables, 869 cu. ft. per lb., whereas the total vapor and condensate in the bomb

must have a specific volume of $3.74/1.43$ or 2.62 cu. ft. per lb. The corresponding quality of the vapor is thus (by Art. 63, Eq. 2) $2.62/869 = 0.003$ or 0.3 per cent, and to all intent the vapor is fully condensed.

The energy emitted as heat by the vapor condensing at constant volume must equal the difference between the internal energy of the dry saturated vapor at the dewpoint and the 0.3 per cent quality mixture at 70°F. or, referring to the steam tables, equals $1113 - 41$ or 1072 B.t.u. per pound of vapor. The heat emitted by condensation of the vapor from one pound of fuel equals 1072×1.43 or 1533 B.t.u. per pound of fuel.

Condition (b)

Air supplied, from above, $= 3.51/0.231 = 15.2$ lb. per pound of fuel.

Products; CO_2 , from above, $= 3.08$ lb. per pound of fuel;

H_2O , from above $= 1.43$ lb. per pound of fuel;

$\text{N}_2 = 15.2 \times 0.769 = 11.70$ lb. per pound of fuel;

Total..... $= 16.2$ lb. per pound of fuel;

Gaseous constituents $= 14.78$ lb. per pound of fuel.

Partial pressure of vapor in products if at or above dewpoint

$$= 14.7 \times \frac{1.43/18.02}{(1.43/18.02) + (3.08/44) + (11.7/28.15)} = 2.06 \text{ lb. per sq. in.}$$

Dewpoint of products mixture is the saturation temperature of water vapor at 2.06 lb. per sq. in., or, from steam tables, equals 127°F.

Partial pressure of vapor and condensate at $70^\circ\text{F.} = 0.36$ lb. per sq. in.

Partial pressure of residual gaseous products at $70^\circ\text{F.} = 14.7 - 0.36 = 14.34$

Volume of gaseous products per pound of fuel, at 70°F. and combined pressure of 14.34 lb. $= M \frac{RT}{P} = 14.78 \times \frac{1545 (3.08/44 + 11.7/28.15)}{14.78 \times 14.34 \times 144} \times 530 = 192$ cu. ft.

As this volume is also the volume of the 1.43 lb. of vapor and condensate in the products at 70°F. their specific volume is $192/1.43$ or 134 cu. ft. per lb. and the quality of the vapor-condensate mixture must be $134/869$ or 0.155 . Alternatively, by a relation developed in problem 15 of Art. 100, final quality $= (0.36/14.34) \times (12.64/2.06) = 0.155$. Therefore 84.5 per cent of the original vapor has condensed out.

The energy emitted in the steady-flow condensation process is the difference between the enthalpy of dry saturated vapor at 127°F. and the 15.5 per cent quality mixture at 70°F. , or $1116 - 201 = 915$ B.t.u. per pound of vapor. The heat emitted by condensation of the vapor from one pound of fuel equals 915×1.43 or 1310 B.t.u. per pound of fuel.

Directly upon the basis of the phenomenon of the condensation of the water vapor in the combustion products, it is the practice to make a distinction between a so-called **higher heating value** and a **lower heating value** for any hydrogenous fuels. By the lower heating value is meant in general the heat emission which would occur if no vapor condensation were to take place, whereas by the higher heating value is meant the heat emission inclusive of that due to complete condensation. Use of

the two heating values is considered necessary for the reasons that the heat of condensation is technically obtainable and so is regarded as properly chargeable in the input account of a furnace and boiler or of an internal-combustion engine, whereas on the contrary it is quite improbable that in practice the products of combustions will be found actually to leave a boiler or engine at a temperature below the dewpoint, for which reason the designer must establish his expectations of the actual energy emission on a basis of no condensation.⁵

At this writing there is unfortunately some lack of agreement as to the exact conditions for determining or computing the two heating values. As regards the higher heating value, that is commonly interpreted simply as the value determined directly by calorimeter, and in fact it would in principle be the value as secured by the oxygen bomb calorimeter. However, Table XII has shown the incompleteness and the *variation* in incompleteness of the vapor condensation in a constant pressure calorimeter. It may be concluded therefore that the direct determination from such a calorimeter is neither the true higher value nor a constant function of that value, but is rather an "apparent" higher value which might readily be several hundred B.t.u. per lb. below the true value.

As regards the lower value, it may be estimated quite closely from the oxygen bomb determination of the higher value by subtracting from that value about 1100 B.t.u. for each pound of vapor produced per pound of fuel burned:

$$\text{L.H.V.} = \text{H.H.V. (by oxygen bomb)} - \frac{18.02}{2.02} \times M_H \times 1100, \quad (14)$$

where M_H = mass of hydrogen per pound of fuel; and

1100 = (closely) the energy released in condensing and cooling the vapor from dewpoint down to calorimeter temperature.⁶

In the case of the constant pressure calorimeter provision is commonly made for collecting and measuring the amount of vapor which is actually

⁵ Rather parallel to the circumstance of vapor condensation in the products is the prevaporization of fuels which must occur prior to combustion. This vaporization is accomplished only by energy absorption from some source, and is properly a debit item against the fuel. However, as the source is commonly the calorimeter itself or the fuel mixture after entering the calorimeters or both, it is automatically debited in the calorimetric determination.

⁶ It has at times been taken that $\text{L.H.V.} = \text{H.H.V. (by calorimeter)} - 9 \times M_H \times 970$. In this relation 970 may be recognized as the enthalpy of vaporization of water at 212° F. and 14.7 lb. The relation is in effect based upon the incorrect hypothesis that the vapor in the products at a (total) pressure of 14.7 lb. would condense wholly and instantly when the products were recooled to or below 212° F. The table and examples show the fallacy of this hypothesis.

condensed out of the products. It may be shown as a *rough rule*, that

$$\begin{aligned} \text{L.H.V.} &= \text{Apparent H.H.V. (by constant pressure calorimeter)} \\ &\quad - M_w \times 1100, \end{aligned} \quad (14a)$$

where M_w = mass of condensed vapor, *as collected*, per pound of fuel burned.

It was noted above that, according to the evidence of the table and by the definition of the higher heating value as that corresponding to *complete* condensation, the constant volume calorimeter using oxygen under pressure should give directly and with adequate accuracy the higher heating value. Correspondingly, the constant pressure calorimeter should give directly and actually the lower heating value *if the mixture and products were respectively to enter and leave the calorimeter at a temperature of, say, 130° F*, or with negligible error if the products only were caused to leave at a temperature above their dewpoint and a corresponding correction were made for the deficit of enthalpy of the entering air. That is,

Correction due to elevated exit temperature

$$= + M_a \times c_{p, \text{air}} \times (t_2 - t_1),$$

where M_a = mass of air per pound of fuel;

$$c_{p, \text{air}} = 0.24;$$

t_2 = exit temperature of products;

t_1 = temperature of air supply.

Table XIII presents accepted values for the higher and lower heating values of various simple fuels or fuel constituents and approximate or average values for various more complex fuels.

An interesting feature which may be observed in the table is the degree of uniformity of the heating value *per pound of mixture* for the more common commercial fuels. A fairly average lower value is about 1200 B.t.u. per pound of mixture.

For gas fuels and for mixtures of air and volatile liquid or gas fuels as used in the internal-combustion engine the heating value *per cubic foot* may become of as much interest as the heating value per pound. For a gas fuel, at given pressure and temperature, the volumetric heating value is given by the obvious relation

$$\text{B.t.u. per cu. ft.} = \text{B.t.u. per lb.} \times \text{density, lb. per cu. ft.} \quad (15)$$

For a fuel mixture at a given total pressure and temperature the calorific value per cubic foot may be computed in several ways but it is probably

TABLE XIII
CALORIFIC VALUE OF FUELS, B.T.U. PER LB.

	B.t.u. per pound of fuel,		Ideal air requirements, pounds per pound	B.t.u. per pound of ideal air-fuel mixture, lower values only
	Higher	Lower		
Hydrogen (H ₂)	61,800 (<i>Q_P</i>) 60,700 (<i>Q_V</i>)	52,100	34.4	1,470
Carbon (C)		14,500	11.5	1,160
Carbon monoxide (CO)		4,350	2.5	1,240
Acetylene (C ₂ H ₂)	21,500	20,000	13.3	1,400
Benzol (C ₆ H ₆)	18,200	17,400	13.3	1,220
Ethylene (C ₂ H ₄)	21,800	20,500	14.8	1,300
Ethane (C ₂ H ₆)	22,200	20,300	16.1	1,180
Methane (CH ₄)	24,000	21,600	17.3	1,180
Ethyl alcohol (C ₂ H ₆ O)	12,800	11,500	9.2	1,130
Gasoline	20,200±	18,900±	15, closely	1,180±
Kerosene	20,700±	19,400±	15, closely	1,210±
Alcohol, denatured	11,600±	10,500±	8, closely	1,170±
Fuel oil	19,000±	17,700±	14±	1,180±
Combustible of coal	14,500 to 15,700	300 to 500 less than H.H.V.	10 to 12	1,200±

most readily done by the relation:

$$\begin{aligned} \text{B.t.u. per cu. ft. of mixture} &= \text{B.t.u. per lb. of fuel} \times \text{lb. of fuel per} \\ &\text{lb. of air} \times \text{density of air at the temperature and proper } \textit{partial} \\ &\text{pressure.} \end{aligned} \quad (16)$$

In this relation the partial pressure of the air will be virtually that of the region in which the mixture exists if the fuel is a solid or a very slightly volatile liquid, but for a gaseous or vapor fuel or a liquid fuel which exerts an appreciable vapor pressure at the temperature of the mixture the partial air pressure will be the total pressure of the mixture less the partial pressure of the fuel constituent. The following example illustrates the methods of computation in the latter condition.

Example 9. Find the lower heating value per cubic foot at 70° F. and a total pressure of 14.7 lb. per sq. in. for (a) hydrogen gas, (b) mixture of acetylene with ideal amount of air.

Solution for (b). — By applying equation 8 (Art. 156) it develops that for 1 lb. of acetylene gas there is ideally required 13.33 lb. of air. In the mixture of 1 lb. of C₂H₂ and 13.33 lb. of air at a total pressure of 14.7 lb. the partial pressure of the air is (by Art. 94, Eq. 3):

$$p_{\text{air}} = 14.7 \frac{13.33/29.0}{13.33/29.0 + 1/26.02} = 13.6 \text{ lb. per sq. in.}$$

$$\text{Density of air in mixture} = \frac{P}{RT} = \frac{13.6 \times 144}{53.3 \times 530} = 0.0694$$

$$\text{B.t.u. per cubic foot of mixture} = 20,000 \times \frac{1}{13.33} \times 0.0694 = 104.$$

Table XIV gives accepted values for the heating values of various simple fuels per cubic foot of fuel and per cubic foot of ideal mixture, and also the approximate values for several more complex gaseous fuels.

TABLE XIV
VOLUMETRIC HEATING VALUES OF FUELS AND OF FUEL-AIR MIXTURES
Volumes based on 70° F. and 14.7 lb.

	Lower Heating Values	
	B.t.u. per cubic foot of gas	B.t.u. per cubic foot of ideal mixture
Hydrogen (H ₂)	270	80
Carbon monoxide (CO)	315	93
Acetylene (C ₂ H ₂)	1400	104
Benzol (C ₆ H ₆)	(liquid) 94	94
Ethylene (C ₂ H ₄)	1485	97
Ethane (C ₂ H ₆)	1570	89
Methane (CH ₄)	895	85
Natural gas	900±	90±
City gas	500±	90±
Coke-oven gas	500±	85±
Producer gas	125±	60±
Blast-furnace gas	95±	55±

159. Energy Equation of Combustion; Temperature Developed. — Combustion processes carried out in a calorimeter, such as those considered in the foregoing, or the processes in a boiler furnace are evidently characterized by energy departure from the system as heat at a rate which approaches in some degree the rate of chemical energy release from the fuel. In distinction to this condition there are a number of circumstances, such as in the internal-combustion engine, in which the combustion is more nearly adiabatic or in which for purposes of analysis it may be desirable to consider it ideally so. In such an adiabatic combustion, due to the absence of energy emission as heat, the released chemical energy goes primarily to increasing the internal energy or the enthalpy of the combustion products, with a large resultant temperature rise. It frequently becomes desirable to predict or estimate the temperatures which may be attained in such processes and to recognize the

factors which will influence the temperature. An analysis of a general combustion energy equation will be useful for that purpose.

A general energy equation for *any* combustion process would be

$$ME_{m,1} + C = ME_{p,2} + Q_{\text{out}} + W_{\text{out}}/J + MC_p, \quad (17)$$

in which C = the chemical energy releasible by complete combustion of a fuel, B.t.u. per pound of fuel;

C_p = any residual chemical energy not released and remaining in the products of combustion due to incompleteness of combustion, B.t.u. per pound of products;

$E_{m,1}$ = molecular energy of mixture as supplied to the system, B.t.u. per pound of mixture;

$E_{p,2}$ = molecular energy of combustion products at any phase of or at end of the combustion process, B.t.u. per pound of products;

M = mass of mixture or products, pounds per pound of fuel;

Q = energy emitted as heat from the system during the combustion, B.t.u. per pound of fuel;

W = any mechanical energy (*mechanical effects*, Art. 36-b) departing from the system during the combustion, foot-pounds per pound of fuel.

Let this general equation be compared with the specialized equation for, say, the constant-volume calorimeter, or (Art. 158)

$$C = Q_V + M(E_p - E_m), \quad (12)$$

in which C and M are as above;

Q_V = the calorific or heating value of fuel, at constant volume;

E_p = molecular energy of products at calorimeter temperature;

E_m = molecular energy of mixture at calorimeter temperature.

If equation 17 is solved for the item $E_{p,2}$ a relation is obtained which expresses in a general but informative manner the ultimate molecular state, and thus indirectly the products temperature, which may result from a combustion. Thus, by equation 17 and 12,

$$\begin{aligned} E_{p,2} &= \frac{C}{M} + E_{m,1} - \frac{(Q_{\text{out}} + W/J)}{M} - C_p \\ &= \frac{Q_V + M(E_p - E_m)}{M} + E_{m,1} - \frac{(Q_{\text{out}} + W/J)}{M} - C_p. \end{aligned} \quad (18)$$

Several significant and practical conclusions become directly evident from this relation. These are that the molecular state of the combustion products and therefore their temperature will depend on (a) the heating

value *per pound of mixture*, rather than simply per pound of fuel, (b) the initial molecular energy and thus the temperature of the mixture supplied to the system, (c) the energy emitted thermally or mechanically during the combustion, and (d) the residual chemical energy which for any reason fails to be released.

The detailed application of the equation to compute the temperature of the combustion products, $T_{p,2}$, is somewhat involved but for those interested the manner of its use is indicated in somewhat more detail by the footnote.⁷ The practical utility of the equation is further handicapped by the phenomenon of a **chemical equilibrium** which is found to occur at the high temperature levels that accompany adiabatic combustion, and which is found to reduce the amount of chemical energy C that may be released during the temperature rise. A development of the subject of chemical equilibrium as associated with combustion is beyond the intended scope of this work.⁸ However some attention is given to the general character of the phenomenon in the following.

⁷ Observe that, if the products and mixture may both be taken as gaseous in character

$$M(E_{p,2} - E_p) = M \int_{T_c}^{T_2} c_{v,p} dT = M \left[a_p(T_2 - T_c) + \frac{b_p}{2}(T_2^2 - T_c^2) + \frac{f_p}{3}(T_2^3 - T_c^3) \right];$$

$$M(E_{m,1} - E_m) = M \int_{T_c}^{T_1} c_{v,m} dT = M \left[a_m(T_1 - T_c) + \frac{b_m}{2}(T_1^2 - T_c^2) + \frac{f_m}{3}(T_1^3 - T_c^3) \right];$$

in which M = mass of fuel mixture or products per pound of fuel;

T_2 = final temperature of products as caused by combustion;

T_1 = initial temperature of fuel mixture at beginning of combustion;

T_c = temperature level of calorimeter during determination of heating value;

c_v = specific heat at constant volume;

a , b , and f = the coefficients in the specific heat expressions for the fuel mixture and for the products (see Art. 82, Eq. 8 and Art. 96).

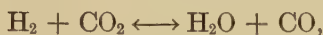
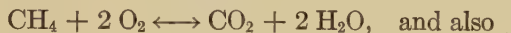
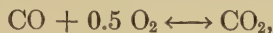
By introducing these terms in equation 18 and rearranging the resulting equation it may be developed that

$$a_p T_2 + \frac{b_p}{2} T_2^2 + \frac{f_p}{3} T_2^3 = \frac{1}{M} \left\{ Q_V + M \left[(a_p - a_m) T_c + \frac{b_p - b_m}{2} T_c^2 + \frac{f_p - f_m}{3} T_c^3 \right] \right\} \\ + \left(a_m T_1 + \frac{b_m}{2} T_1^2 + \frac{f_m}{3} T_1^3 \right) - (W/J + Q_{out})/M - C_p. \quad (19)$$

From this relation, given or having available all data except T_2 , that temperature may be determined by a cut-and-try solution assisted by graphical aids such as employed in example 14 of Art. 87. The longer bracketed term involving Q_V , et cetera, which appears in this relation has on occasion been referred to and employed as an effective or equivalent value of the chemical energy released per pound of fuel.

⁸ For full presentation and bibliography of the subject of chemical equilibrium as it pertains to the problems of combustion the reader is directed to Goodenough and Felbeck, An Investigation of the Maximum Temperatures and Pressures Attainable in the Combustion of Gaseous and Liquid Fuels, University of Illinois Engineering Experiment Station Bulletin 139.

The chemical reactions occurring in combustion are not unidirectional, as perhaps has been implied in the earlier discussions, but under proper circumstances they may proceed in directions reverse to those indicated. Thus, considering several oxidation reactions,



in which the double arrow indicates the possibility for the reaction to take place in either direction. Under temperatures which are sufficiently high to cause ignition of a fuel mixture, but still are relatively moderate, there is a "driving force" tending to cause the reactions to proceed in the right-hand direction, with a *release* of chemical energy as a result of the reaction. In distinction to this, it would be found that if by some external means a sufficiently high temperature were maintained the reactions would tend to proceed in the reverse direction, with a dissociation of the products, accompanied by and effected by an *absorption* of energy. From these considerations it would follow that for any of the above reactions there is a certain maximum temperature of the burning material at which a chemical equilibrium would occur. At that equilibrium temperature both combustible material and products will exist simultaneously in the mixture but in such proportions that any further energy emission by continued combustion would tend to produce a corresponding dissociation of products, with equal energy absorption.

The evident consequence of this phenomenon is that it acts to impose a limit on the amount of chemical energy which may be released in a *high temperature* combustion and thus a virtual "ceiling" above which the temperature may not rise. The location of this ceiling will depend on the combustible material employed, varying for different ones and for different mixtures of various combustibles. It may be computed by the use of certain additional thermodynamic functions which are developed in Part V, in connection with data on the dissociation characteristics of the several combustion products.

The following tabulation is presented as giving an interesting comparison between the maximum temperature actually obtained on test of a given fuel under certain conditions of combustion and the predicted values of the temperature as computed by various methods. The fuel was a coal gas with a lower heating value of 16,650 B.t.u. per lb. and it was burned in 5.25 per cent excess air giving a lower heating value of

1173 B.t.u. per pound of mixture.⁹ The conditions of the actual combustion were not adiabatic but were such that 8.9 per cent of the calorific value was emitted from the region as heat, and for that reason certain of the computed temperatures are based upon that heat emission.

- (a) Temperature computed upon a basis of complete constant volume and adiabatic combustion and a constant specific heat of 0.173 ($= c_v$ for air at atmospheric temp.) 6,840° F.
- (b) Temperature computed upon a basis of complete and adiabatic combustion but with specific heats of the mixture and products selected in conformity with best available data on the proper coefficients 4,690° F.
- (c) Temperature computed upon a basis of complete combustion but 8.9 per cent energy emission as heat and with proper attention to actual specific heats 4,240° F.
- (d) Temperature computed upon a basis of 8.9 per cent heat emission and with proper attention to attainment of chemical equilibrium and to actual specific heats . . . 4,100° F.
- (e) Maximum temperature actually obtained . 3,960° F.

The first computation, (a), is reported solely to indicate the hopeless inadequacy of the basis, which at one time was regarded as the only available one. Comparison of (a) and (b) shows the very material influence of specific heat and specific heat variation with temperature. Computation (c) is done according to equation 19. There is quite fair agreement between (d) and (e). It is computed that upon reaching chemical equilibrium there still remained about 10 per cent of the chemical energy of the fuel which was unreleased. It was thought that in the actual combustion the equilibrium condition was not quite reached.

Although the chemical equilibrium phenomenon acts to limit the maximum temperatures attainable by combustion it is not to be regarded as reducing in any degree the amount of chemical energy which may ultimately be released from a fuel. In this connection observe that just as dissociation is induced only by high temperature, so the tendency

⁹ The experiment was that of Dr. W. T. David, as reported in a paper on "The Internal Energy of Inflammable Mixtures of Coal Gas and Air after Explosion," Proc., Royal Soc., Vol. 98A, page 303, 1921. The temperature computed with attention to chemical equilibrium is that of Goodenough and Felbeck, Univ. of Ill. Bulletin 139, page 63, 1926.

toward dissociation or toward limitation of the energy release decreases with decrease of temperature. The result is that if during combustion there is a simultaneous energy departure from the region either as work or as heat, with consequent tendency toward a reduction of the internal energy and temperature of the products below the equilibrium temperature, combustion may proceed in a corresponding degree, and a continued energy delivery with ultimate reduction of the temperature to a moderate level will permit completion of the combustion. This action is one of the reasons for the so-called "after-burning" which occurs during the expansion stroke of the internal-combustion engine or during the passage of furnace gases through the heat-absorption portion of a boiler.

160. Distribution of Combustion Losses. — In analyzing the test performance of a boiler and furnace installation or of an internal-combustion engine valuable information is secured by ascertaining the individual character and magnitudes of the various energy losses in the products of combustion leaving the machine. These losses may be divided into two general classes: (a) chemical energy losses by reason of the presence of incompletely burned combustible materials, and (b) molecular energy losses due to the relative amount and temperature of the exit gases.

A first and obvious requisite for the determination and distribution of these losses is the analysis of the combustion products, usually to determine the CO_2 , O_2 , CO and N_2 and, much less frequently, the H_2 , CH_4 , et cetera. The methods by which, on the basis of the analysis, the computations of the several losses may be made are in part parallel to the methods already employed in Art. 157 for ascertaining the relative masses of the various components and of the total of the combustion products. Reference should be made to that article.

Chemical Energy Losses. — Losses of chemical energy by reason of the presence of unburned combustible material are in general directly attributable to unfortunate furnace design or operation in a boiler installation or to improper mixing and injection in an engine. In particular the losses will arise from the appearance of CO , H_2 or CH_4 in the gases and of uncombined coke or carbon as soot or smoke or as a part of the ashpit refuse from the grates or stokers of a coal-fired furnace.

To evaluate the losses it is necessary simply to ascertain the amount of each material per pound of fuel and to multiply by the calorific value per pound of material. In expressing the relative amounts of the several materials it will again be convenient to employ the symbols CO_2 , O_2 , CO , N_2 , et cetera to represent the volumetric percentages of those gases in the gaseous combustion products, and it is also again to be recalled

that the relative mass of gas equals its relative volume multiplied by its molecular weight. It is believed that the development of the following relations will be self explanatory.

Loss by CO, B.t.u. per lb. of fuel supplied,

$$\begin{aligned}
 &= \text{Lb. of CO per lb. of carbon burned} \times \text{lb. of carbon burned} \\
 &\quad \text{per lb. of fuel supplied} \times 4350 \\
 &= \frac{28 \text{ CO}}{\frac{1}{4} (44 \text{ CO}_2) + \frac{1}{2} (28 \text{ CO}) + \frac{1}{8} (16 \text{ CH}_4)} \times (\text{carbon in fuel, by} \\
 &\quad \text{analysis} - \text{loss to ashpit, etc.}) \times 4350 \\
 &= 10,150 \frac{\text{CO}}{\text{CO}_2 + \text{CO} + \text{CH}_4} \times (\text{lb. of C per lb. of fuel, by analysis,} \\
 &\quad - \text{lb. of C lost to ashpit, etc. per lb. of fuel}) \quad (20)
 \end{aligned}$$

Loss by unburned carbon, B.t.u. per lb. of fuel supplied,

$$= \text{Carbon to ashpit, etc., lb. per lb. of fuel supplied,} \times 14,500. \quad (21)$$

Loss by H₂ and CH₄, B.t.u. per lb. of fuel supplied,

$$\begin{aligned}
 &= \frac{2 \text{ H}_2 \times 52,100 + 16 \text{ CH}_4 \times 21,600}{12 (\text{CO}_2 + \text{CO} + \text{CH}_4)} \times (\text{carbon in fuel, by analysis,} \\
 &\quad - \text{loss to ashpit, etc.}) \\
 &= \frac{8,670 \text{ H}_2 + 28,800 \text{ CH}_4}{\text{CO}_2 + \text{CO} + \text{CH}_4} \times (\text{lb. of C per lb. of fuel, by analysis,} \\
 &\quad - \text{lb. of C lost to ashpit, etc., per lb. of fuel}). \quad (22)
 \end{aligned}$$

It will be recognized that equations 20 and 22 are developed upon the same basis as was employed in the development of equation 10 of Art.157. For a liquid fuel of unknown analysis the methods of equation 11 of the same article may be employed and an alternative expression developed for the losses in the CO, H₂, and CH₄. Without detailed development, the expression becomes

Loss in CO, H₂ and CH₄, B.t.u. per lb. of fuel actually burned,

$$= \frac{30,600 \text{ CO} + 26,400 \text{ H}_2 + 87,000 \text{ CH}_4}{2 \text{ CO}_2 - \text{O}_2 + 2.5 \text{ CO} + 0.268 \text{ N}_2 + 0.5 \text{ H}_2 + 4 \text{ CH}_4} \quad (23)$$

It may be taken as a rough approximation that each volumetric per cent of CO or H₂ appearing in the gases would represent a loss of some 3 to 5 per cent of the energy of the fuel and similarly for CH₄ each per cent would represent a loss of some 9 to 15 per cent.

Molecular Energy Losses. — The energy losses associated with the molecular energy in the hot products of combustion are measured in terms of the excess of energy passing out in the products at their actual exit temperature over that which they would contain if recooled to

the temperature at which the fuel and air entered the system. As the composite processes in the furnace or in the engine are effectively steady-flow in character the losses are evaluated by determination of the difference of enthalpy of the products at the two temperatures.

It is convenient and customary to divide the total of these losses into at least two parts, the one comprising the losses in the gaseous or so-called "dry" portion of the products and the second comprising those due to failure to cool and condense the vapor portions of the products. The first is frequently further divided, and with advantage, into the components associated with the air (and dry products) ideally required for combustion and a component associated with any excess air supply. Further divisions have also been suggested.

For evaluation of the actual amount of the gaseous products per pound of fuel test data on the air supply and the fuel analysis may be used, or it may be computed from the gas analysis. By methods essentially parallel to those already used,

Loss in gaseous portion of combustion products, B.t.u. per pound of fuel supplied,

$$\begin{aligned}
 &= \text{Pounds of gaseous products per pound of carbon burned} \times \text{carbon} \\
 &\quad \text{burned per pound of fuel supplied} \times c_{p, \text{ products}} \times (t_{\text{exit}} - t_{\text{air supply}}) \\
 &= \frac{44 \text{ CO}_2 + 32 \text{ O}_2 + 28 \text{ CO} + 28.15 \text{ N}_2 + 2.02 \text{ H}_2 + 16.04 \text{ CH}_4}{12 (\text{CO}_2 + \text{CO} + \text{CH}_4)} \times \\
 &\quad (\text{carbon in fuel, by analysis} - \text{loss to ashpit, etc.}) \times 0.24 \times \\
 &\quad (t_{\text{exit}} - t_{\text{supply}}) \\
 &= \frac{11 \text{ CO}_2 + 8 \text{ O}_2 + 7 \text{ CO} + 7.05 \text{ N}_2 + (0.5 \text{ H}_2 + 4 \text{ CH}_4)}{3 (\text{CO}_2 + \text{CO} + \text{CH}_4)} \times \\
 &\quad (\text{pounds of C per pound of fuel, by analysis} - \text{lb. of C lost to} \\
 &\quad \text{ashpit, etc., per pound of fuel}) \times 0.24 \times (t_{\text{exit}} - t_{\text{air supply}}). \quad (24)
 \end{aligned}$$

For a liquid fuel of unknown analysis, by methods parallel to those employed in the development of equation 11,

Loss in gaseous portion of combustion products, B.t.u. per pound of fuel burned

$$\begin{aligned}
 &= \frac{11 \text{ CO}_2 + 8 \text{ O}_2 + 7 \text{ CO} + 7.05 \text{ N}_2 + (0.5 \text{ H}_2 + 4 \text{ CH}_4)}{2 \text{ CO}_2 - \text{O}_2 + 2.5 \text{ CO} + 0.268 \text{ N}_2 + (0.5 \text{ H}_2 + 4 \text{ CH}_4)} \times 0.24 \\
 &\quad \times (t_{\text{exit}} - t_{\text{air supply}}). \quad (24a)
 \end{aligned}$$

The loss due to failure to cool and condense the water vapor portion of the combustion products is rather analogous to the difference between the higher and lower heating values of the fuel. An exact evaluation would require computation of the mass of vapor passing out per pound of fuel and of the difference of enthalpy of (a) the superheated vapor at the partial pressure at which it exists in the products mixture at the exit

pressure and temperature of the mixture and (b) the partially condensed condition at which the vapor would exist after cooling to the initial supply temperature of the air. The later computation is a rather involved one, along the lines of example 8. In lieu of this technically exact procedure it is considered in practice to be adequately exact to employ the following arbitrary formula:

$$\begin{aligned} \text{Loss in vapor portions of combustion products, B.t.u. per pound of fuel,} \\ = (w + 9H) (0.46 t_2 + 1090 - t_1)^{10} \end{aligned} \quad (25)$$

in which w = mass of moisture in fuel, pounds per pound of fuel;

H = mass of hydrogen in fuel (other than in moisture), pounds per pound;

t_2 = exit temperature of products of combustion, degrees Fahrenheit;

t_1 = temperature of air supply, degrees Fahrenheit.

In the reporting of a furnace and boiler test or the test of an internal-combustion engine it is the practice to present a tabular compilation of the energy going to the water in a boiler or in the jacket of an engine, the energy to work in an engine, the energy to each of the above loss items, et cetera. The values are commonly expressed either or both in B.t.u. per pound of fuel supplied or in per cent of the energy supplied, the latter as determined by the higher heating value of the fuel. Such a report of the energy distribution in the machine is known as its **energy** (or "**heat**") **balance**.

Example 10. A furnace and boiler installation employed a coal fuel of the following analysis, in per cent by weight as fired (including moisture):

$C = 77.3$; $H = 4.1$; $O = 7.3$; $S = 0.6$; moisture = 3.0; ash = 7.7.

The higher calorific value of the coal as determined by bomb calorimeter was 13,070 B.t.u. per lb. as fired. Data are given for two distinctive operating conditions.

	Condition A	Condition B
Coal fired per hour, pounds	2,740	3,240
Water fed and evaporated per hour, pounds	25,000	25,000
Temperature of water to boiler, °F.	200	200
Steam pressure, lb. per sq. in. abs.	365	365
Steam temperature, °F.	500	500
Temperature of air to furnace, °F.	85	85
Temperature of stack gases, °F.	600	650
CO ₂ in stack gases, per cent by volume	14.3	9.4
O ₂ in stack gases, per cent by volume	5.1	10.6
N ₂ in stack gases, per cent by volume	80.6	80.0
Carbon losses, pounds per pound of fuel	negligible	0.033

¹⁰ This formula is in effect based on the presumptions (a) that the dewpoint of the products is about 125° F. and (b) that the specific heat of the superheated vapor in the products is 0.46 and of the condensed vapor is 1.0, all of which are reasonable, but (c) that irrespective of the dewpoint or of t_1 the vapor would have wholly condensed on being cooled to t_1 . The last condition is somewhat improbable.

Compute, in B.t.u. per pound of fuel and in percentage of the energy supplied in the fuel, the energy delivered to the water, the items of chemical energy loss and of molecular energy loss, and any remaining unaccounted-for loss. Note that the per cent delivered to the water expresses the boiler and furnace efficiency.

161. Summary. — The combustion of the fuels, by an oxidation reaction for which the atmosphere is used to supply the oxygen, is the sole process which man has been able to devise and control by which he may tap the energy stores of nature and effect a partial transformation of the energy so released to the mechanical energy which he so urgently requires for all of his industrial and social activities.

The essential combustible constituents of the fuels are carbon and hydrogen. These occur in wide ranges of combinations in the various solid fuels, in petroleum, and in the natural gases. From these natural fuels many other fuels are prepared in forms that better adapt them to certain specific uses. When fully oxidized the carbon in a fuel passes to CO_2 , requiring ideally 11.53 lb. of air to furnish the requisite oxygen and forming 3.67 lb. of CO_2 per pound of carbon burned. The oxidation of hydrogen produces water vapor in the amount of 8.9 (roughly, 9) lb. of vapor per pound of hydrogen, requiring ideally 34.3 lb. of air per pound of hydrogen. Atmospheric air is 23.1 per cent oxygen by weight (20.9 per cent by volume), with the remainder primarily nitrogen. The nitrogen is inert in the combustion process and acts only as a diluent.

In general a marked reduction of efficiency in combustion will follow the supplying of air either in an amount less than or in any considerable degree greater than that ideally required for the combustion of a given fuel. A most convenient and ready index of the propriety of the air supply is a volumetric analysis of the gaseous portion of the products of combustion, determining at least the amount of CO_2 but preferably the CO_2 , O_2 , CO , and N_2 , with perhaps in addition any unburned combustible gases such as H_2 , CH_4 , et cetera. From the more complete analyses one may compute with some degree of confidence the mass of air actually supplied per pound of fuel. Even the CO_2 determination, associated with information concerning the character of the fuel, will in itself offer useful evidence of the relative excess of air being supplied.

The heating value of a fuel is a figure expressing the quantity of heat energy which will be emitted from a calorimeter in which the fuel is being burned and the combustion products are being recooled to the initial temperature of the fuel and air (or oxygen) supply. Two heating values are ascribed to a fuel containing hydrogen. The *higher* value evaluates in effect the amount of heat energy emitted if the combustion and calorimeter conditions are such that the water vapor formed from the hydrogen is wholly condensed during the recooling of the products, whereas

the *lower* value evaluates the energy emitted under conditions such that no condensation may occur. For exact calorimetry due care needs be exercised to ascertain the degree to which the condensation may have progressed. The higher value is regarded as the proper one upon which to base the energy input and thus the efficiency of a fuel-consuming machine or plant. The lower value is of the major interest to a designer. The heating value is an indirect index of the amount of chemical energy released as a result of the combustion. The heating values per pound of ideal air-fuel mixtures exhibit a unique degree of uniformity at about 1200 B.t.u. per pound of mixture (lower value) for a wide range of distinctly different commercial fuels. A like condition applies also to the heating value per cubic foot of ideal mixture for the vapor and gas fuels.

The temperature attained upon combustion are influenced in general by: (1) the heating value per pound of actual mixture, (2) the rate of energy departure from the system as work or as heat during the combustion, (3) the initial temperature of the fuel mixture at ignition, (4) the specific heat and manner of variation of the specific heat of the products, and (5) the chemical equilibrium characteristics of the products, because those affect the temperatures above which there develop predominant tendencies toward dissociation of the products and chemical energy absorption. About 4000° F. probably represents the maximum temperature attainable by a constant pressure combustion of normal fuel mixtures, with air as the oxygen-supplying agent, with the practicable minimum of air excess and of heat loss and with the combustion originating at about atmospheric temperature.

For the critical analysis of the performance of a furnace and boiler or of an internal-combustion engine it is desirable to ascertain the individual magnitudes of the various chemical energy losses due to the presence of unburned combustible material in the substances leaving the system and also the various energy losses in the hot departing products of combustion. A tabular compilation of these loss items, with other pertinent items, serves to show advantageously the manner in which the chemical energy supplied is ultimately distributed as a result of the processes in the machine. Such a compilation is known as the energy (or "heat") balance of the machine.

162. Review Questions and Topics. —

1. (a) What features of the combustion process make it a particularly vital one; what class of substances enter into the combustion process?
- (b) List the major commercial fuels.
2. (a) What constituents comprise the combustible portion of the fuels and what other constituents or classes of constituents are found in the common fuels?
- (b) What is the constitution of our atmosphere?

(c) Write and interpret completely the chemical equations for the complete ideal combustion of carbon, of hydrogen, and of sulphur, each burning with air.

(d) Which of the components of the combustion products from the above reactions are invariably gaseous?

(e) The provision of an excess of air over that ideally required for combustion would introduce what additional component in the combustion products and would influence the percentage of CO_2 in those products in what manner? For a given excess of air supply how would the existence of hydrogen in the fuel influence the percentage of CO_2 in the gaseous portion of the products?

(f) Outline the effect of the provision of air in an amount less than that ideally required for complete combustion.

3. (a) How may the relative air supply in an actual combustion be estimated quite readily?

(b) Delineate the steps leading up to equation 10 and illustrate the application of that equation.

(c) Delineate the steps leading up to equation 11 and illustrate the application of that equation.

4. (a) Define the term *heating value* of a fuel and write the energy equations involving the heating value term as the equations apply to the several distinctive types of fuel calorimeter.

(b) Distinguish between the higher and the lower heating values of a fuel which contains hydrogen, and justify the use of the two.

(c) State a fair average value for the (lower) heating value per pound of ideal mixture for commercial fuels, and discuss briefly.

(d) Delineate a method of computation of the heating value per cubic foot of an air-fuel mixture and report a fair average value for the (lower) heating value per cubic foot of ideal mixture at 70°F . and 14.7 per sq. in.

5. (a) State the more evident factors which influence the temperatures obtainable by the combustion of a fuel.

(b) Describe briefly the phenomenon of chemical equilibrium and indicate its general influence on the temperatures obtainable by combustion; quote a fair maximum value for the temperature obtainable by the combustion of a normal air-fuel mixture.

6. State and classify the chemical energy losses and molecular energy losses which may accompany the usual combustion process in a boiler and furnace or an internal-combustion engine.

Symbols and Abbreviations, Chapter XV

C = chemical energy released by combustion, B.t.u. per pound of fuel; and the element *carbon*.

C_p = residual chemical energy remaining in combustion products, B.t.u. per pound of products.

CO = gaseous or molecular carbon-dioxide; and the volumetric percentage of carbon-monoxide in a gas mixture.

CO_2 = gaseous or molecular carbon-dioxide; and the volumetric percentage of carbon-dioxide in a gas mixture

c = specific heat, B.t.u. per lb.

E = molecular energy, B.t.u. per lb.

H = enthalpy, B.t.u. per lb.; and the element hydrogen.

H_2 = gaseous or molecular hydrogen; and the volumetric percentage of hydrogen in a gas mixture.

H_2O = water vapor.

J = Joule's equivalent, foot-pounds per B.t.u. (= 778)

M = mass of mixture or products, pounds per pound of fuel.

m = molecular weight and, as subscript, to designate a property of a mixture, usually of fuel and air.

N_2 = gaseous or molecular nitrogen; and the volumetric percentage of nitrogen in a gas mixture.

O = the element oxygen.

O_2 = gaseous or molecular oxygen; and the volumetric percentage of oxygen in a gas mixture.

P = pressure, pounds per square foot.

p = pressure, pounds per square inch, and, as subscript, to designate a property of the products of a combustion.

Q = energy departing from a system as heat (during combustion), B.t.u. per pound of fuel.

QP = heating value at constant pressure, B.t.u. per pound of fuel.

QV = heating value at constant volume, B.t.u. per pound of fuel.

R = the gas constant.

S = the element sulphur.

SO_2 = gaseous or molecular sulphur-dioxide; and the volumetric percentage of sulphur-dioxide in a gas mixture.

T = absolute temperature, degrees Rankine (= degrees Fahrenheit + 460).

t = temperature, degrees Fahrenheit.

V = specific volume, cubic feet per pound.

W = any mechanical energy (not shaft-work only) departing from a system during combustion, foot-pounds per pound of fuel.

w = mass of water in fuel, as moisture.

x, y, z = subscripts designating the number of atoms of an element in a molecule of a compound of the elements.

CHAPTER XVI

THE INTERNAL-COMBUSTION ENGINE

The internal-combustion engine constitutes one of the two general types of devices which man has been able to develop for the purpose of realizing as shaft-work some portion of the available energy associated with the high temperature products of the combustion of a fuel, the other device being represented by the composite arrangement of furnace, steam boiler, and steam engine or turbine. It will be recalled that in the latter arrangement the stored chemical energy of the fuel is released by a process of combustion carried out in a furnace at high temperature levels but at constant atmospheric pressure, that the energy so released is then in part delivered by conduction to moderate temperature steam which is formed in the boiler under high pressure, and that this steam acts as a carrier through the medium of which energy is delivered to the engine, in which there is finally accomplished a partial transformation of the energy to shaft-work and from which the unavailable residue of energy is discarded at about atmospheric temperature. In distinction to this serial procedure in the steam plant the internal-combustion engine effects a more direct utilization of the chemical energy released by the combustion by so confining the burning fuel mixture that a *high pressure is produced (or maintained) in the hot combustion products themselves*, whereby through a suitable mechanism these products may act rather directly upon a piston to motivate a shaft against whatever resisting force may be imposed by the external load.

By reason of the simplicity, compactness, and lightness of the engine mechanism which may be devised for such direct utilization of the fuel energy, and also by reason of the highly concentrated character of the liquid fuels to which this engine may be adapted with particular advantage, it has a marked and at present a practically uncontested monopoly of utility in small portable power plants such as those of the automobile and aircraft. By further reason of an attainable thermal efficiency which even the most modern steam plant may only approach the internal-combustion engine has a distinct vogue in moderate capacity power installations both ashore and afloat. In the extremely large capacity installations, however, such as those of the large central station ashore and the large and fast ship, the superior practical utility of the steam turbine is as yet rather undisputed and may continue to be so

unless or until the engineer may succeed in devising an internal-combustion engine or turbine which is comparable with the large steam turbine plant in compactness, efficiency, and reliability.

The internal-combustion turbine is as yet in an experimental stage and its development into a successful commercial machine seems to await certain advancements in other technical lines. Consequently our attention will be limited to the **reciprocating engine**.

163. General Characteristics and Classifications.—It will be recalled that the usual reciprocating engine is made up essentially of a cylinder within which is a movable piston, this being joined by a connecting rod to a rotatable crank-shaft whereby a reciprocating motion of the piston may be transformed to a unidirectional rotation of the shaft. The continuous motivation of this mechanism against an external shaft loading is accomplished by the imposition of a greater average unit pressure and force on the piston face during the power stroke of the piston and the reduction of the average force on the face during the additional stroke or strokes of the engine cycle.

In the steam engine (Art. 134) the excess pressure during the power stroke is effected by the admission of high-pressure steam during a portion of that stroke and an expansion of the steam during the remainder of the stroke, with release of pressure at or about the end of the stroke by opening of an exhaust valve. The reduced pressure during the major portion of the return stroke is about that of the region (as the atmosphere or a condenser) to which the spent steam is discarded.

In distinction to this simple operating schedule in the steam engine the characteristic conditions for which the internal-combustion engine must be designed are somewhat more complex and variable. They may be summarized as follows:

(A) A major portion of the material which must be introduced into the engine cylinder, is the atmospheric air which supplies the oxygen necessary for the combustion. It is therefore necessary that the engine shall provide for the induction of air from the atmosphere to the engine cylinder during some portion of the engine cycle. This may be accomplished in various ways. A very common method is to employ one complete piston stroke for this purpose. An engine which does so is said to operate upon a **four-stroke** cycle. That cycle is described in the following. The alternative or **two-stroke** cycle is considered briefly later.

In a continuous graphic record of (a) the pressure existing within the space enclosed by the cylinder and piston at any point of the piston travel and (b) the total volume of that space at the same piston position, such as is furnished by the engine *indicator* (see also Art. 135),

this induction or **suction** stroke would be represented by a line such as the line ab in Fig. 100. During this entry of the fluid through the *intake* valve the pressure within the cylinder will be less than that at the entrance to the induction system by the amount of the pressure drop necessary to accelerate the fluid and to overcome fluid friction in the intake passages.

(B) For both practical and theoretical reasons, the latter of which will be made more evident in subsequent discussions, it is necessary that prior to the combustion of the fuel the fuel mixture or at least the air portion of this mixture shall have been brought to a pressure and temperature materially above those of the atmosphere. This is accom-

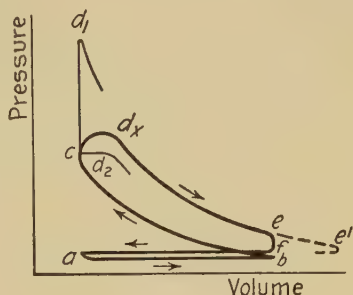


FIG. 100

plished by a compression which is caused to take place by a return or **compression** stroke of the piston with all valves closed. For this stroke the simultaneous fluid pressures and volumes would be represented by points on the line bc .

(C) For motivation of the engine it is necessary that during the next or **power** stroke of the piston the average fluid pressure shall be materially greater than that during the other strokes. This pressure increase is effected by a pronounced temperature increase of the fluid during and resulting from combustion while confined within the cylinder. This combustion is initiated at or about a point in the piston travel which would be represented by point c in the figure.

The rate of combustion, and thus the rates of energy release and of temperature and pressure rise as a result of combustion, is a characteristic which is in effect the major basis for classification of modern engine types. Suitable control of the conditions under which the combustion takes place might permit or cause it to be so rapid and explosive in character as to take place at virtually constant volume and produce a more or less "vertical" rise of temperature and pressure, such as indicated by the line cd_1 in Fig. 100. Alternatively the combustion might be caused to proceed with such lesser rapidity that it would continue through a portion of the power stroke, with the production of a somewhat lesser temperature and pressure rise than would occur under the first conditions. Lines cd_x or cd_2 would be representative of the latter condition.

The exact rate of energy release under this second schedule might be caused to be virtually anything from the one producing almost con-

stant volume combustion to one so relatively slow as simply to maintain a constant temperature and thus to permit a gradual pressure drop following the initiation of the combustion. The line cd_x may be taken as a general one representing any sort of combustion conditions which might obtain. However, the prototype of one of the standard cycles which are employed in present-day internal-combustion engine practice, that is, the **Diesel cycle**, is one in which it is presumed that the pressure shall be maintained constant during and by the combustion, producing a combustion line as represented by cd_2 in the figure. The constant volume combustion of line cd_1 , is a basic characteristic of the **Otto cycle**.

To enable the constant volume combustion of the Otto cycle it is necessary that at point c in the cycle the engine cylinder shall contain the fuel and air in intimate mixture and thus in effective condition for ignition (by electric spark) and *explosive* combustion. This condition is obtainable by (1) the use of either gaseous fuels or relatively volatile liquid fuels and (2) the atomization and intimate mixing of the fuel with the air in some portion of the cycle prior to the completion of the compression. This mixing is ordinarily done outside of the engine cylinder in a mixing valve or **carburetor** and during the induction of the air on stroke ab . It is to be noted however that this very presence of a combustible mixture in the cylinder during the compression stroke imposes certain definite limits on the extent of the compression which may be employed in the Otto cycle engine, this because of the temperature rise produced by the compression and the resultant tendency toward spontaneous pre-ignition of the mixture, and also by reason of the tendency of numerous fuels toward an abnormally rapid combustion and the production of abnormally and seriously high combustion temperature and pressure if compression is carried beyond certain limits. This latter phenomenon is known as **detonation**.

The progressive, constant pressure combustion of the conventional Diesel cycle is securable (1) by withholding the fuel from the engine cylinder until about the completion of the compression stroke, which implies that only air shall have entered during the suction stroke, and (2) by a subsequent progressive injection of an atomized jet of the (liquid) fuel into the cylinder during the combustion period *at a properly controlled rate*. Except in various smaller engines operating on a so-called "semi-Diesel" cycle, the ignition of the fuel is accomplished by compression of the air to so high a pressure P_c that at the end of compression it shall have attained a temperature high enough to induce spontaneous ignition upon the fuel injection. Even under this condition, however, to accomplish suitable ignition and combustion requires that the fuel must be injected in a very finely atomized state.

(D) Returning to a consideration of the events of the cycle — except as there is some incompleteness of combustion at point *d* by reason of chemical equilibrium (see Art. 159) and thus a persistence of combustion, or *after-burning*, beyond that point, the *remainder* of the power or **expansion** stroke provides solely for an expansion of the high temperature products of combustion. This continues until the end of the stroke is reached, at or somewhat before which point the exhaust valve is opened. Line *de* is representative of this expansion phase. It will be observed that an expansion to the same total volume as that at the beginning of compression (V_b) predicates a release of the combustion products at a pressure which is materially higher than the intake or atmospheric pressure, and also their rejection or exhaust at a temperature very considerably above atmospheric temperature. The available energy loss associated with this temperature and pressure excess might be reduced by arranging for a continuation of the expansion to some greater volume such as the volume V_e of the figure. Such cycles have been and are frequently proposed but have not been developed commercially.

(E) Directly upon opening of the exhaust valve there occurs a rapid drop of pressure within the cylinder due to the escape of a portion of the combustion products through the exhaust passages, and upon the following or **exhaust** stroke in a four-stroke cycle the remainder of the products are ejected by the returning piston, except as a portion is left in the **clearance space** between the piston and the head end of the cylinder. These two exhaust processes are represented by lines *ef* and *fa* in Fig. 100. During the exhaust stroke the pressure within the cylinder would exceed that of the exterior by the amount necessary to accelerate the fluid and to overcome fluid friction in the exhaust system. The particular mass of products which remains in the clearance space at the end of any given exhaust stroke is replaced by an equivalent mass at the end of the next cycle; this clearance residue may therefore be regarded as continuously entrapped within the cylinder.

The performance of a complete cycle has thus been described. Observe that for its accomplishment four strokes of the piston have been required and therefore, as noted above, the cycle would properly be designated as a **four-stroke** cycle. An engine operating on this four-stroke schedule would be described as a **four-cycle** engine whether it might conform to the Otto cycle or the Diesel cycle or any modification of either as regards the degree of compression or the character of combustion.

In lieu of the four-stroke schedule it is possible and wholly practicable to accomplish the introduction of the air or the mixture into the cylinder,

its compression, the combustion and expansion, and a sufficient clearing of the combustion products from the cylinder, in two piston strokes. A cycle so carried out is designated, as remarked above, as a **two-stroke** cycle and an engine operating on the cycle is a **two-cycle** engine. The

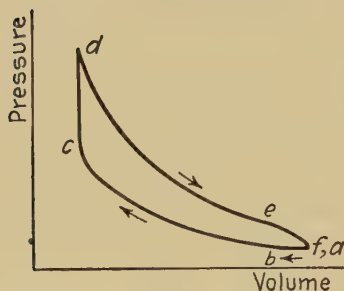


FIG. 101

arrangements by which the cycle is accomplished differ in detail in various engines but the general procedure is in effect (a) to cause the exhaust valves (or ports) to open rather early in the *expansion* stroke, thus facilitating as far as possible the escape of the combustion products without the provision of a separate exhaust stroke, and (b) to force the air (or the mixture) into the cylinder and jointly to sweep out the residual combustion products by the use of a low

pressure **scavenger** blower which is caused to discharge to the cylinder during the early portion of the compression stroke, thus eliminating a separate intake stroke. The general appearance of the pressure-volume diagram of a two-stroke cycle is shown in Fig. 101, in which equivalent phases in the two- and the four-stroke cycles are denoted by the same letters as were used in Fig. 100.

164. The Engine Energy Equation. — Observe that the internal-combustion engine is a mechanism to which there is steadily delivered a mass of fuel and air, from which energy departs as shaft-work output and as heat to cooling water and the surrounding atmosphere,¹ and from which there also depart steadily the hot products of combustion, with perhaps a residue of chemical energy in those products due to incompleteness of combustion. Upon these observations one may write directly a general energy equation which will be applicable in principle to any engine whatsoever. Thus, in B.t.u. per pound of fuel supplied and regarding all kinetic energies as negligible,

$$MH_{m,1} + C = W_{\text{out}}/J + Q_{\text{out}} + MH_{p,2} + MC_p, \quad (1)$$

in which M = mass of mixture or of products per pound of fuel;
 $H_{m,1}$ = enthalpy of the entering air and fuel mixture, B.t.u. per pound of mixture;
 C = chemical energy entering in fuel and releasable by complete combustion, B.t.u. per pound of fuel;

¹ The reader is presumably familiar with the physical necessity for cooling the engine cylinder by water-jacketing or by direct air cooling.

W_{out} = shaft-work output, foot-pounds per pound of fuel;

Q_{out} = heat emission, B.t.u. per pound of fuel;

$H_{p,2}$ = enthalpy of the departing products of combustion,
B.t.u. per pound of products;

C_p = any residual chemical energy in incompletely burned
products, B.t.u. per pound of products.

Just as it was desirable and valuable to establish ideal but approachable standards of performance for the steam plant or steam engine (see Arts. 122, 125, 129, etc.) by reference to which one might judge more intelligently the performance of the actual machine, so it would be advantageous if similar standards might be set up for the internal-combustion engine. It will be the purpose of the following to analyze the bases of development of such standards. Although it will be discovered that it is not practicable to evolve any general formulations for ideal engine efficiency, certain valuable evidence and information will develop from the analyses.

The idealized conditions of operation of an internal-combustion engine may be taken to involve the following general features:

- (a) Zero resistance and thus zero pressure drop through the intake system, whereby the cylinder pressure during admission might be that of the atmosphere, P_a ;
- (b) A reversible adiabatic compression of the air or mixture, the adiabatic feature being in distinction to the actual condition of energy transfer as heat between the fluid and the water-cooled or air-cooled cylinder of the real engine;
- (c) A reversible adiabatic combustion phase, which again is in distinction to the actual condition of energy escape as heat from the burning mixture to the cylinder during combustion;
- (d) A reversible adiabatic expansion of the combustion products, with, however, the possibility of some persistence of combustion during and after expansion if conditions of chemical equilibrium were encountered during combustion;
- (e) An instantaneous pressure drop upon opening of the exhaust valve *at the end* of the expansion stroke, and zero resistance and thus zero pressure drop through the exhaust system, whereby the cylinder pressure during exhaust might be that of the atmosphere, P_a .

A pressure-volume diagram for an Otto cycle conforming to these ideal features would be like that of Fig. 102. Observe that in the figure the increase of cylinder volume during the intake stroke ab measures the

volume of air or mixture induced during that stroke and that this volume also equals the piston displacement during the exhaust stroke fa .

The general energy equation of the engine, equation 1, as modified to conform to the above idealized conditions, would initially appear as follows:

$$MH_{m,1} + C = (W_{\text{out}}/J)_{\text{ideal}} + MH_{p,2} + MC_p,$$

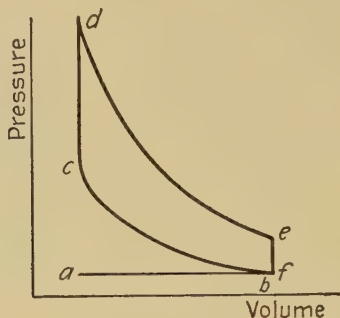


FIG. 102

or, arranged in solution for the shaft-work term,

$$(W_{\text{out}}/J)_{\text{ideal}} = C + M(H_{m,1} - H_{p,2}) - MC_p. \quad (2)$$

For further modification of this relation to more convenient form observe that

$$H_{m,1} = E_{m,1} + P_a V_{m,1}/J,$$

in which P_a = the atmospheric pressure ideally existing in the cylinder during and at the end of the intake stroke.

Also observe that the enthalpy of the departing products $MH_{p,2}$ must equal the internal energy of the products at the point of opening of the exhaust valve, or $ME_{p,e}$, plus the energy imparted to the products by the work done on them by the piston during the exhaust stroke. This ejection work done by the piston equals the pressure P_a times the piston displacement (see also footnote 6, Art. 136), and it was noted above that the latter is also equal to the piston displacement during the induction of the mixture or $MV_{m,1}$. Thus we may write that

$$MH_{p,2} = ME_{p,e} + M(P_a V_{m,1}/J).$$

Introducing these values of $H_{m,1}$ and $H_{p,2}$ into equation 2,

$$\begin{aligned} (W_{\text{out}}/J)_{\text{ideal}} &= C + M(E_{m,1} + P_a V_{m,1}/J - E_{p,e} - P_a V_{m,1}/J) - MC_p \\ &= C + M(E_{m,1} - E_{p,e}) - MC_p. \end{aligned}$$

² This relation is very slightly in error for the Diesel engine, where air only and not mixture enters the cylinder during the intake stroke.

In the bracketed term of this relation let a term $E_{p,1}$ be subtracted and added, giving the relation

$$(W_{\text{out}}/J)_{\text{ideal}} = [C + M(E_{m,1} - E_{p,1})] - M(E_{p,e} - E_{p,1}) - MC_p.$$

But, from equation 12, Art. 158, $C + M(E_{m,1} - E_{p,1}) = Q_V = Q_f$.
Therefore

$$(W_{\text{out}}/J)_{\text{ideal}} = Q_f - M(E_{p,e} - E_{p,1}) - MC_p, \quad (3)$$

in which Q_f = the (constant-volume) heating value of the fuel as determined by calorimeter, B.t.u. per pound of fuel.

If in accordance with the usual custom the calorific or heating value of the fuel is assigned as the energy chargeable to the engine per pound of fuel, then the thermal efficiency of the ideal engine becomes

$$\begin{aligned} \text{Thermal efficiency, ideal} &= \frac{Q_f - M(E_{p,e} - E_{p,1}) - MC_p}{Q_f} \\ &= 1 - \frac{(E_{p,e} - E_{p,1}) + C_p}{Q_f/M}. \end{aligned} \quad (4)$$

Referring to equation 3, recall that during a constant volume process the energy departure as heat equals the change of internal energy of the fluid (Eq. 8, Art. 36), whence it appears that the term $(E_{p,e} - E_{p,1})$ might be also regarded as the heat energy departure that would be associated with the cooling of the products of combustion at constant volume from their temperature at the end of expansion to the initial air supply temperature. This circumstance justifies a common procedure of regarding the pressure drop from e to f , Fig. 102, which accompanies the escape of a portion of the combustion products as the apparent and effective equivalent of a constant volume cooling of the entire products. Alternatively, equation 3 may be written in the modified form,

$$(W_{\text{out}}/J)_{\text{ideal}} = Q_f - M \int_{T_1}^{T_e} (c_v dT)_p - MC_p. \quad (3a)$$

The ideal efficiency relation of equation 4 is simple enough in form and, if one might presume complete combustion, would appear to indicate that the cycle efficiency is dependent only on the heating value of the fuel mixture and on the temperature excess of the combustion products at the end of the expansion stroke $(T_e - T_1)$. However, the temperature T_e is in turn dependent on the mixture heating value and is also a function of the magnitude and manner of variation of the specific heats of the fuel mixture and combustion products at the elevated temperatures and pressures encountered in the cycle, a function of the magnitude and influence of the chemical equilibrium or dissociation effects and a func-

tion of the amount of compression employed in the cycle prior to the combustion and the amount of expansion allowed subsequent to the combustion.

It would thus appear that the actual computation of the ideal cycle efficiency would not only be very complex indeed and would give somewhat diversified results for the various fuel mixtures which may be employed, but it would also involve the use of principles of chemical equilibrium (as influencing dissociation and the prolongation and incompleteness of combustion) which are beyond the scope of this material. For these reasons the detailed computations of the ideal efficiencies of the various cycles will not be considered here. However, it is to be noted that they have been carried out very skillfully by competent specialists and, although the results of those computations probably may not be taken to provide ultimate general expressions for the ideal cycle efficiencies, they do afford very significant information and will therefore be considered further in subsequent articles.

It was remarked above that the extent of the compression and expansion employed in the several cycles is a feature which affects the final products temperature and thus the efficiency of the cycle. It is interesting that the outstanding influence of these features was originally indicated by quite artificial and unreal analyses which are known as the **Air Standard** analyses. Consequently, in spite of the artificiality and otherwise limited utility of these analyses, it is desirable that they be developed in the following discussion by reason of the important evidence they furnish. In these analyses it is presumed (1) that air alone is caused to pass through the typical state changes of the cycles, (2) that the temperature and pressure rises which in the actual cycles are produced by the chemical energy release during combustion shall in this hypothetical cycle be produced by a like supply of energy to the air as heat from some external source, (3) that in accordance with equation 3a et priori the energy discarded by the cycle ($E_{p,e} - E_{p,1}$) equals the energy emission during a constant-volume cooling of the air from the temperature T_e to the original temperature T_1 , and (4) that the specific heat of the air may be regarded as constant and as that of air at normal atmospheric temperature.

With the foregoing observations in mind we are in a position to proceed with somewhat more detailed consideration of the several typical engine cycles.

165. Otto Cycle, Efficiency Analysis. — Recall that the distinctive characteristic of the Otto cycle engine is the induction and compression of an externally prepared fuel mixture, whereby that mixture may be ignited explosively when the piston is at the head end of the compression

stroke. In the ideal cycle the explosion is presumed to occur at constant volume.

It will be discovered in the following that an outstandingly significant item associated with the *internal combustion engine* cycle is the ratio between (1) the total volume in the cylinder at the end of intake and beginning of compression and (2) the clearance or residual volume in the cylinder when the piston is at the end of the compression stroke. This ratio is known as the **ratio of compression** and will be denoted as r_c . Referring to Fig. 103, which is representative of the events in the ideal Otto cycle,

$$\text{Compression ratio, } r_c = \frac{V_b}{V_c}. \quad (5)$$

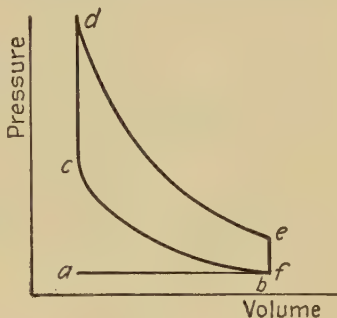


FIG. 103

A second generally significant ratio is that between the volume at the end of expansion and the volume at the beginning of expansion. This ratio is known as the **ratio of expansion** and will be denoted as r_e . Referring again to Fig. 103,

$$\text{Expansion ratio, } r_e = \frac{V_e}{V_d}. \quad (6)$$

It is observed that for the ideal Otto cycle the expansion ratio and the compression ratio are equal.

The Air Standard analysis of the Otto cycle, although of distinctly limited utility in many features, does provide very useful evidence of the character of the influence of the compression ratio on the cycle efficiency. The analysis is therefore made in the following. The subscripts employed correspond to the figure.

By equation 15 of Art. 87, for the ideally isentropic compression of the air,

$$T_c = T_b \left(\frac{V_b}{V_c} \right)^{k-1} = T_b (r_c)^{k-1}. \quad (a)$$

For the constant volume pressure and temperature rise the required energy supply, which is hypothetically supplied as heat to the air in the Air Standard cycle, is:

$${}_cQ_d = Q_{\text{supplied}} = c_v(T_d - T_c). \quad (b)$$

For the ideally isentropic expansion of the air,

$$T_e = T_d \left(\frac{V_d}{V_e} \right)^{k-1} = \frac{T_d}{(r_e)^{k-1}} = \frac{T_d}{(r_c)^{k-1}} \quad (c)$$

The energy rejected by the cycle was noted above to equal the heat emission required to cool at constant volume from T_e to T_b , or

$${}_eQ_b = Q_{\text{rejected}} = c_v(T_e - T_b). \quad (d)$$

From equations (a) and (c)

$$(r_c)^{k-1} = \frac{T_c}{T_b} \text{ and also } = \frac{T_d}{T_e},$$

whence

$$\frac{T_d}{T_c} = \frac{T_e}{T_b}, \text{ and also } \frac{T_d - T_c}{T_c} = \frac{T_e - T_b}{T_b}. \quad (e)$$

From the latter relation (e), from (a), (b), and (d) above, and from the general concept of thermal efficiency,

$$\begin{aligned} \text{Thermal efficiency, Air Standard Otto cycle} &= \frac{\text{Work output}}{\text{Energy supplied}} \\ &= \frac{{}_cQ_d - {}_eQ_b}{{}_cQ_d} = 1 - \frac{{}_eQ_b}{{}_cQ_d} \\ &= 1 - \frac{c_v(T_e - T_b)}{c_v(T_d - T_c)} \end{aligned}$$

(compare with Eq. 4)

$$\begin{aligned} &= 1 - \frac{T_b}{T_c} \\ &= 1 - \frac{1}{(r_c)^{k-1}} = 1 - \frac{1}{(r_c)^{0.4}} \text{ for air. } (7) \end{aligned}$$

The evidence offered by equation 7 is that, for the Air Standard Otto cycle, the thermal efficiency is determined solely by the compression or expansion ratio. Its influence is indicated graphically by the curve *A* of Air Standard thermal efficiency vs. compression ratio in Fig. 104. In the figure there are also represented, in curves *B*, *C*, and *D*, the results of computations made by Goodenough and Baker³ for the ideal (indicated) thermal efficiency of an Otto cycle employing real mixtures of gasoline and air. Their computations were based in general on equation 4 and careful attention was given to the effect of specific heat variation and of chemical equilibrium during combustion, employing the then (1926) best available data. Curve *E* shows the actual thermal efficiencies (based on indicated horsepower) attained on test of a very high grade, single-cylinder Otto engine in which the compression ratio could be varied at will, the results being those reported by Ricardo⁴ when using a non-detonating fuel and a fuel mixture containing air 15 per cent in

³ Goodenough and Baker, A Thermodynamic Analysis of Internal Combustion Engine Cycles, Univ. of Illinois Engineering Experiment Station Bulletin 160 (1927).

⁴ See Ricardo, Engines of High Output, Macdonald and Evans, London (1926).

excess of the amount theoretically required for complete combustion of the fuel.

The results of the ideal efficiency computations (curves *B*, *C*, and *D*) and the actual efficiency results (curve *E*) distinctly corroborate the evidence offered by the Air Standard Cycle computations in so far as those predict the material influence of compression and expansion ratio on efficiency.⁵ In addition the ideal efficiency curves indicate a material influence of the character of the fuel mixture on the ideal efficiency. Test results on actual engines operating with mixture strengths which are readily explosive further corroborate the evidence of the ideal curves, to wit, that decrease of the mixture strength acts to increase efficiency quite distinctly.

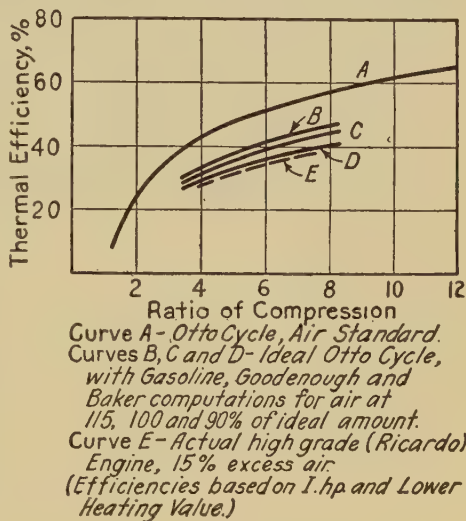


FIG. 104

The indications of the above curves well justify the continuing efforts of designers toward the development of engines and fuels which will permit the use of higher compression ratios and also the use of less rich fuel mixtures. Progress in the use of distinctly weak mixtures is handicapped by the difficulty in ignition and rapid burning of the mixture and by the material reduction in the power capacity of a given size engine with reduction in mixture strength, maximum power being secured both in the ideal cycle and in the actual engine with mixtures in which the air supply is some 5 to 10 per cent less than the theoretical requirement. Progress in the direction of the higher compression ratio is handicapped by the detonation tendency of many fuels but similarly is facili-

⁵ It is interesting to note that the relations of curves *B*, *C*, *D*, and *E* may be represented fairly accurately by the following empirical equations:

$$\text{Curve B} - \text{Eff.} = 1 - 1.02/(r_c)^{0.31};$$

$$\text{Curve C} - \text{Eff.} = 1 - 1.03/(r_c)^{0.296};$$

$$\text{Curve D} - \text{Eff.} = 1 - 0.99/(r_c)^{0.246};$$

$$\text{Curve E} - \text{Eff.} = 1 - 1.02/(r_c)^{0.253}.$$

The interesting feature is the real similarity in form as between these expressions and equation 7.

tated by the development and utilization of fuels which are less detonative in character (such as gasoline-benzol blends, gasoline treated with various catalysts, alcohol, et cetera). The tendency of the efficiency curves toward a gradual flattening with increase of compression ratio and a very pronounced increase of the peak pressure of the cycle (see the curve of P_d , Fig. 105) without commensurate increase of the engine power will undoubtedly act to limit the ratio for which it is economically worth while to strive. With ordinary gasolines, values of the ratio in excess of about 5 can not be used to advantage due to detonation.

The actual efficiency results shown by curve E indicate at least a gratifying degree of success in engine design, if we may presume the ideal curves to be based on adequate and accurate data. The departure of actual engine efficiency from the ideal is attributed to a number of factors, such as (1) a failure to burn portions of the fuel which are in intimate contact with the cylinder walls, (2) untimely burning of portions of the fuel during the compression stroke, this occasioned by localized contacts with hot portions of the cylinder or exhaust valve, and (3) energy conduction through the cylinder walls as heat during the combustion and expansion. Regarding the latter, it is claimed that a very considerable portion of the energy which passes through the cylinder

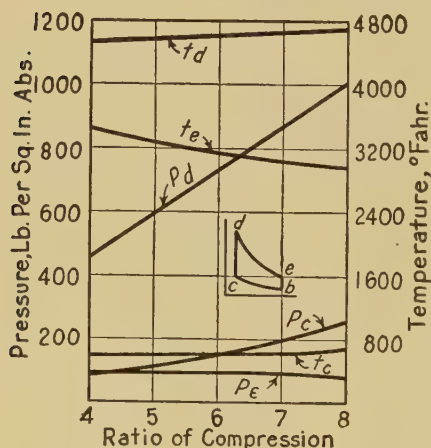


FIG. 105

walls does so during the exhaust stroke and that the loss during combustion and expansion is not so large a factor as might perhaps be anticipated. The efficiency of multiple-cylinder engines is frequently decreased very materially by a poor distribution of a properly proportioned fuel mixture, that is, by the delivery of over-weak mixture to certain cylinders and over-rich mixture to others. The efficiency of two-cycle Otto engines is commonly rather less than that of a four-cycle engine by reason of a direct wastage of fuel mixture through

the exhaust during the dual exhaust-induction phase of that cycle.

For general interest, Fig. 105 presents the results of the computations of Goodenough and Baker for the pressures and temperatures at various points of the ideal Otto cycle for gasoline at various compression ratios and a mixture containing exactly the theoretical air requirement.

Example 1. Compute for the Air Standard Otto cycle the successive temperatures and pressures which would occur presuming a temperature and pressure at the beginning of compression (point *b*, Fig. 103) of, respectively, 60° F. and 14.7 lb. per sq. in., a compression ratio of 5.0, and energy supply during the constant volume phase of 500 B.t.u. per lb. Also compute the work supplied per pound of air during the compression, that delivered during expansion, the net work output, the energy rejection and the efficiency of the cycle. Compare the latter with the efficiency computed directly by use of equation (7).

(63,000 ft-lb.; 248,000 ft-lb.; 263 B.t.u.; 0.475.)

Example 2. Compute the specific fuel consumption, pounds per (indicated) horsepower-hour, for an engine operating on gasoline at a compression ratio of 5.0, presuming (*a*) Air Standard efficiency; (*b*) the ideal efficiency computed by Goodenough and Baker at 100 per cent air, and (*c*) the actual efficiency reported by Ricardo (Fig. 104).

166. Diesel Cycle, Efficiency Analysis; Dual Combustion.— Recall that the distinctive characteristics of the Diesel cycle are the induction and compression of air only but also a degree of compression sufficient to raise the air to a temperature well above the ignition temperature of the fuel, whereby upon injection of an atomized fuel oil jet at the end of the compression stroke ignition will automatically occur and combustion will continue so long as fuel injection is continued and the oxygen or air supply in the cylinder does not become depleted. The resulting temperatures and pressures attained by the products during the fuel admission will depend primarily on the rate of fuel injection and combustion. In the *conventional* present-day concept of the Diesel cycle the rate is presumed to be such that the pressure which was reached at the end of compression shall be maintained during the piston travel to the end of the injection phase. Line *cd* of Fig. 106 is representative of such a condition.

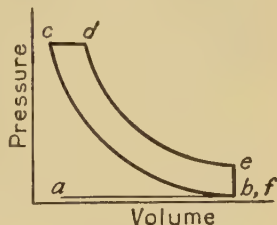


FIG. 106

The remainder of the cycle of the figure conforms to the idealized conditions noted in Art. 164. In the figure, V_b/V_c is again the **ratio of compression**, and V_e/V_d is the **ratio of expansion**. It is obvious that in the conventional Diesel cycle the expansion ratio is less than the compression ratio. For any given compression ratio the duration of the energy supply phase, *cd*, would be increased and the expansion ratio would be correspondingly decreased with increase in the *amount* of energy supplied to the engine per cycle. Interpreted in association with the actual engine, the amount of energy supplied per cycle is measured by the amount of fuel supplied per cycle and thus by the rate and duration of the fuel injection. Note in this connection that the ideal

limiting amount of fuel which might properly be injected per cycle is established by the amount of oxygen and thus of air which had been inducted into the engine cylinder on the intake stroke of the cycle. Fuel injection in that limiting amount would be said to provide a mixture of zero per cent air excess; injection to one half of that amount would be equivalent to the provision of air 100 per cent in excess of that ideally required for combustion; injection to one third of the maximum would be equivalent to 200 per cent excess air; et cetera. Thus, summarizing, decreased fuel injection per cycle implies increased expansion ratio and increased air excess.

Again, the Air Standard analysis of the engine cycle contributes certain useful evidence of the general factors which influence the cycle efficiency and so will be presented in spite of its otherwise inadequacy.

For the ideally isentropic compression of the air (Eq. 15, Art. 87),

$$\frac{T_c}{T_b} = \left(\frac{V_b}{V_c} \right)^{k-1} = (r_c)^{k-1}. \quad (a)$$

For the constant pressure temperature rise during the energy supply phase,

$${}_cQ_d = Q_{\text{supplied}} = c_p(T_d - T_c). \quad (b)$$

and

$$\frac{T_d}{T_c} = \frac{V_d}{V_c} = \left(\frac{V_d}{V_e} \right) \left(\frac{V_b}{V_c} \right) \text{ (since } V_e = V_b) = \frac{r_c}{r_e}. \quad (c)$$

For the ideally isentropic expansion of the air,

$$\frac{T_e}{T_d} = \left(\frac{V_d}{V_e} \right)^{k-1} = \frac{1}{(r_e)^{k-1}}. \quad (d)$$

The energy rejected by the cycle was noted in Art. 164 to be equivalent to the heat emission required to cool at constant volume from T_e to T_b , or

$${}_eQ_b = Q_{\text{rejected}} = c_v(T_e - T_b). \quad (e)$$

$$\begin{aligned} \text{Thermal efficiency, Air Standard} &= \frac{{}_cQ_d - {}_eQ_b}{{}_cQ_d} = 1 - \frac{{}_eQ_b}{{}_cQ_d} \\ \text{Diesel cycle} &= 1 - \frac{c_v(T_e - T_b)}{c_p(T_d - T_c)}. \end{aligned}$$

Certain manipulations are desirable in order to express this relation in a more useful form. Thus, noting that $c_v/c_p = 1/k$, dividing both numerator and denominator of the fraction by $(T_d r_e)$ and making further rearrangements as noted in the following,

$$\begin{aligned}
 \text{Thermal efficiency, Air Standard Diesel cycle} &= 1 - \frac{1}{k} \frac{(T_e - T_b)/(T_d r_e)}{(T_d - T_c)/(T_d r_e)} \\
 &= 1 - \frac{1}{k} \frac{\frac{T_e}{T_d} \frac{1}{r_e} - \frac{T_b}{T_c} \frac{1}{T_d r_e}}{\frac{1}{r_e} - \frac{T_c}{T_d} \frac{1}{r_e}} \\
 &= 1 - \frac{1}{k} \frac{\left(\frac{1}{r_e}\right)^k - \frac{1}{(r_c)^{k-1}} \frac{r_e}{r_c} \frac{1}{r_e}}{\frac{1}{r_e} - \frac{r_e}{r_c} \frac{1}{r_e}} \\
 &= 1 - \frac{1}{k} \frac{\left(\frac{1}{r_e}\right)^k - \left(\frac{1}{r_c}\right)^k}{\left(\frac{1}{r_e}\right) - \left(\frac{1}{r_c}\right)}; \quad (8)
 \end{aligned}$$

or in an alternative arrangement which is somewhat parallel to the expression for the Air Standard efficiency of the Otto cycle and which is obtained by extracting the term $\frac{1}{(r_c)^{k-1}}$ from the fraction,

$$= 1 - \frac{1}{(r_c)^{k-1}} \frac{\left(\frac{r_c}{r_e}\right)^k - 1}{k \left(\frac{r_c}{r_e} - 1\right)}. \quad (8a)$$

The evidence offered by equations 8 or 8a is that for the Air Standard Diesel cycle the efficiency is determined jointly but solely by the compression and expansion ratios.

The influence of the two is indicated graphically by the solid lines in Fig. 107. The figure also shows, that for a given compression ratio the Diesel efficiency is less than that of the Otto and departs progressively from the Otto with decreasing values of the expansion ratio r_e . This last fact is due directly to the increase of the temperature T_e

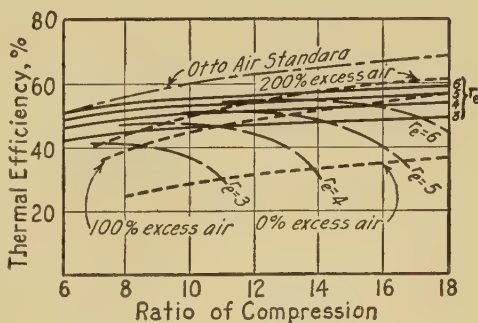


FIG. 107

(at the end of the expansion and the beginning of the energy rejection phases of the cycle) with decrease of the expansion ratio. This temperature increase is in turn due jointly to the increasing tempera-

ture T_d resulting from greater energy supply to the fluid during the longer injection phase and the concurrently lesser opportunity for temperature drop during the expansion phase. In spite of this apparent handicap the actual Diesel engine is in general able to attain thermal efficiencies higher than those obtained by the Otto, due to the compensating advantage that the detonating character of the fuel used with the Otto engine and the very high maximum pressures encountered in that cycle limit its practicable compression ratio to about 5 or 6 (see Art. 165), whereas the Diesel engine may and commonly does employ compression ratios of 12 or over.

In Fig. 107 there are also presented two allied sets of curves which are based upon the results of computations made by Goodenough and Baker for the ideal indicated efficiency of a Diesel cycle employing a real fuel. As with the corresponding curves of Fig. 104, the computations for these curves were based in general on equation 4 (Art. 164) but careful attention was given to the influence of specific heat variations and of chemical equilibrium during combustion, employing the then (1926) best available data on those phenomena. The two sets of curves are observed to be curves of ideal indicated thermal efficiency vs. compression ratio at (a) a variety of expansion ratios (dash lines) and (b) a variety of air excess percentages (dotted lines). The first set is thus directly comparable with the set of Air Standard curves.

The evidence of these curves is particularly significant. Scrutinizing the set which involves the expansion ratios, those show the marked effect of decreased expansion ratio at any given compression ratio. However they also show a unique critical relation between the compression ratio and the suitable expansion ratio. Thus optimum indicated efficiencies are observed to be uniformly obtainable only at a range of expansion ratios from about 40 to 60 per cent of the compression ratio, with a maximum at about 50 per cent and a fairly sharp falling off at expansion ratios less than about 40 per cent of the compression ratio. This outstanding influence of the relative expansion ratio when real fuels are considered results not only from its effect on the temperature T_c at which the combustion products are released but also from the degree of incompleteness of combustion or of delay of combustion occasioned by chemical equilibrium.

The curves involving the air-excess percentages furnish simply an alternative manner of portraying the influence of the expansion ratio. They indicate a very marked reduction of ideal indicated efficiency if, with *constant pressure* combustion, fuel injection is attempted up to an amount approaching that for which oxygen or air is present in the engine cylinder. Optimum indicated efficiencies appear to be uniformly obtain-

able at an air excess of or moderately greater than 200 per cent (or 33 per cent or less fuel injection). Recall however that this conclusion pertains to the constant pressure type of combustion phase. The last paragraphs of this article point out that by a modification of the combustion phase a more complete utilization of the available oxygen may be obtained with much less serious effect on the thermal efficiency.

Both curves show the benefit of increasing compression ratio but a futility in increased compression ratio if improper expansion ratios are employed.

In Fig. 108 are presented the results of the computations of Goodenough and Baker for the pressures and temperatures at various points of the ideal Diesel cycle (with kerosene as fuel) at various compression ratios and for amounts of fuel such that the air supply is 0, 100 and 200 per cent in excess of that ideally required for the fuel which is injected. The data of these curves are comparable with those of Fig. 105 for the ideal Otto cycle.

It is to be stated that in the Diesel engine a very thorough atomization of the fuel oil jet is necessary as well as a jet momentum sufficient to cause the fuel particles to penetrate the mass of highly compressed air, and also there needs be a considerable turbulence in that air, all in order that the fuel and oxygen molecules may rapidly become intimately associated and rapid combustion may thereby be facilitated. It has been the usual practice to assist the atomization of the fuel and the mixing by introducing with the fuel jet a jet of very highly compressed air which was furnished by an accessory compressor. With more recent advances in the art, fuel nozzles are being developed which provide adequate pulverization and dissemination of the fuel without the aid of the air jet. The two methods of introducing the fuel are known respectively as **air injection** and **airless injection** or, as the latter is frequently although rather inaptly termed, **solid injection**.

With the advances in the art of jet formation and of the fuel injection timing and metering, the actual Diesel engine has tended toward a modification of the conventional cycle. In this development, instead of fuel

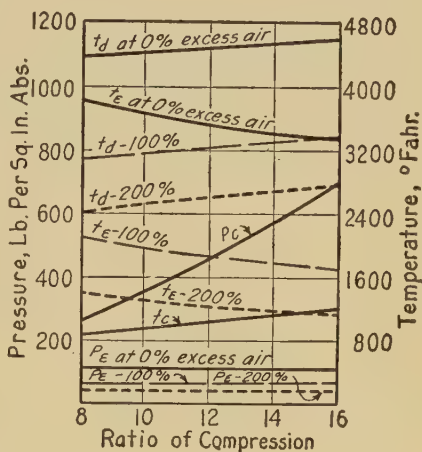


FIG. 108

injection at a rate sufficient that the pressure should merely remain constant during combustion, the rate of injection and combustion has been so accelerated that the pressure rises above that occurring at the end of compression and beginning of combustion. The resulting cycle may in some respects be regarded as a compromise between the conventional Otto and Diesel cycles.

A strictly compromise cycle would perhaps be one in which a portion of the energy is supplied (or released) at constant volume and the remainder at constant pressure. Such a cycle is commonly known as a **Dual-combustion** cycle and is depicted in Fig. 109. An obvious disadvantage of the cycle is the increased pressures which are encountered if the same

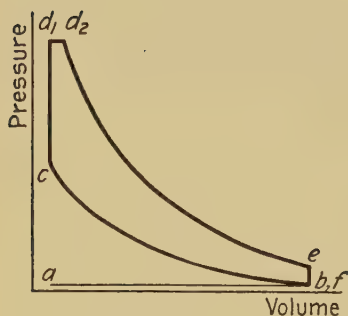


FIG. 109

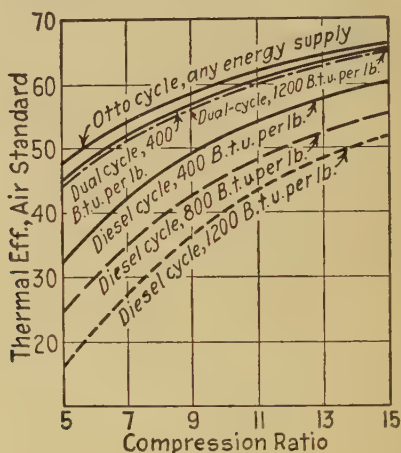


FIG. 110

compression ratio is employed. Its advantage lies in the increased expansion ratio and efficiency attainable at a given compression ratio and energy supply. Computations of the ideal efficiency of the cycle made with due attention to the actual specific heats and the effect of chemical equilibrium are not available at this writing but an Air Standard analysis again will serve to indicate the advantageous features of the cycle. The results of such an analysis are shown in Fig. 110 in which for purposes of comparison thermal efficiency is plotted against compression ratio for the Otto cycle; for the Diesel cycle at energy supplies of 400, 800 and 1200 B.t.u. per pound of air; and for the Dual-combustion cycle at energy supplies of 200 B.t.u. at constant volume and 200 at constant pressure and at supplies of 600 B.t.u. at constant volume and 600 at constant pressure, or respective totals of 400 and 1200 B.t.u. per pound of air. The Diesel cycle curves are comparable or equivalent to those for various percentages of air excess which appeared in Fig. 107. The

evidence of the curves is (a) that the efficiency of the Dual-combustion cycle with the 50-50 energy distribution between the constant volume and constant pressure phases closely approaches that of the Otto, and (b) that the efficiency is virtually independent of the amount of energy supply per pound of air. In both features the cycle offers marked advantage over the conventional constant pressure Diesel cycle. Although the true Dual-combustion cycle is rather impracticable of actual attainment there is thus presented (and there is otherwise obtainable) valid evidence of the benefits accruing from operation of the Diesel engine at such rates of fuel injection and combustion as to obtain some pressure rise and thus some approach to the characteristics of the compromise cycle.

Example 3. Compute for the Air Standard Diesel cycle the successive pressures and temperatures which would occur presuming the pressure and temperature at the beginning of compression to be 14.7 lb. per sq. in. and 70° F. Use a compression ratio of 14 and an energy supply of 500 B.t.u. per pound of air. Also compute the work supplied during the compression, that delivered during the energy supply phase and during the expansion, the net work output, the heat energy supplied and the energy discarded, all items to be *per pound of air*. From the above compute the efficiency and compare with the efficiency computed directly by the use of equation 8 or 8a. What was the expansion ratio of the cycle?

Example 4. Estimate the *specific* fuel consumption (pounds per indicated horsepower-hour) for an engine operating on oil of 19,500 B.t.u. per lb. lower calorific value on the conventional Diesel cycle at a compression ratio of 12 and a fuel-air ratio of one half of the ideal maximum, presuming (a) the ideal efficiency computed by Goodenough and Baker and (b) an engine efficiency of 0.80 relative to the Goodenough and Baker ideal efficiency. Also estimate for case *b* the fuel consumption per shaft horsepower-hour on the presumption of a mechanical efficiency of 0.80, and compare your result with representative actual engine performances.

(0.27; 0.34; 0.42.)

Example 5. Compute for an Air Standard Dual-combustion cycle the successive pressures and temperatures which would occur presuming the pressure and temperature at the beginning of compression to be 14.7 lb. per sq. in. and 70° F. Use a compression ratio of 14 and an energy supply of 500 B.t.u. per pound of air, with 250 B.t.u. supplied at constant volume and 250 B.t.u. at constant pressure. Also compute the work supplied during the compression, that delivered during the several energy supply phases and during the expansion, the net work output and the energy discarded, all items to be *per pound of air*. From the above compute the efficiency and compare with the data of Fig. 110. What was the expansion ratio of the cycle?

167. Power Capacity, Internal-Combustion Engine Cylinder. — The power output of a given engine cylinder is determined by six primary items: (1) the heating value of the fuel supplied to the engine, (2) the mass of fuel per pound of air (or of mixture), (3) the mass of air (or of fuel mixture) which is introduced into the cylinder during a complete cycle of the engine, (4) the number of cycles per unit time, (5) the effi-

ency with which the energy entering and released is transformed to work at the piston face, and (6) the efficiency with which the work energy delivered to the piston face is further delivered to the engine shaft. These items act jointly to establish the rate of energy delivery, or the shaft horsepower output of the engine, in accordance with the following evident relation:

$$\text{S.hp.} = \frac{J \left[\left(Q_f \times \frac{M_f}{M_a} \right) \times M_a \times N \times \text{Eff.}_{\text{indic.}} \times \text{Eff.}_{\text{mech.}} \right]}{33,000}. \quad (9)$$

where

S.hp. = shaft horsepower output of engine;

Q_f = heating value of fuel, B.t.u. per pound (at constant volume);

M_f/M_a = mass of fuel per pound of air induced into the engine, pounds per pound;

M_a = mass of air induced *per complete cycle* of the engine, pounds;

N = number of complete cycles per minute;

$\text{Eff.}_{\text{indic.}}$ = *indicated* thermal efficiency of engine, that is, the proportion between the energy delivered as work at the piston face (presumably as determined by the engine indicator) and the energy supplied as heating value in the fuel;

$\text{Eff.}_{\text{mech.}}$ = mechanical efficiency of the engine, that is, the proportion between the energy delivered as shaft-work and the work energy delivered to the piston face.

These several items are considered individually in reverse order in the following paragraphs.

In Art. 135 it was noted that the rate of energy delivery as work to the piston face of a reciprocating engine as determined by the engine indicator is designated as the **indicated power**, the rate of energy dissipation by mechanical friction between the piston face and the engine shaft is known as the **friction power** and the net rate of work delivery through the shaft is known as the **shaft** or **brake power**. In the internal-combustion engine there is commonly included in the friction power any work energy diverted for purposes of pumping the charge of air or fuel into the engine cylinder. The ratio between the shaft and indicated power is the **mechanical efficiency**. Its magnitude is determined by the mechanical nicety of the design and finish of the engine bearings and of the rubbing surfaces between the reciprocating piston and the cylinder, the adequacy of the lubrication of such bearings and surfaces, the relative

weight of the engine parts for a given power delivery, the pumping requirements, et cetera. In a high grade Otto type engine of relatively light weight, mechanical efficiencies of 90 per cent are obtained or exceeded, but this figure is much reduced in heavier and more cumbersome engines such as have been more typical of the Diesel type. It is a unique circumstance that the frictional torque of a given engine is nearly the same at any suitable load or speed at which the engine may operate.

Considerable attention has been given in the foregoing articles to the indicated thermal efficiencies obtained or obtainable in the several types of internal-combustion engine. It is to be noted in this connection that due to the practical difficulties in securing reliable indicator cards at the higher rotative speeds and with the high peak pressures which are characteristic of the Otto engine the thermal efficiency of internal-combustion engines is frequently expressed in terms of its **over-all thermal efficiency**. In principle,

$$\left. \begin{array}{l} \text{Brake or} \\ \text{over-all} \end{array} \right\} \text{thermal efficiency} = \text{Eff.}_{\text{indic.}} \times \text{Eff.}_{\text{mech.}} \quad (10)$$

The number of cycles performed per unit time depends on the rotative speed at which the engine may chance to be operating and on the engine character as regards whether it may operate on the two-stroke or the four-stroke cycle. It is recalled that *for an individual end of a cylinder* the number of cycles performed equals the number of revolutions of the crank-shaft if operation is on the two-stroke cycle, whereas it equals one half of the crank-shaft revolutions if operation is on the four-stroke cycle.⁶ The maximum value of the rotative speed of the crank-shaft is fixed by limitations associated with the inertia forces of reciprocating parts and the centrifugal forces of rotating parts and thus by the relative masses of those parts, by considerations of balance and vibration, by lubrication difficulties and also by a reduced effectiveness of the air or mixture induction through the intake system at high rates of fluid flow. In general the lighter forms of the Otto engine, such as those of the automobile and aircraft, may operate at maximum rotative speeds of 2500 revolutions per minute (r.p.m.) or over, whereas the heavier Diesel type engine has been characterized by speeds of 200 r.p.m. or less. However, higher speed Diesels are being developed. Frequently speed designation is made alternatively and to advantage in terms of the total distance traveled by the piston per unit time, or the **piston speed**. Evidently,

$$\text{Piston speed} = \text{r.p.m.} \times 2 \times \text{length of stroke.} \quad (11)$$

⁶ In the lighter and higher speed engine only the one end of the cylinder is commonly used, whereas the larger and lower speed engines are very frequently built double-acting.

Piston speed in excess of, say, 1200 ft. per minute is not usual except in particularly well designed automotive and aircraft engines, in which even double this speed may be exceeded.

The mass of air (or of mixture) inducted into the engine cylinder per intake stroke or per cycle depends in general on the piston displacement per stroke and on the density of the air (or mixture) after introduction into the cylinder. The density after introduction into the cylinder is influenced and determined by a considerable variety of items, the predominant of which are: (a) the density of the air outside of the engine intake system; (b) the degree by which the designer is able to reduce the fluid friction loss accompanying the flow of the air (or mixture) through the intake system, thereby minimizing the pressure depression in the cylinder at the end of the intake stroke; (c) the effectiveness of the engine exhaust system in minimizing the pressure at the end of the exhaust stroke and thus the mass of hot combustion products remaining in the cylinder at that point and tending to warm and rarify the incoming fluid; (d) the thermal efficiency of the engine cycle, with its consequent influence on the temperature and mass of the combustion products remaining in the cylinder; (e) the effectiveness of the engine cooling and the absence of major contact of the incoming fluid with any particularly hot surfaces such as exhaust valve heads or other hot spots; and (f) for an Otto engine using liquid fuel, the latent heat of vaporization of the fuel and the degree of vaporization which occurs at any point in the induction system, this by reason of the fact that the energy required for the vaporization may come primarily from the air supply and thus acts to reduce the ultimate temperature of the mixture and so increase its density. The so-called **supercharger**, which is frequently employed with the aircraft engine and may perhaps become a more common accessory of the automotive and marine engine, is simply a compact compressor which acts to increase the density of the air or mixture supply to the engine intake system.

The designer finds it convenient to evaluate the effectiveness of the intake stroke of the piston for air induction in terms of the so-called **volumetric efficiency**. This efficiency may be defined as

Volumetric efficiency

$$\begin{aligned}
 &= \frac{\text{volume of "free air" entering cylinder per intake stroke}}{\text{piston displacement per stroke}} \\
 &= \frac{\text{mass of air entering per intake stroke or cycle } (M_a)}{\text{piston displacement per stroke} \times \text{density}} \\
 &\quad \text{of air supplied to engine intake system}
 \end{aligned}
 \tag{12}$$

For later use in equation 9 let this relation be rearranged in the form,

$$\begin{aligned} &\text{Mass of air entering per intake stroke or cycle } (M_a) \\ &= \text{piston displacement per stroke} \times \text{density of external air} \\ &\quad \text{supplied} \times \text{volumetric efficiency,} \end{aligned}$$

or, in symbols,

$$M_a = L \times A \times D_{\text{air}} \times E_{\text{vol.}}, \quad (12a)$$

where

$$\begin{aligned} L &= \text{length of engine stroke, feet;} \\ A &= \text{area of engine piston, square feet;} \\ D_{\text{air}} &= \text{density of external air as supplied, pounds per cubic foot;} \\ E_{\text{vol.}} &= \text{volumetric efficiency.} \end{aligned}$$

A well-designed four-cycle engine will exhibit volumetric efficiencies up to about 90 per cent, but this figure is susceptible to marked reduction in a two-cycle engine or in any engine if the intake system is poorly formed or design is otherwise poor. Reduction of volumetric efficiency by throttling in the intake line is a conventional method of controlling the power output of an Otto cycle type of engine. The maximum power capacity obtainable with an Otto engine is rather more influenced by the attainable volumetric efficiency than is the case with a Diesel engine, due to the fact that in the former the mass of air inducted rather closely controls the mass of fuel and thus the amount of energy supplied per cycle. It is recalled that in the Diesel engine the mass of fuel injected per cycle is in general rather less than that for which air is available in the cylinder.

The heating value of the fuel (Q_f) and the fuel-air ratio (M_f/M_a) have been given attention in the discussions of combustion and in the foregoing articles of this chapter. Observe that the product of the two, $Q_f \times \frac{M_f}{M_a}$, is the energy furnished per pound of air supplied to the engine. It was indicated in Art. 158 that with the ideal fuel-air mixture the energy released shows a uniquely uniform magnitude of about 1200 B.t.u. (lower heating value) per pound of air for a considerable variety of fuels. The large influence of the relative energy release per pound of air on the obtainable thermal efficiency of the conventional Diesel cycle has also been pointed out in the foregoing articles.

All items involved in equation 9 and influencing the power capacity of the internal-combustion engine have thus been considered individually. It is desirable to incorporate the items of equation 12a in equation 9 to give a correspondingly comprehensive expression for the items

which control power capacity. Thus:

$$\text{S.hp.} = \frac{J \left[\left(Q_f \frac{M_f}{M_a} \right) \times D_{\text{air}} \times L \times A \times N \times E_{\text{vol.}} \times E_{\text{indic.}} \times E_{\text{mech.}} \right]}{33,000} \quad (9a)$$

Quite as with the reciprocating steam engine the **mean effective pressure** (Art. 135) is an outstandingly useful index of the relative effectiveness with which a given cylinder or given piston displacement is being utilized for its main objective of work energy delivery. Due to the difficulty of measuring the indicated power of an internal-combustion engine, particularly when operating at high speed, attention is more commonly given to the shaft (or "brake") m.e.p. Recall from the above reference (Art. 135, equations 8 and 9) that

$$\frac{\text{Shaft m.e.p., lb. per sq. ft.}}{\text{lb. per sq. ft.}} = (a) \frac{\text{shaft-work per cycle, ft.-lb.}}{\text{piston displacement, cu. ft. per stroke}} \quad (13)$$

$$\text{or} = (b) \frac{\text{shaft hp.} \times 33,000}{L \times A \times N}. \quad (13a)$$

An alternative and useful relation expressing the shaft m.e.p. obtainable, in terms of the fundamental items which control it, is secured by correlating equations 13a and 9a:

$$\frac{\text{Shaft m.e.p., lb. per sq. ft.}}{\text{lb. per sq. ft.}} = J \left[\left(Q_f \frac{M_f}{M_a} \right) \times D_{\text{air}} \times E_{\text{vol.}} \times E_{\text{indic.}} \times E_{\text{mech.}} \right]. \quad (14)$$

The indicated m.e.p. is expressed in the same manner except for the omission of the mechanical efficiency term.

Equation 13 may also be reinterpreted from the useful viewpoint that

$$\begin{aligned} \frac{\text{Necessary piston displacement, cu. ft. per stroke}}{\text{cu. ft. per stroke}} &= \frac{\text{work per cycle, ft.-lb.}}{\text{m.e.p., lb. per sq. ft.}} \\ &= \frac{33,000 \times \text{S.hp.}}{\text{S.m.e.p.} \times N} \end{aligned} \quad (13b)$$

In the modern high grade Otto cycle a shaft m.e.p. of 125 lb. per sq. in. (18,000 lb. per sq. ft.) and more is obtainable and at rotative speeds of 2000 r.p.m. or more. In contrast, due among other factors to the thermal efficiency limitations which accompany high energy supplies per pound of air (as indicated in Art. 166), the shaft m.e.p. obtained with the conventional Diesel cycle engine commonly does not exceed about 70 lb. per sq. in. (10,000 lb. per sq. ft.). Also, pending advances in the ability to so pulverize and disseminate the fuel jet that adequately rapid combustion might be obtained, the rotative speed of the Diesel engine has in general not exceeded some 300 to 400 r.p.m. and com-

monly has been much less. Considering these features in connection with equation 13*b*, one realizes the cause of the relatively large physical size and weight which have been characteristic of the Diesel engine. However, trends in the way of Dual-combustion (Art. 166) and advances in the art of jet formation are causing rapid increase in the efficiently attainable pressures and the rotative speeds of the modern Diesel engine, with correspondingly great reductions in its size and weight per horsepower of capacity.

Example 6. Compute by the use of equation 13 the indicated m.e.p. (lb per sq. in.) for the Air Standard Otto and Diesel cycles of examples 1 and 3 respectively.
(122; 125 lb. per sq. in.)

Example 7. The following test data apply to a 5-in. (bore) $\times$ 7-in. (stroke), twelve-cylinder, four-stroke Otto cycle engine operating at 1700 r.p.m.:

Torque delivered, 1300 lb.-ft.;	Atmospheric pressure, 14.7 lb. per sq. in.
Atmospheric temperature, 70° F.;	Air intake per minute, 52.0 lb.
Gasoline per hour, 230 lb.;	Compression ratio, 5.5.
Heating value of fuel: higher, 20,500 B.t.u. per lb.; lower, 19,200.	
Ideal air-fuel ratio, 15.1—1.	

Find: (a) volumetric efficiency of engine (including carburetor); (0.86)
 (b) brake or shaft horsepower; (421)
 (c) specific fuel consumption, pound per shaft horsepower-hour; (0.55)
 (d) over-all thermal efficiency of engine (based on higher value); (0.23)
 (e) actual air-fuel ratio and percentage of ideal; (13.6/1)
 (f) shaft m.e.p. developed, lb. per sq. in., computed by equation 13 from the above cylinder dimensions, engine power and r.p.m. and checked from the above results by equation 14. (119)
 (g) engine efficiency relative to the Goodenough and Baker ideal efficiencies of Fig. 104. (0.70)

Example 8. For the Diesel engine of example 4 estimate the shaft m.e.p., presuming a volumetric efficiency of 0.85.

168. T-S Analysis of the Internal-Combustion Engine Cycles. — In connection with the several types of internal-combustion engine it is desirable also to give attention to the fundamental thermodynamic causes of the influence of compression ratio, expansion ratio, et cetera. The same causes further account for the generally higher efficiency of the internal-combustion engine relative to that of the steam power plant.

It has frequently and emphatically been indicated in the foregoing that the temperature of energy reception by the working fluid and the temperature of unavailable energy rejection from the fluid constitute controlling factors in determining the ideally obtainable as well as the actual efficiency of any temperature engine cycle. Recall further that the temperature-entropy (*T-S*) diagram offers an outstandingly useful graphical device for portraying jointly the state changes of a fluid and

the energy quantities passing to or departing from the fluid (other than as shaft-work) during any mechanically reversible state change. In Fig. 111 there is shown a typical T - S diagram for a gas, with several representative constant volume and constant pressure contour lines (see also Figs. 31 and 32, in the latter of which the ordinate was for convenience plotted to logarithmic instead of linear scale). The diagram is to approximate scale with air as the fluid, with entropy values *per pound* and with respect to 14.7 lb. and 32° F. as a reference state. Recall

(Art. 55, Eq. 3c) that the instantaneous slope of a state change line on these coördinates, or dT/dS , equals T/c , where c is the specific heat.

In the figure there are also outlined two conventional Air Standard Otto cycles, the one designated as $bcdefb$ having the relatively high compression ratio of 12 and the one designated as $bc'd'e'f'b$ having the more representative Otto cycle compression ratio of 5. The compression phases of the two cycles are represented respectively by the lines bc and bc' . The energy supplied during the constant volume energy reception phases (cd or $c'd'$) of the two cycles is represented respectively by the areas $gcdh$ and $gc'd'i$. For convenience in comparisons equal energy quantities (1200 B.t.u.

per lb.) are presumed to be received in each instance, so that the above areas are equal. The isentropic expansions are denoted by lines de or $d'e'$. The mechanically irreversible state change during the escape of the gases from the engine cylinder is represented by lines ef or $e'f'$ and thus the respective states of the exhaust gases at their departure from the engine are represented by points f or f' . The energy discarded from the cycle would be measured by the energy departure as heat from the gases in recooling at constant (atmospheric) pressure to the original supply temperature and therefore is measured respectively by areas $gbfj$ and $gbf'k$. The excess discard in the cycle

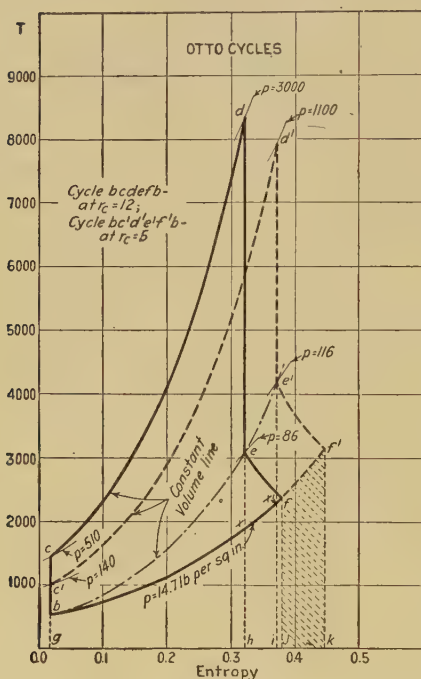


FIG. 111

of lower compression ratio is represented by area $jff'k$. It will be recalled (Art. 164, equation 3a et priori) that these energy quantities are equal to and may be regarded as equivalent to the energy which would be discarded in a *constant volume* cooling from e (or e') to b , or thus might alternatively be measured by the respective areas $gbeh$ and $gbe'i$.⁷

By the use of equation 7 (Art. 165) or by reference to Figs. 104 or 110 it appears that the cycle efficiency at compression ratios of 12 and 5 is respectively 0.63 and 0.475. From inspection of Fig. 111 it becomes directly evident that fundamentally the benefit of the increased compression ratio is due jointly to the higher average temperature level of the energy supply phase cd , with respect to the average temperature of $c'd'$, and the lower average temperature level of the energy rejection phase fb relative to that of $f'b$. Recall that equal energy quantities were supplied in both cases.

In Fig. 112a are shown for purposes of comparison an Otto cycle and a conventional Diesel cycle, both Air Standard, both with the same compression ratio (12) and both with the same

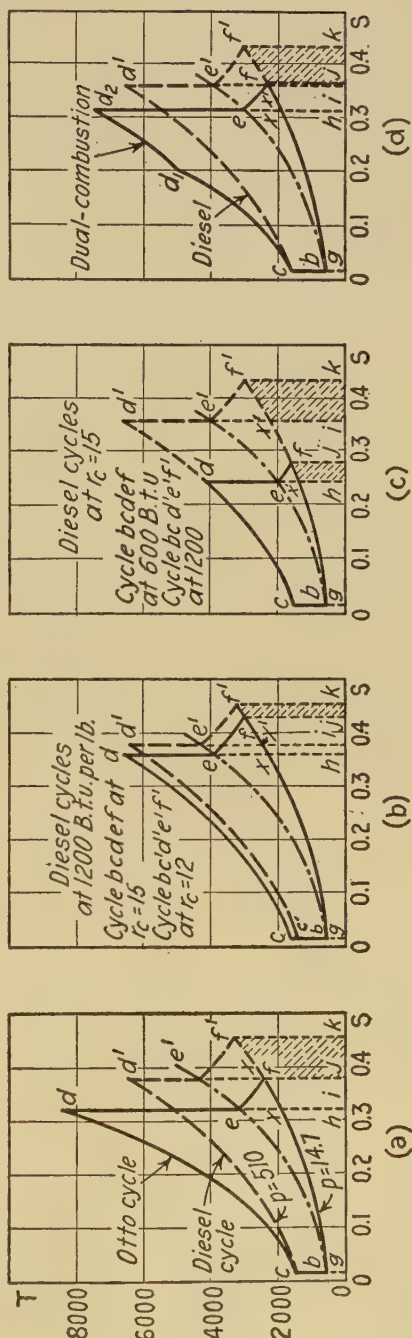


FIG. 112

⁷ The properties at state f are determinable from the equality of $c_v(T_e - T_b)$ and $c_p(T_f - T_b)$, from which $T_f = T_b + (T_e - T_b)/k$.

energy supply per pound of air (1200 B.t.u.). The basic thermodynamic reasons for the lesser efficiency of the Diesel cycle *at equal compression ratio and equal energy supply* becomes obvious from the figure. The excess energy discard from the Diesel cycle is represented by the area $jff'k$. In diagram (b) appear two Diesel cycles with equal energy supplies per pound (1200 B.t.u.) but with compression ratios of 12 and 15. The excess energy discard from the cycle of lower compression ratio is represented again by area $jff'k$. In diagram (c) appear two Diesel cycles, both with a 15 compression ratio but with energy supplies of 1200 and 600 B.t.u. per lb. In the comparison of these it develops that the major influence is the lower degree of irreversibility and entropy increase in the cycle of lesser energy supply and greater expansion ratio. In this diagram, as in each of the other diagrams, the areas $hxfj$ and $ix'f'k$ represent the increments of unavailable energy which are occasioned by the failure to expand the fluid reversibly to the (atmospheric) pressure at which it is ejected (see also Art. 55 and compare with Fig. 78 of Art. 136). The disproportionately larger loss due to the irreversible phase ($e'f'$) in the cycle of greater energy supply is evident.

In diagram (d) are represented a Diesel cycle and a Dual-combustion cycle, each with the same compression ratio of 15 and each with the same energy supply of 1200 B.t.u. per lb., but with the supply divided equally between the constant volume and constant pressure phases (cd_1 and

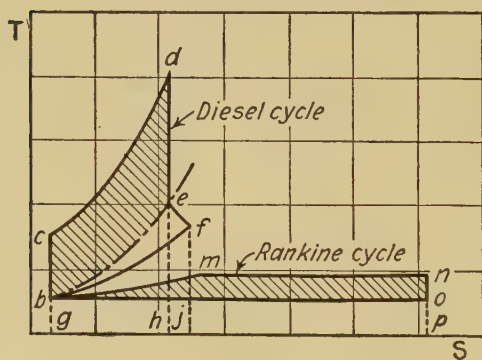


FIG. 113

d_1d_2) in the Dual-combustion cycle. The relatively more efficient cycle may be readily detected and its higher efficiency may again be seen to be due intrinsically to its higher average temperature of energy reception and lower average temperature of energy rejection. The excess energy discard in the Diesel cycle is represented by the area $jff'k$.

By way of indicating graphically the thermodynamic reasons for a generally superior efficiency for the internal-combustion engine relative to that of the steam power plant there is shown on T - S coördinates in Fig. 113 an Air Standard Diesel cycle and a representative complete expansion Rankine cycle. In the figure the entropy is that of the energy itself rather than that per pound of any particular fluid, and

also for purposes of comparison the energies supplied to each cycle have been taken to be the same. Thus areas $gcdh$ and $gbmnp$, which measure the energy supply in the respective cycles, are equal. Areas $bcdeb$ and $bmnob$ measure correspondingly the energy ideally deliverable as shaft-work in the two cycles. The relative superiority of the internal-combustion cycle is evident. However, in connection with this comparison it is significant to observe the two inherently diverse handicaps and advantages respectively suffered and enjoyed by the two cycles.

The handicap upon the steam cycle is the thermally irreversible transfer of energy from the high temperature combustion products in the furnace whereby the energy is received by the steam at a quite moderate temperature. The advantage enjoyed by the vapor cycle lies in the feature that the average temperature of energy rejection may be brought virtually to the *atmospheric temperature* by reason of the fact that the vapor may be expanded to and *condensed* at that temperature, and thus the liquid may be repumped to the upper pressure of the cycle at the expense of a relatively small feed-pump work expenditure. In contrast with these features of the steam or vapor cycle, by using the combustion products directly in the engine the internal-combustion engine avoids occasion for the thermal irreversibility in the furnace and boiler of the steam plant, with the consequent marked advantage of a very high average temperature of energy reception. However, due in a sense to the impossibility of condensing the gaseous combustion products, those products may be expanded in the engine only to the *atmospheric pressure* and must leave at a temperature which still is quite high, whereby the average temperature of energy rejection in the internal-combustion engine is unfortunately high. Plants have been developed in practice in which a steam plant is activated by the otherwise discarded energy from an internal-combustion engine, employing both the energy in the exhaust gases and in the cooling water of the engine. Such plants operate with very high over-all thermal efficiency but in general have been commercially impracticable due to high first cost and various operating difficulties.

169. Summary. — The internal-combustion engine constitutes the second of the two general types of devices available for transforming to work energy a portion of the stored chemical energy in the fuels. It does so more directly than does the steam plant by so confining the fuel-air mixture within a combustion chamber that high pressure is either produced or maintained therein, whereby a shaft may be motivated through a suitable engine mechanism. The practical present-day mechanism is a reciprocating engine.

The primary phases of the internal-combustion engine cycle are: (a) the induction of air or an air-fuel mixture into the engine cylinder, (b) the compression of this fluid within the cylinder, (c) the combustion of the fuel, (d) the expansion of the high temperature combustion products, and (e) the departure of the products from the cylinder. The motivation of the engine piston results from the superior pressure which exists within the cylinder during the combustion and expansion phases of the cycle. The characteristic feature of the Otto class of engine is the compression and combustion of a previously prepared mixture of air with a gaseous or volatile fuel, whereby the combustion may proceed with such rapidity that the volume of the fluid may be caused to remain virtually constant during the combustion. The characteristic feature of the conventional Diesel class of engine is the compression of air only but to such a pressure and temperature that upon a progressive injection of a suitably atomized jet of only moderately volatile liquid fuel a progressive combustion may occur and maintain a virtually constant pressure during the first part of the power stroke of the piston. The five primary phases of the engine cycle may be carried out in the four (intake, compression, expansion and exhaust) strokes of the four-cycle engine or, by the provision of a separate pump for positive delivery of the air (or mixture) to the cylinder, the cycle may be effectively carried out in the two piston strokes of the two-cycle engine.

The basic energy equation of the internal-combustion engine is, in energy quantities per pound of fuel,

$$W_{\text{out}}/J = C - M(H_{p,2} - H_{m,1} + C_p) - Q_{\text{out}}. \quad (1)$$

For the idealized conditions of adiabatic processes and frictionless reversible inflow and exit of the fluid the equation may be shown to become

$$(W_{\text{out}}/J)_{\text{ideal}} = Q_f - M(E_{p,e} - E_{p,1}) - MC_p. \quad (3)$$

It follows that

$$\text{Thermal efficiency, ideal} = 1 - \frac{(E_{p,e} - E_{p,1}) + C_p}{Q_f/M}. \quad (4)$$

Accurate computation of the ideal efficiency of any given internal-combustion engine cycle upon the basis of equation 4 becomes quite difficult indeed when due attention is given to the exact specific heat and manner of specific heat variation for the fuel mixture or combustion products and to the influence of chemical equilibrium in affecting the temperature and possible incompleteness of combustion of the products at the point of their release. However, it is found that an analysis of the

several types of engine cycles upon a simpler, although quite artificial, basis known as the Air Standard analysis affords valuable evidence of certain factors which outstandingly influence the efficiency. The Air Standard analysis presumes air of constant specific heat to be conducted through a cycle which is parallel to that of the actual internal-combustion engine. Concretely the hypothetical cycle is one in which the air is compressed, the temperature of the air is caused to rise by supplying energy as heat from some external source, the air is expanded and, to close the cycle, the air is recooled (at constant volume) to its original state by the departure of energy as heat to some external receiver.

An Air Standard analysis of the Otto cycle would indicate that its thermal efficiency depends only upon the compression ratio of the cycle and increases with increase in compression ratio in accordance with the relation

$$\text{Thermal efficiency, Air Standard Otto cycle} = 1 - \frac{1}{(r_c)^{0.4}}. \quad (7)$$

Although more exact estimates of ideal performance as well as actual engine tests show that efficiency is likewise influenced by the proportions of the fuel-air mixture, with improved efficiency at mixtures with some excess of air, the evidence of the Air Standard analysis is well corroborated with respect to the effect of the compression ratio.

For the conventional Diesel cycle with constant pressure combustion the Air Standard analysis would indicate that the cycle efficiency is influenced jointly by the compression ratio and the expansion ratio in accordance with the relation

$$\begin{array}{l} \text{Thermal efficiency, Air Standard} \\ \text{Diesel cycle} \end{array} = 1 - \frac{1}{(r_c)^{0.4}} \frac{(r_c/r_e)^{1.4} - 1}{1.4 (r_c/r_e - 1)}. \quad (8a)$$

The Air Standard efficiency of the Diesel cycle is increased by increasing compression but at a given compression ratio it is less than the efficiency of the Otto cycle and departs progressively therefrom with decreasing values of the expansion ratio. The expansion ratio is in effect an indirect index of the amount of energy supplied per pound of air and thus the Air Standard analysis would indicate that at a given compression ratio the Diesel efficiency is reduced progressively with increased energy supply. All of this evidence is well corroborated by more exact estimates of ideal performance and by actual engine tests, except that the influence of large energy supply is even more marked and more serious than would be anticipated from the Air Standard analysis. Like analysis of the Dual-combustion cycle, in which a por-

tion of the energy is supplied at constant volume and the remainder at constant pressure, indicate a much less detrimental effect of increasing energy supply per pound of air.

The power output of a given engine cylinder is determined by the amount of energy supplied per pound of air introduced, by the mass of air introduced per cycle, by the number of cycles performed per unit time, by the thermal efficiency with which the energy supply is transformed to work at the piston face, and by the mechanical efficiency of the engine. In the Diesel cycle the influence of the relative amount of energy supplied per pound of air on the indicated thermal efficiency of the cycle controls outstandingly the amount which may profitably be supplied. The mass of air inducted per cycle depends, for a given piston displacement, on the volumetric efficiency of the induction. The mean effective pressure obtained with the engine is a valuable index of the effectiveness with which a given piston displacement is being utilized for power production.

The T - S diagram as usual serves advantageously for graphic representation of the engine cycle and for indicating thereby the fundamental influence of the average temperatures of energy reception and energy rejection on the thermal efficiency of the cycle.

170. Review Questions and Topics. —

1. Distinguish between the steam plant and the internal-combustion engine with reference to the general characteristics of the processes of energy release in the two; compare the particular adaptabilities of the two.

2. Describe the function and character of the several phases which constitute the internal-combustion engine cycle, indicating that feature of the general cycle which would characterize an Otto cycle and that characterizing the Diesel cycle; describe or distinguish between the four-stroke and two-stroke methods of carrying out the cycle.

3. (a) Write the general steady-flow energy equation as applied to the internal-combustion engine.

(b) Indicate the conditions of operation of an ideal engine cycle.

(c) Write and interpret the energy equation expressing the work output ideally obtainable per pound of fuel supplied and also the expression for the ideal thermal efficiency of the engine. May quantitative evaluation of these relations be made easily when actual fuels and fuel mixtures are considered, and if not, why not?

(d) Describe the so-called Air Standard method of consideration or analysis of the internal-combustion engine cycle, and discuss briefly both its limitations and its utility.

4. (a) Define compression ratio and expansion ratio.

(b) Quote the relation expressing the thermal efficiency of the Air Standard Otto cycle and discuss with reference to the evidence which it offers and the validity of that evidence. Interpret the evidence of an ideal Otto cycle analysis made with due attention to the actual characteristics of the fuel mixture and the combustion.

(c) Indicate and discuss the compression ratios and mixture proportions which

are practicable and suitable for the Otto cycle engine; discuss briefly the additional conditions which influence actual engine efficiency.

5. (a) Quote the relation expressing the thermal efficiency of the Air Standard Diesel cycle and discuss the expression with reference to the evidence which it offers and the validity of that evidence. Interpret the evidence of an ideal Diesel cycle analysis made with due attention to actual characteristics of the fuel mixture and the combustion.

(b) Describe the Dual-combustion cycle and indicate the advantages of such combustion.

6. (a) List the primary items which control the power output of a given engine cylinder and quote the expression which relates these items. Discuss the several individual items.

(b) Define volumetric efficiency and indicate the conditions which influence the volumetric efficiency of an engine.

(c) Define mean effective pressure. Quote and interpret the relation by which the obtainable m.e.p. may be predicted from estimates on the operating characteristics of an engine.

(d) Compare and justify the relative size and weight which have been representative of the Otto engine and the Diesel engine.

7. Depict the Air Standard Otto and Diesel cycles on T - S coördinates and indicate the general thermodynamic causes which predominantly influence the relative thermal efficiencies of those cycles. Do likewise for the internal-combustion engine and the steam power plant.

Symbols and Abbreviations, Chapter XVI

a = a subscript referring to air or designating atmospheric conditions.

C = chemical energy in fuel and releasable by combustion, B.t.u. per pound of fuel.

C_p = chemical energy remaining in incompletely burned combustion products, B.t.u. per pound of products.

c = specific heat.

D = density, pounds per cubic foot.

E = internal (molecular) energy, B.t.u. per lb.

f = a subscript referring to fuel.

H = enthalpy ($E + PV/J$), B.t.u. per lb.

J = Joule's equivalent, foot-pounds per B.t.u. (= 778).

k = ratio of specific heats, c_p/c_v .

M = mass of mixture (fuel plus air) or of products, pounds per pound of fuel.

m = a subscript referring to the fuel-air mixture.

m.e.p. = mean effective pressure, pounds per square foot.

N = number of complete cycles performed per minute.

P = pressure, pound per square foot absolute, or a subscript denoting constancy of pressure.

p = a subscript referring to the products of a combustion.

Q = energy in transition as heat (by radiation or conduction), B.t.u. per pound of fuel supplied.

Q_f = calorific or heating value of a fuel (by constant volume calorimeter), B.t.u. per lb.

r_c = ratio of compression (by volume).

r_e = ratio of expansion (by volume).

S.hp. = shaft horsepower.

T = temperature, degrees Rankine (Fahrenheit absolute).

V = specific volume, cubic foot per pound, or a subscript denoting constancy of volume.

W = shaft-work, foot-pounds per pound.

CHAPTER XVII

COMPRESSORS AND BLOWERS

In the foregoing portions of Part IV the concern has been primarily with processes and machines in which a fluid to which energy has been imparted at a higher temperature and pressure is permitted to expand to a lower temperature and pressure, usually for the purpose of transforming into shaft-work a portion of the energy carried by the fluid. In this chapter, and also in the one which follows, attention is given to the somewhat opposite processes in which energy is supplied as shaft-work to a machine for the purpose of effecting the delivery of a fluid from a region of lower pressure to one of relatively higher pressure. Invariably there will be more or less rise in the temperature of the fluid during and as a result of the process, but in general the primary objective is the pressure increase. The device in which such a process is carried out is known in general as a **compressor**.

171. Compressor Classifications. — In the following the term *compressor* will be employed in a broad generic sense to denote *any* device whatsoever by which a fluid may be delivered from a region of lower pressure to one of higher pressure. However, it is to be recognized that the term frequently is employed in a more limited meaning and that other terms are commonly used to designate various particular types of machines.

One general but necessarily flexible classification of the types of machines used for the purpose of compression is made upon the basis of the pressure range involved. Thus for the condition of an intake pressure which is well below atmospheric and a discharge pressure which may be about atmospheric the machine used would commonly be called a **pump**. The air or vacuum pump of the steam condenser is illustrative. With intake and discharge pressure which differ by only a few ounces or pounds per square inch and each of which is about atmospheric a **fan** or **blower** is employed. The ordinary ventilating fan or the metallurgical furnace blower are examples. For pressure ranges of tens to hundreds of pounds per square inch the general term **compressor** is customarily employed. Typical circumstances under which such pressure ranges are encountered are those of the supplying of air to pneumatic tools or to the nozzles of the air-injection type of oil engine, of the compression of vapors in refrigerating machines, et cetera.

A second classification of the types of machines is based upon their manner of operation. The outstanding types are:

(a) **Positive displacement** type, which is in turn subdivided into the reciprocating piston type (Fig. 114) and the rotating impeller type (Fig. 115). The former is generally more applicable where relatively moderate volume rates but higher pressure ranges are involved. The cylinder of the compressor is commonly cooled by a water-jacket. For pressure ranges above about 75 lb. per sq. in. the compression is com-

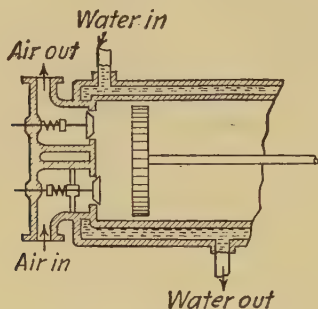


FIG. 114

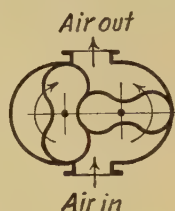


FIG. 115

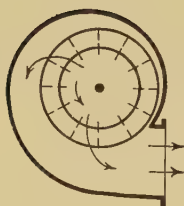


FIG. 116

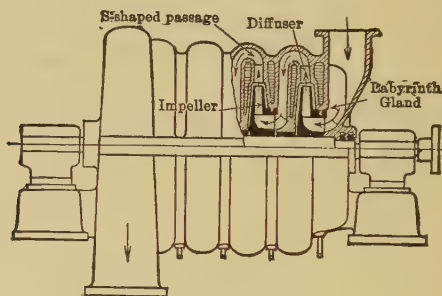


FIG. 117

monly done in several stages, and air coolers are provided between stages. The rotating impeller type which is illustrated is an ingenious arrangement consisting of a casing and two inter-meshing rotors which act to entrap, compress, and deliver the gas. This type is better adapted for use with pressure ranges not exceeding some 10 lb. per sq. in. and for moderate volume rates. It is usually not water jacketed.

(b) **Centrifugal** type, which is subdivided into the fan class (Fig. 116) and the compressor class (Fig. 117). Both act initially to impart a kinetic energy increase to the fluid by centrifugal actions in a rapidly rotating impeller. By the use of diffuser channels in the discharge

stream (see Art. 117) a partial deceleration of the fluid is effected, which serves to assist in the delivery of the fluid against the discharge pressure. Both classes are more particularly applicable where relatively larger volume rates are required and both enjoy the characteristic advantage of relatively low weight and space per unit volume rate of delivery. Fans are usually single-stage, unjacketed, operate through pressure ranges not exceeding perhaps 6 in. of water ($\frac{1}{4}$ lb. per sq. in.) and deliver the gas at considerable velocity. Centrifugal compressors operate at high speeds (3600 r.p.m. and up), are usually multi-stage, often are provided with inter-stage cooling arrangements and may operate through pressure ranges up to some 5 or 10 lb. per sq. in. per stage.

In the several immediately following articles (Arts. 172–176) analyses are made of the general compression process and of several idealized processes. It is to be remarked that these analyses pertain to both the positive displacement and centrifugal types of compressor.

172. Compressor Energy Equations and General Energy Analysis. — As the compressor is actually or in effect a steady-flow device the familiar steady-flow energy equation may be employed to advantage. Thus (Art. 31),

$$\frac{U_1^2}{64.34} + JE_1 + P_1V_1 + W_{\text{in}} = \frac{U_2^2}{64.34} + JE_2 + P_2V_2 + JQ_{\text{out}}, \quad \text{or}$$

$$\frac{U_1^2}{64.34} + JH_1 + W_{\text{in}} = \frac{U_2^2}{64.34} + JH_2 + JQ_{\text{out}},$$

where as usual the subscripts 1 and 2 refer to conditions respectively at the intake and discharge regions of the compressor. Under the usual conditions of compressor operation any energy transition by conduction or radiation (as heat) is an outward one, so that the single Q_{out} term is introduced in these relations.

Recalling that for actuating the compressor shaft-work input is necessary and anticipating that methods for expressing that work will be desired it is evident that the equation may be rearranged with advantage in the form:

$$W_{\text{in}}, \text{ ft-lb. per lb. of fluid} = \frac{U_2^2 - U_1^2}{64.34} + J(H_2 - H_1) + JQ_{\text{out}}. \quad (1)$$

For a gas and under conditions of negligible specific heat variation (see also Arts. 77 and 79) the equation becomes

$$\begin{aligned} W_{\text{in}}, \text{ ft-lb. per lb. of fluid} &= \frac{U_2^2 - U_1^2}{64.34} + Jc_p(T_2 - T_1) + JQ_{\text{out}}, \quad \text{or} \quad (1a) \\ &= \frac{U_2^2 - U_1^2}{64.34} + \frac{k}{k-1}(P_2V_2 - P_1V_1) + JQ_{\text{out}}. \quad (1b) \end{aligned}$$

Although equation 1 and its modifications will be of distinct utility in subsequent analyses it does not in itself afford adequate information whereby one might predict the work energy requirements for actual compression or might establish a standard performance for an ideal compression. Its inadequacy lies primarily in its failure to indicate the existence or character of any mutual relation between the heat energy emission (Q_{out}) and the increase of enthalpy of the fluid during its passage through the compressor. In fact a superficial scrutiny of the equation might readily lead to the erroneous conclusion that an increase of the heat emission would act to increase the requisite work input. We shall discover shortly that on the contrary the effect of energy departure as heat is to so reduce the enthalpy increase as actually to decrease the necessary work.

To validate this important conclusion it is convenient to refer to that modified form of the steady-flow equation which was developed in Art. 36*b*, where it was shown (equation 9*b*) that for a *mechanically reversible* process the composite **mechanical effects** are uniquely evaluated by $-\int_1^2 P dV$, or

$$\frac{U_1^2 - U_2^2}{64.34} + (P_1 V_1 - P_2 V_2) + W_{\text{in}} = -\int_1^2 P dV, \quad \text{or}$$

$$W_{\text{in}} = \frac{U_2^2 - U_1^2}{64.34} + \left(P_2 V_2 + \int_2^1 P dV - P_1 V_1 \right).^1 \quad (2)$$

To interpret this relation let reference be made to the pressure-volume diagram of Fig. 118. In that diagram point *c* represents the final state,

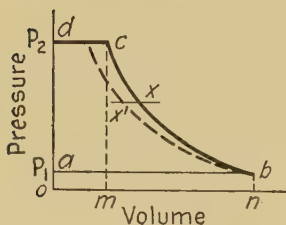


FIG. 118

or state 2, of the fluid leaving the compressor, and thus the area *odcm* measures the product $P_2 V_2$, which is the flow-work required for delivery of the fluid from the compressor. Similarly the point *b* represents the initial state 1 of the fluid and the area *oabn* measures the product $P_1 V_1$, which is the flow-work energy supplied in the passage of the fluid to the compressor. Finally, the line *bc* represents the state change of the

fluid during the compression and the area *mcbn* measures the $-\int_1^2 P dV$

¹Consideration of the idealized condition of mechanical reversibility is legitimate as we are here concerned only with the influence of heat emission (Q_{out}) on the enthalpy increase (ΔH), and for that item any conclusion reached for the ideal condition will also hold for actual conditions.

or $+\int_2^1 P dV$, which is the work energy ideally required for effecting the actual compression within the compressor. Thus, aside from the work required for and going to increasing the kinetic energy, the net area $abcd$ measures the net shaft-work required for the induction, compression, and delivery of the fluid under conditions which are ideal in the feature of absence of fluid or mechanical friction, that is, mechanically reversible. Mathematically this area may be expressed alternatively as $\int_1^2 V dP$, whence we may write that

$$W_{\text{in, ft-lb. per lb. of fluid}} = \frac{U_2^2 - U_1^2}{64.34} + \int_1^2 V dP. \quad (2a)$$

Comparing this relation for mechanically reversible compression with the general relation of equation 1, it develops that

$$\int_1^2 V dP = J(H_2 - H_1 + Q_{\text{out}}). \quad (3)$$

From inspection of equation 2a and of Fig. 118 it is apparent that any condition which will act to further reduce the specific volume of the fluid during the compression state change, giving a state change line such as the dash line from b instead of the line bc , will act to reduce the work requirement. Considering a point at any pressure plane along the compression curve, such as point x , it is likewise evident that a volume decrease to a state such as x' might have been effected only by energy departure as heat prior to or upon reaching that pressure. Thus it must be concluded that energy departure as heat during the compression has the fortunate effect of reducing the requisite work supply. Considering this conclusion in connection with equation 1, or equation 3, it is evident that energy emission as heat (Q_{out}) *during compression* has the unique characteristic of reducing the enthalpy change (increase) by an amount which *exceeds* the amount of heat departing.

This significant conclusion needs be given due attention in arriving at a suitable basis for evaluating the ideally minimum shaft-work requirement for a compressor. In conformity with the conclusion one would correctly anticipate minimum work with the maximum practicable heat extraction. In the compression of a vapor a maximum conceivable extraction would be one which would cause the fluid to leave the compressor at the upper pressure in the liquid state. Such a condition may be regarded, however, as rather impracticable of actual attainment. In the compression of gases the usual operating conditions are such that the fluid is supplied to the compressor at or about at atmospheric tem-

perature. Under such a condition it is thermodynamically impracticable and uneconomical to attempt to obtain any lesser temperature during compression. As a result the accepted engineering practice is to regard an *isothermal* compression as the optimum or ideal for air (or gas) compressor operation. It is in fact unattainable even with the most effective water-jacketing but it is considered as an ideal standard with which actual compressor performance may properly be compared.

Example 1. The following test data were taken during operation of a given compressor:

Rate of air delivery — 1.25 lb. per second.

Entering air in intake line — 70° F. and atmospheric pressure.

Leaving air in discharge line — 250° F. and 100 lb per sq. in. gage.

Cooling water — 375 lb. per minute, entering at 60° F. and leaving at 69.5° F.

Pipe sizes — 8-in.-diameter intake and 3-in.-diameter discharge.

Compute the kinetic energy increase, the internal energy increase, and the enthalpy increase per pound of air passing the compressor; the flow-work supplied per pound on entry and that delivered on discharge; the energy emission as heat to the cooling water per pound of air. (17.4; 24,200; 33,800; 28,200; 37,800; 37,000.)

Estimate the shaft-work required per pound of air delivered and the horsepower required to drive the compressor, presuming that in addition to the energy passing as heat to the cooling water 5 per cent of the work input is dissipated by radiation from the bearings, et cetera. (74,500; 170 hp.)

By what per cent would the work estimate be in error as the result of a 0.5° F. error in determining the temperature rise of the water? (2.6 per cent.)

173. Isothermal Air Compression. — In establishing the ideal or standard performance for isothermal compression it is further and properly taken that all processes shall also be ideal in the feature of absence of any fluid or mechanical friction, that is, mechanically reversible.² For this condition the work ideally required per pound is readily evaluated by combining equation 2a with the characteristic equation of state of a gas (Art. 74):

$$PV = RT, \text{ or } \frac{PV}{T} = \text{the gas constant, } R,$$

² It is to be observed that, in establishing as a standard of compressor performance one in which maximum energy emission as heat occurs, no attention is given to the energy which might be carried out with the departing fluid if heat emission were reduced. This is justified by the practical circumstance that pressure rise is usually the sole useful objective, any temperature or enthalpy increase in the fluid commonly being of no practical utility as the corresponding energy would shortly be dissipated as heat to the atmosphere. An exception is the compressor of the refrigeration plant (Chapter XVIII), in which a temperature rise of the fluid from the cold region temperature to about atmospheric temperature is necessary in order to effect an energy discard from the fluid as heat to the atmosphere.

or rather with the special isothermal form of the equation of state:

$$(P_2 V_2)_T = (P_1 V_1)_T = (PV)_T.$$

Thus, employing the latter in connection with equation 2a,

$$\begin{aligned} W_{\text{in, reversible isothermal}} \\ \text{compression, ft-lb. per lb.} &= \frac{U_2^2 - U_1^2}{64.34} + \int_1^2 V dP \\ &= \frac{U_2^2 - U_1^2}{64.34} + \int_1^2 \frac{P_1 V_1}{P} dP \\ &= \frac{U_2^2 - U_1^2}{64.34} + P_1 V_1 \log_e \frac{P_2}{P_1}, \end{aligned} \quad (4)$$

$$\text{or} \quad = \frac{U_2^2 - U_1^2}{64.34} + RT_1 \log_e \frac{P_2}{P_1}. \quad (4a)$$

The kinetic energy term in these expressions is something of a nuisance and might be eliminated to advantage. It may of course be eliminated directly and with propriety when the pressure range is of such magnitude that the kinetic energy change will be relatively negligible. Such will in general be the case when the pressure rise exceeds about 10 per cent of the initial pressure, that is, when $P_2/P_1 > 1.1$. Thus it may be taken with a reasonable degree of confidence that in connection with the positive displacement type of compressor the kinetic energy term may invariably be omitted.

In the case of the fan or low pressure blower or compressor the kinetic energy item may easily be of such *relative* magnitude that it might not be omitted with propriety. However, it is to be observed that it may be accounted for automatically if, in connection with the test of a blower the performance of which is to be compared with the ideal, the measured test pressures and temperature are *impact* pressures and temperatures, that is, they are taken respectively with an impact tube and with a thermometer with bulb placed directly in the stream. The correctness of this procedure has been indicated in Art. 112 but it may again be stated that procedure on this schedule constitutes in effect the measurement of the pressure and temperature which the gas would have if flowing from the intake region or to the discharge region without appreciable velocity. Upon this basis the above equations may now be written without the kinetic energy term and in the correspondingly simpler form,

$$W_{\text{in, reversible isothermal}} \\ \text{compression, ft-lb. per lb.} = P_1 V_1 \log_e \frac{P_2}{P_1} \quad (4b)$$

$$= RT_1 \log_e \frac{P_2}{P_1}; \quad (4c)$$

in which T_1 , P_1 and P_2 are to be regarded as the *impact* temperature and pressures respectively, except as they are actually taken in regions of negligible velocity or as the operating pressure range is sufficiently great that the kinetic energy item in the energy equation is of negligible relative magnitude. In subsequent expressions for ideal work input the kinetic energy term will as a rule be omitted, but the several pressures and temperatures shall be interpreted as above if the kinetic energy terms can not be neglected with propriety.

Observing that the term $\log_e \frac{P_2}{P_1}$ equals $\int_1^2 \frac{dP}{P}$, it is to be noted that for the circumstance of a pressure range ΔP (or $P_2 - P_1$) which is relatively small, such as would be the general case with a fan, P might be regarded as approximately constant and $\log_e P_2/P_1$ would approximately equal $\Delta P/P$. By reference to a table of natural logarithms or to the log-log slide rule it will appear that the two differ by not more than about 0.5 per cent for values of $\Delta P/P$ up to about 0.01. As a consequence, for values of $(P_2 - P_1)/P_1$ which do not exceed about 1 per cent the logarithmic term of equation 4 *et sequi* may be replaced by the simpler term $(P_2 - P_1)/P_1$. Thus for use in this circumstance equation 4 might be rewritten in the form

$$W_{\text{in, ideal, if } (P_2 - P_1)/P_1 = \frac{U_2^2 - U_1^2}{64.34} + (P_2 - P_1)V_1. \quad (4d)$$

does not exceed 0.01

In analyzing the performance of fans and low pressure blowers, which usually do not operate through a pressure range in excess of about 1 per cent of P_1 , the several terms of equation 4d are frequently regarded as representing and measuring the *useful energy output* of the machine, in foot-pounds per pound of air delivered. Scrutinizing the several terms of the equation it is apparent that, if the state change of the air through the fan is so slight as to affect the specific volume negligibly, the term $(P_2 - P_1)V$ evaluates the net flow-work which is required for and goes to delivery of the air against the pressure differential, and the kinetic energy term measures the energy per pound going to acceleration of the air. Following hydraulic practice the two terms are very frequently designated as the *static head* and *velocity head* delivered by the fan. The energy aspect of the terms would seem to be the more significant one.

The power requirements for ideal compression evidently depend directly and jointly on the work required per pound of gas delivered, as evaluated for isothermal compression in equations 4 to 4d, and the mass rate of delivery (M') in pounds per unit time. Thus, writing for illustration expressions parallel to equations 4b, 4c and 4d,

$$\text{Hp. input, reversible isothermal compression} = \frac{M' P_1 V_1 \log_e P_2 / P_1}{550} = \frac{P_1 V_1' \log_e P_2 / P_1}{550}; \quad (5b)$$

$$\text{or} = \frac{M' R T_1 \log_e P_2 / P_1}{550}; \quad (5c)$$

$$\text{or} = \frac{M' [(P_2 - P_1) V_1 + (U_2^2 - U_1^2) / 64.34]}{550},$$

if $(P_2 - P_1)$ does not exceed $0.01 P_1$. $(5d)$

In these relations,

M' = pounds of gas delivered per second.

$V_1' = M' V_1$ = the volume of gas, in cubic feet per second, at the state under which it exists in the region from which the compressor suction is taken, that is, the **free air** taken in per second.

P and T = *impact* pressures and temperature except as the kinetic energy change is negligible or is otherwise accounted for (as it is in equation 5d).

For the low pressure fan or blower the horsepower as evaluated by equation 5d is also commonly regarded as its useful power output and is designated as the **air horsepower** of the fan.

In connection with the reversible isothermal compression of a gas it is interesting to observe that, since $\Delta H = c_p \Delta T$ and $\Delta T = 0$, it follows from equation 1a of Art. 172, that

$$W_{\text{in, reversible isothermal compression}} = \frac{U_2^2 - U_1^2}{64.34} + JQ_{\text{out}}.$$

This relation discloses the unique circumstance that, except for that portion of the work input which goes to increasing the kinetic energy of the gas, all energy input as work passes out wholly and concurrently as heat (Q_{out}) to the cooling medium.

Example 2. Compute the work ideally required for the isothermal compression and delivery of 1 lb. of air from a supply region at atmospheric pressure and 70° F. to a discharge region at a pressure of 100 lb. per sq. in. gage; also the horsepower required for 1000 cu. ft. of free air delivered per minute. (58,000 ft.-lb.; 131.5 hp.)

Example 3. Compute the work ideally required per pound of air for compression and delivery, the air delivered in pounds per second and cubic feet per minute and the air horsepower of a fan operating under the following test conditions:

Air intake region —

Pressure, atmospheric; temperature, 70° F.

Discharge duct —

Static pressure (above atmosphere), 3.50 in. water.

Impact pressure at point of mean velocity, 4.94 in. water.

Diameter, 12 in.

[For convenience note that, closely, 1 in. water = $(62.4/12 =)$ 5.2 lb. per sq. ft.]
(341 ft.-lb. per lb.: 4.74 lb. per sec.; 3780 c.f.m.; 2.94 hp.)

174. Adiabatic Compression, Single and Multi-stage. — It will be recalled that in those cyclic processes in which shaft-work output is to be obtained from a fluid of elevated temperature and pressure the reversible adiabatic expansion is one of outstanding significance. Also isentropic compression would be regarded as an ideal manner of returning the fluid, after expansion, to the upper temperature as in the temperature engine cycles the primary objective of the compression would be an advantageous or reversible return of the fluid from the lower to the upper *temperature* of the cycle. In distinction to this condition, in the isolated compressor the objective is only the delivery of the fluid against a superior *pressure*, and it has been shown in the foregoing that energy extraction as heat during compression and consequent reduction of the temperature rise is beneficial. Nevertheless, perhaps due to some confusion of thought but also in recognition of the fact that in the actual fan, blower, or centrifugal compressor no appreciably effective heat extraction may be accomplished during compression, a reversible adiabatic compression is frequently designated as a **second standard** of performance with which to compare actual compressor performance. Furthermore, due again to the practical circumstance that in any multi-stage compression a fairly effective recooling of the fluid may be effected by inter-coolers located between stages and a material work saving may thereby be secured, a **third standard** of performance is also employed in which isentropic compression is presumed in each individual stage but complete recooling of the gas to the original supply temperature prior to its entry to any succeeding stage. The shaft-work input ideally required for these two conditions of operation is considered in the following.

Complete Isentropic Compression. — By introducing the conventional T - P relationship for the reversible adiabatic (Art. 87, Eq. 14), that is,

$$\frac{T_2}{P_2^{(k-1)/k}} = \frac{T_1}{P_1^{(k-1)/k}} = \frac{T}{P^{(k-1)/k}},$$

into the energy relation of equation 1 (Art. 172), again omitting, however, the kinetic energy term upon the basis that if it is not negligible it may readily be reintroduced or it may be automatically accounted for by employing *impact* data, it develops that

$$\begin{aligned} W_{\text{in, reversible adiabatic}} &= J(H_2 - H_1)_S \\ \text{compression, ft-lb. per lb.} &= Jc_p(T_2 - T_1)_S \\ &= Jc_p T_1 [(P_2/P_1)^{(k-1)/k} - 1], \end{aligned} \quad (6)$$

$$\text{or} = \frac{k}{k-1} P_1 V_1 [(P_2/P_1)^{(k-1)/k} - 1]. \quad (6a)$$

The power required is as usual the product of the work required per pound times the mass-rate of delivery, or

$$\frac{\text{Hp. input, reversible}}{\text{adiabatic compression}} = \frac{M' J c_p T_1 [(P_2/P_1)^{(k-1)/k} - 1]}{550}, \quad (7)$$

$$\text{or} = \frac{\frac{k}{k-1} P_1 V'_1 [(P_2/P_1)^{(k-1)/k} - 1]}{550}, \quad (7a)$$

where M' = mass rate of delivery, pounds per second,

V'_1 = volumetric capacity of compressor, cubic feet of free air per second.

For the condition of operation through a pressure range such that P_2/P_1 does not greatly exceed unity an accurate evaluation of the term $[(P_2/P_1)^{(k-1)/k} - 1]$ in the above relations becomes bothersome. By performing a series expansion on the term by the binomial theorem (see also Art. 106, footnote 2, page 228) a simpler relation is obtainable which is adequately accurate for any value of $(P_2 - P_1)$ which does not exceed about 10 per cent of P_1 , or

$$W_{\text{in, reversible adiabatic compression, ft.-lb. per lb.}} = R T_1 \frac{P_2 - P_1}{P_1} \left(1 - \frac{1}{2k} \frac{P_2 - P_1}{P_1} \right), \quad (6b)$$

$$\text{or} = V_1 (P_2 - P_1) \left(1 - \frac{1}{2k} \frac{P_2 - P_1}{P_1} \right). \quad (6c)$$

Also,

$$\frac{\text{Hp. input, reversible}}{\text{adiabatic compression}} = \frac{M' R T_1 \frac{P_2 - P_1}{P_1} \left(1 - \frac{1}{2k} \frac{P_2 - P_1}{P_1} \right)}{550}, \quad (7b)$$

$$\text{or} = \frac{V'_1 (P_2 - P_1) \left(1 - \frac{1}{2k} \frac{P_2 - P_1}{P_1} \right)}{550}. \quad (7c)$$

Referring to equation 6c (or 7c), observe that the last term in the relation $\left(1 - \frac{1}{2k} \frac{P_2 - P_1}{P_1} \right)$ will become of relatively negligible magnitude when $(P_2 - P_1)/P_1$ does not exceed about 0.01. If under this circumstance the term is omitted the resulting expression is found to be the same as equation 4d, excepting that in the latter the kinetic energy term is included explicitly. It thus would appear that the ideal work or power requirements for compression and delivery with relatively so small a pressure range is virtually the same whether the compression is isothermal or isentropic. This condition is in fact the case, for the reason that the temperature or internal energy change accompanying even

isentropic compression through the very small pressure range is relatively so slight as to be negligible.

Isentropic Compression, with Complete Inter-cooling. — Letting P' represent the discharge pressure from the first stage and the intake pressure to the second stage of a two-stage compressor in which the air is recooled to the initial intake temperature by passage through an inter-cooler, the ideal shaft-work requirements in the two stages and thus for the entire compressor may be written directly by use of equation 6, or

$$\begin{aligned} W_{\text{in, two-stage isentropic compression with inter-cooling}} &= Jc_p T_1 [(P'/P_1)^{(k-1)/k} - 1] \\ &\quad + Jc_p T_1 [(P_2/P')^{(k-1)/k} - 1] \\ &= Jc_p T_1 [(P'/P_1)^{(k-1)/k} \\ &\quad + (P_2/P')^{(k-1)/k} - 2]. \end{aligned} \quad (8)$$

From this relation it is obvious that, for a given intake and final discharge pressure, variation of P' will vary the entire bracketed term and thus the work required. It would be found further by trial that the term would have a minimum value at one particular value of P' . By regarding the term as a dependent function of P' and by then employing the familiar mathematical device of setting the first derivative of the function equal to zero that value of P' may be found which will give the minimum value of the term. Performing the differentiation,

$$\begin{aligned} &\frac{d[(P'/P_1)^{(k-1)/k} + (P_2/P')^{(k-1)/k} - 2]}{dP'} \\ &= \frac{k-1}{k} \left(\frac{1}{P_1} \right)^{(k-1)/k} (P')^{-1/k} - \frac{k-1}{k} (P_2)^{(k-1)/k} (P')^{(1-2k)/k} = 0; \end{aligned}$$

or, solving for P' ,

$$\begin{aligned} P' &= (P_1 P_2)^{1/2}, \quad \text{or} \\ P'/P_1 &= P_2/P' = (P_2/P_1)^{1/2}. \end{aligned} \quad (9)$$

Introducing the latter relation into equation 8 *et priori* it develops that for minimum work

$$W_{\text{in, ideal two-stage isentropic compression}} = 2 Jc_p T_1 [(P_2/P_1)^{(k-1)/2k} - 1], \quad (10)$$

$$\text{or} = \frac{2k}{k-1} P_1 V_1 [(P_2/P_1)^{(k-1)/2k} - 1]. \quad (10a)$$

Considering equation 9 in connection with equation 8 it appears that for minimum work the inter-stage pressure is also such as to give an equal distribution of work as between the two stages.

It may be shown that for N stages of isentropic compression, with complete recooling between stages, the values of the proper intermediate pressures for securing minimum work and equal work distribution between stages will follow the schedule:

$$P' = [(P_1)^{(N-1)}(P_2)]^{1/N}; \quad P'' = [(P_1)^{(N-2)}(P_2)^2]^{1/N};$$

$$P''' = [(P_1)^{(N-3)}(P_2)^3]^{1/N}; \dots; P^{N-1} = [(P_1)(P_2)^{(N-1)}]^{1/N}.$$

The corresponding minimum work requirement for the entire compressor is

$$W_{in} = NJc_p T_1 [(P_2/P_1)^{(k-1)/Nk} - 1], \quad (10b)$$

$$\text{or} = \frac{Nk}{k-1} P_1 V_1 [(P_2/P_1)^{(k-1)/Nk} - 1]. \quad (10c)$$

Example 4. Compute the work ideally required for the complete isentropic compression and delivery of 1 lb. of air from a supply region at atmospheric pressure and 70° F. to a discharge region at a pressure of 100 lb. per sq. in. gage; also the horsepower required per 1000 cu. ft. of free air delivered per minute. Compare results with those of example 2 and discuss. (79,000 ft-lb.; 179 hp.)

Example 5. Compute the minimum work required for the compression and delivery of 1 lb. of air from a supply region at atmospheric pressure and 70° F. to a discharge region at 100 lb. gage, the compression being done reversibly and adiabatically in (a) two stages, (b) three stages, with complete inter-cooling between stages. Also compute the inter-cooler pressure or pressures for minimum work and the horsepower required under each condition per 1000 cu. ft. of free air delivered per minute. Compare results with those of examples 2 and 4. (a - 67,500 ft-lb., 153 hp.) (b - 64,000 ft-lb., 145 hp.)

175. Graphic Representations.—The state changes and energy transitions associated with the several types of compression which were considered in the foregoing may as usual be represented graphically with

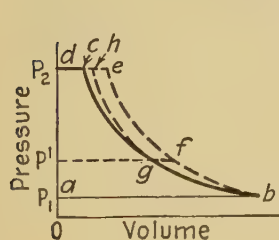


FIG. 119

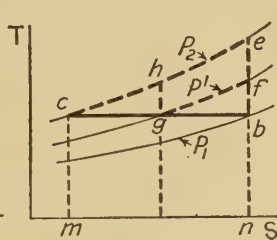


FIG. 120

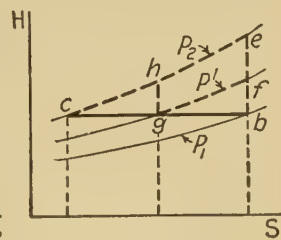


FIG. 121

advantage. In the P - V diagram of Fig. 119 the line bc represents isothermal compression and the area $abcd$ measures the net work required for the composite intake, compression, and delivery. In the same figure the line be represents completely isentropic compression and the area $abed$ measures the corresponding work required, with work require-

ments in excess of that for reversible isothermal compression in the amount of the area *bce*. Finally, for multi-stage compression with complete recooling between the isentropic compression stages, the line *bf* represents the compression in the first stage, *fg* the recooling in the inter-cooler, and *gh* the compression in the second stage. The work saving over the complete isentropic is measured by the area *fghe* but the excess over that required for the reversible isothermal is represented by the sum of the two areas *bgf* and *gch*.

On the *T-S* and *H-S* coördinates of Figs. 120 and 121 the several significant states and state changes in the various compressions are denoted by letters corresponding to those employed in the *P-V* diagram.

Example 6. Show by areas on the *T-S* diagram of Fig. 120 the work required per pound and the heat extraction for reversible isothermal compression, and the heat extraction in the inter-cooler in multi-stage compression.

Example 7. Show by linear distances on the *H-S* diagram of Fig. 121 the work required for complete isentropic compression and the work in the individual stages and total for the multi-stage isentropic compression with inter-cooling.

176. Index of Actual Compressor Performance. — The performance of the various types of actual compressors departs from the ideal performance for a variety of reasons. All exhibit however the common characteristic of requiring a shaft-work input which exceeds in greater or less degree the ideal requirements for reversible isothermal compression. In subsequent paragraphs the individual features occasioning this departure will be considered for several characteristic types of compressors.

For judging actual compressor performance it is the practice to employ as an index of performance the ratio of the ideal work requirement to the actual. Although, in the exact technical meaning of the word, this ratio is not in any sense an **efficiency**³ it is nevertheless the practice to so designate it. By reason of the several bases upon which ideal performance may be predicated several so-called "efficiencies" are likewise employed. They are defined as follows:

Over-all efficiency

$$= \frac{\text{Work for reversible isothermal compression}}{\text{Actual work input}^4}, \quad (11)$$

Compression efficiency

$$= \frac{\text{Work for completely isentropic compression}}{\text{Actual work input}^4}, \quad (12)$$

³ It will be recalled that efficiency is strictly definable as the ratio of the useful output of a machine to the energy input.

⁴ In the A.S.M.E. Power Test Code the actual input for use in computation of

Equivalent compression efficiency, multi-stage compressor

$$= \frac{\text{Work for multi-stage isentropic compression with complete inter-cooling}}{\text{Actual work input}^4} \quad (13)$$

Also

Fan (or blower) efficiency

$$= \frac{\text{Air horsepower output}}{\text{Horsepower input}^4} \quad (14)$$

Again it may be remarked that, except as the purpose of a compression might be that of mechanically effecting a temperature rise in a fluid concurrently with the pressure rise, reversible isothermal compression would seem to be distinctly the more suitable ideal standard with which to compare actual performance. The isentropic standards are justifiable only on the basis that, unfortunately, compression in the actual unjacketed compressor more nearly approaches adiabatic conditions.

177. Mechanical Performance, Reciprocating Compressor. — The general character of one stage of the ordinary reciprocating-piston type of compressor or pump is indicated diagrammatically in Fig. 122. In the actual compressor the intake and discharge valves may be spring-loaded and automatic in their action, or they may be mechanically actuated. The higher pressure compressor is water-jacketed; the lower pressure pump or blower usually is not. The compressor may be double-acting, as in the figure, or may be single-acting.

For mechanical reasons it is not practicable that the piston should

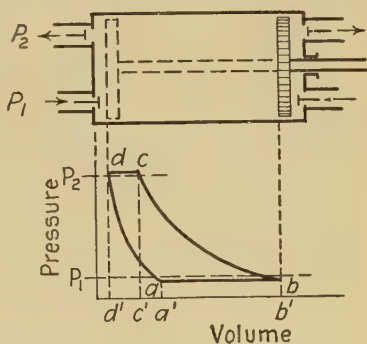


FIG. 122

travel to the extreme ends of the cylinder. In the figure the piston is indicated at the two end positions of its stroke. From this it appears that not all of the gas in the cylinder is ever discharged. The volume

Over-all Efficiency is taken as the input to the compressor shaft in all cases except when the compressor is driven by a reciprocating engine; in that case the indicated horsepower of the engine cylinder is specified as the input. The actual input for purposes of computation of the Compression Efficiency is likewise taken as the shaft-work input except in the case of the reciprocating compressor, in which case the indicated power in the compressor cylinder is specified. The lack of uniformity in the specified point of measurement of the input is unfortunate but in some degree is necessary.

remaining at the end of the discharge stroke is known as the **clearance volume** (V_c). Its relative amount is customarily evaluated in terms of the proportion it bears to the space or volume swept by the piston per stroke, or the **piston displacement**. Thus, in the figure,

$$\text{Volumetric clearance } (C) = \frac{V_d}{V_b - V_d} \quad (15)$$

In larger and high grade compressors the clearance may be as low as 2 per cent, in smaller, high speed and poorly designed machines it may be 10 per cent or more.

Directly below the compressor there is shown by curve *abcd* a graphic record of the simultaneous pressures and volumes as they would exist in one end of the cylinder during one complete cycle of the compressor. Such a diagram, when taken to small scale on an operating compressor, is recalled to be the familiar "indicator diagram," except for the addition of abscissa and ordinate axes drawn at absolute zero pressure and zero volume, respectively. Thus the pressure at any point is the absolute pressure, and the corresponding volume is the *total* volume (including clearance) in that end of the cylinder when the piston is at the given point.

The pressures in the intake and discharge regions are P_1 and P_2 respectively. These are set by operating conditions external to the compressor. It will be noted that the pressure within the cylinder during intake, P_{a-b} , is shown as slightly less than P_1 , this due to acceleration and throttling through the intake line and valve. For a like reason the pressure during discharge, P_{c-d} , exceeds P_2 . It is not a requisite that $P_b = P_a$, nor that $P_d = P_c$, but for many practical purposes this may be taken to be virtually the case and will be so considered in the following.

Although the general significance of points *b* and *c* and of lines *ab*, *bc*, and *cd* in the indicator diagram of Fig. 122 is the same as that of the like-lettered point or lines in Fig. 118 the two diagrams differ in particular. Referring to Fig. 122, the point *a* in that figure depicts the state of the clearance air which remained in the cylinder *after its reëxpansion* and at the instant of opening of the intake valve. It is apparent that the piston must first have moved from the end of the stroke to point *a* in order that the pressure of the clearance air within the cylinder might be sufficiently reduced that during the remainder of the stroke a fresh charge of air might enter the cylinder from the exterior. The length of line *ab* thus measures the change of cylinder volume and the volume of fresh charge which may be induced during the effective portion of the suction stroke. The line *bc* as usual represents the state change during compression, but

in this instance the fluid compressed is the *total* mass of air including that which remains in the clearance. Line *cd* is simply a record of the cylinder pressure during delivery after compression. Line *da* represents the reëxpansion of the clearance air.

The individual work items and the net indicated work which must be furnished at the piston face in a complete cycle are represented in the figure by areas as follows:

Work required for compression	=	$b'bcc'$
Work required for delivery	=	$c'cdd'$
Work returned during reëxpansion	=	$- d'daa'$
Work returned during air entry	=	$- a'abb'$
Net work required per cycle	=	$abcd$
	=	area enclosed by diagram.

As usual the area enclosed by the diagram and the net work per cycle may be regarded as the product of a **mean effective pressure** (P_m) and the piston displacement per stroke (see also Art. 135, Eq. 8). It will be further recalled that in practice the indicator diagram is employed to obtain P_m , doing so by dividing the diagram area by its length and multiplying by the pressure scale, and also that the *indicated power* delivered via the piston face to the air is computable by the familiar relation

$$\text{Indicated hp.} = \frac{P_m L A N}{33,000}. \quad (16)$$

The length of the diagram ($= d'b'$) is a scalar measure of the piston displacement per stroke, LA .

As mechanical irreversibility must exist by reason of the friction between moving parts of the compressor the shaft-work required must exceed the work at the piston face as measured by the indicator diagram. Expressed in terms of horsepower the difference between the indicated horsepower of the compressor and the shaft horsepower is as usual known as the **friction horsepower** of the compressor, and the ratio of the indicated horsepower to the shaft horsepower is known as the **mechanical efficiency** of the compressor.

For purposes of design and estimate the graphic representation of Fig. 122 may be fabricated, rather than taken in actuality by an indicator, and used as a basis for predicting and analyzing compressor performance, selecting cylinder sizes, et cetera.

In that figure the character of the curves *bc* and *da* will depend materially on the provision or absence of water-jacketing and, if that is provided, on its effectiveness. With no cooling the curves would tend to approach the reversible adiabatic. With extremely effective cooling

they would tend to approach the reversible isothermal. Actually they will lie between these. There is no reason to anticipate that they will be simple exponential curves (and in fact the opposite might be expected) but it is usually taken as an allowable approximation that they may be represented by the reversible polytropic relation

$$PV^n = \text{constant},$$

where n is between 1.00 and k (see Art. 90, Eq. 23 et sequi). With the ordinary water-jacketed air compressor and in the absence of piston or valve leakage n may frequently be about 1.35 for the compression curve.⁵ Under like conditions the effective value of n for the reëxpansion curve will usually be rather less than that of the compression curve, or perhaps about 1.2.

During the discharge, from c to d , there will be a considerable heat interchange from the warm gas to the cylinder, the magnitude depending again on the effectiveness of the cooling provided but amounting to perhaps 80 per cent or more of the total amount of heat rejected. This heat interchange during delivery evidently has no effect on the work required per cycle but will influence the temperature of the clearance air and thereby the capacity of the compressor. During intake, from a to b , there is again some heat interchange, from the cylinder walls to the entering air, and also between the entering gas and that remaining in the clearance volume.

For estimating the anticipated indicated power requirements of a compressor from an indicator card which has been fabricated along the foregoing lines the card may of course be laid down on paper, the mean effective pressure measured, and the power computed by the use of equation 16. However, given the intake and discharge pressures, the exponents of the compression and reëxpansion curves, and the volumetric clearance, the power per cycle may be computed directly. Thus the area of the card may be regarded as

$$\int_b^c V dP - \int_a^d V dP,$$

where the volumes are the total (not specific) volumes in the cylinder. More particularly, if $P_b V_b^n = P_c V_c^n = PV^n$ for the compression curve bc and $P_d V_d^{n'} = P_a V_a^{n'} = PV^{n'}$ for the reëxpansion curve da , then the several integrals will take the concrete forms:

$$\begin{aligned} \int_b^c V dP &= (P_b)^{1/n} V_b \int_b^c \frac{dP}{P^{1/n}} = \frac{n}{n-1} P_b V_b \left[\left(\frac{P_c}{P_b} \right)^{(n-1)/n} - 1 \right]; \\ \int_a^d V dP &= (P_a)^{1/n'} V_a \int_a^d \frac{dP}{P^{1/n'}} = \frac{n'}{n'-1} P_a V_a \left[\left(\frac{P_d}{P_a} \right)^{(n'-1)/n'} - 1 \right]. \end{aligned}$$

⁵ See footnote 13, Art. 90.

If it is taken as a sufficiently accurate approximation that $n' = n$, that $P_b = P_a$, and that $P_d = P_c$, these several expressions may be combined to give the single expression for evaluation of the work per cycle:

$$\text{Indicated work per cycle, ft.-lb.} = \frac{n}{n-1} P_b (V_b - V_a) \left[\left(\frac{P_c}{P_b} \right)^{(n-1)/n} - 1 \right].$$

In this expression $V_a = V_d(P_d/P_a)^{1/n'} = V_d(P_c/P_b)^{1/n}$. For direct estimate of the indicated work from more conventional compressor data the expression may advantageously be modified by introducing the above value of V_a , by subtracting and adding V_d in the middle term, and by recalling (Eq. 15) that the clearance fraction $C = V_d/(V_b - V_d)$. The resulting expression is

$$\begin{aligned} \text{Indicated work per cycle, ft.-lb.} &= \frac{n}{n-1} P_b (V_b - V_d) \left[1 + C - C \left(\frac{P_c}{P_b} \right)^{1/n} \right] \\ &\quad \times \left[\left(\frac{P_c}{P_b} \right)^{\frac{n-1}{n}} - 1 \right]. \end{aligned} \quad (17)$$

The volume $V_b - V_d$ is recognized as the piston displacement per stroke.

Example 8. Estimate the work per cycle for the *head* and the *crank* end of the compressor, the indicated horsepower for each end and the total for the compressor and the shaft horsepower for an 18-in. bore $\times$ 20 in. stroke double-acting single-stage air compressor (with 3-in. piston rod through crank end) operating at 192 r.p.m. The intake region pressure is atmospheric and the discharge region pressure is 100 lb. per sq. in. gage. Assume a pressure drop through the suction and the discharge valves of 0.5 lb. per sq. in., a volumetric clearance of 0.04 (4 per cent), a mechanical efficiency of 0.87 and a value of n for both compression and reëxpansion of 1.3.

$$(79 + 77 = 156 \text{ I.hp.; } 180 \text{ S.hp.})$$

Example 9. If the compressor of example 8 delivered 75 lb. of air per minute on the estimated shaft-horsepower input compute the over-all efficiency and compression efficiency. (Note that the pressure conditions are the same as those of examples 2 and 4.)

178. Volumetric Efficiency, Reciprocating Compressor. — In the preceding article it was observed that the existence of a clearance space in the reciprocating compressor and the necessary reëxpansion of the clearance air to somewhat less than the intake region pressure before fresh air intake may begin must make it definitely impossible for the compressor to take in from the exterior an air volume equal to the piston displacement. This condition is aggravated by other items. The term **volumetric efficiency** is employed as an index for quantitative evaluation of this inability of the positive displacement type of compressor actually to induce and deliver the air at a rate corresponding to the rate of piston displacement.

Volumetric efficiency is conventionally and rather naturally defined upon a *volume* basis. However, for various reasons, one of which is the fact that in the customary test determinations of the actual capacity of an operating compressor the rate of delivery as measured by Venturi meter, orifice, or tank is a mass-rate of flow determination, it is advantageous and somewhat more significant also to define the volumetric efficiency upon a *mass* basis. Definitions from both aspects are (see also page 446):

Volumetric efficiency

$$= \frac{\text{volume of "free air" entering cylinder and delivered per intake stroke}}{\text{piston displacement per stroke}}; \quad (18)$$

$$= \frac{\text{mass of air delivered per intake stroke, or cycle}}{\text{piston displacement per stroke} \times \text{density of air supplied to intake system}}; \quad (18a)$$

$$\text{or} = \frac{\text{mass of air delivered per unit time}}{\text{piston displacement per stroke} \times \text{intake strokes per unit time} \times \text{density of air supplied}}. \quad (18b)$$

For purposes of estimating or predicting the volumetric efficiency of a compressor, or for analysis of the factors which predominantly influence it, it is convenient to recognize that aside from the air which might fail of delivery by reason of valve or piston-ring leakage the mass of air which might be delivered per cycle must equal the difference between the total mass of fresh and clearance air in the cylinder at point *b* (Fig. 122) and the mass of clearance air remaining in the cylinder at points *a*. To express this difference:

Mass delivered per cycle, *M* (lb.)

$$\begin{aligned} &= M_b - M_a = \frac{1}{R} \left(\frac{P_b V_b}{T_b} - \frac{P_a V_a}{T_a} \right) \text{ or, if } P_b \text{ and } P_a \text{ do not} \\ &\quad \text{differ materially,} \\ &= \frac{P_b}{R} \left(\frac{V_b}{T_b} - \frac{V_a}{T_a} \right) \text{ or, as } V_a = V_d \left(\frac{P_d}{P_a} \right)^{1/\gamma'} = V_d \left(\frac{P_c}{P_b} \right)^{1/\gamma'}, \\ &= \frac{P_b}{R} \left[\frac{V_b}{T_b} - \frac{V_d}{T_a} \left(\frac{P_c}{P_b} \right)^{1/\gamma'} \right] = \frac{P_b}{R T_b} \left[V_b - V_d \frac{T_b}{T_a} \left(\frac{P_c}{P_b} \right)^{1/\gamma'} \right]. \end{aligned}$$

The mass of air which would correspond to full and wholly effective intake throughout the whole of the suction stroke is $\frac{P_1}{R T_1} (V_b - V_d)$. It follows that the volumetric efficiency would be:

$$\text{Volumetric efficiency} = \frac{P_b}{P_1} \frac{T_1}{T_b} \frac{V_b - V_d \frac{T_b}{T_a} \left(\frac{P_c}{P_b} \right)^{1/\gamma'}}{V_b - V_d}.$$

By subtracting and adding V_d in the numerator of the last fraction and recalling that $V_d/(V_b - V_d)$ is the clearance fraction, C , it is possible to rewrite this equation in the more useful form

$$\text{Volumetric efficiency} = \frac{P_b}{P_1} \frac{T_1}{T_b} \left[1 + C - C \frac{T_b}{T_a} \left(\frac{P_c}{P_b} \right)^{1/n'} \right]. \quad (19)$$

In this relation the fraction P_b/P_1 serves to show quite directly the deleterious influence of depression of the interior pressure P_b due to valve friction and the ratio T_1/T_b shows the disadvantage of the rise of the interior air temperature during entry of the air into the warm cylinder. The pressure ratio P_c/P_b which appears in the subtractive item of the bracketed term indicates the disadvantageous effect of increased pressure range. Also, because the pressure ratio is invariably considerably greater than unity in the reciprocating compressor, increased clearance must likewise reduce the volumetric efficiency. As regards the influence of the temperature of the reexpanded clearance air T_a , it is difficult to draw a direct conclusion because that temperature also influences the final mixture temperature T_b . Also it is to be remarked that neither temperature may be accurately ascertained or predicted.

In connection with the influence of the pressure range it is to be noted that by multi-stage compression the pressure ratio for any individual stage is reduced and the volumetric efficiency of the cylinder is correspondingly increased.

Example 10. Referring to the compressor of example 8 and taking its operating conditions estimate the volumetric efficiency, the mass-rate of air delivery, and the free air capacity of the compressor. Take the air supply region temperature as 70° F. and assume the temperature T_b to be 120° F. and the temperature T_a to be 40° F. (0.71.)

179. Centrifugal Blower and Compressor Performance. — In the centrifugal blower or compressor the excess of actual work input requirements over the ideal is occasioned to some degree by the usual mechanical friction between the moving parts of the machine but to a considerably greater degree by the excess discharge temperature and enthalpy increase which results from the high order of turbulence and fluid friction associated with and caused by the higher velocities at which the gas flows and the tortuous channels through which it passes in the machine. As a result of these features the centrifugal blower or compressor exhibits in general a rather lower index of performance, or "efficiency," than does the high grade reciprocating compressor. This condition may be further aggravated by the practical difficulty of obtaining any effective cooling during compression. However this type of compressor enjoys the characteristic advantage of less weight and

space requirements which accompany high rotative speeds and thus is frequently indicated where larger volumetric capacities are desired. The centrifugal compressor has a further mechanical advantage in that its rate of discharge may be reduced in any degree by closing the discharge line valve without encountering the dangerous pressures which would result under like circumstances with the reciprocating compressor.

As the principles controlling the proper design of the impellers and diffusers of the centrifugal compressor lie more in the science of hydro-mechanics than in the thermodynamics no attention will be given here to such design. For the same reason no adequate rational basis for estimating or predicting actual compressor performance can be presented. It may be observed, however, that the actual power requirements of a compressor is frequently estimated by employing the approximation that the state change of the gas during its irreversible compression is representable by a relation of the form

$$P_1 V_1^n = P_2 V_2^n = P V^n, \text{ or} \\ T_1/P_1^{(n-1)/n} = T_2/P_2^{(n-1)/n} = T/P^{(n-1)/n}.$$

It will be recalled from Art. 88 that the exponent n for an irreversible adiabatic compression must exceed the value of the specific heat ratio k . It will also be found to exceed k in the actual centrifugal compressor even when some slight cooling is effected during the compression, frequently exhibiting an effective value of 3 or more.

Presuming that by test or experience an adequately accurate value of n may be selected and that an estimate may be made of the heat emission from the air via the compressor casing and from the compressor bearings, the necessary work input may then be predicted by direct use of the energy equation. Thus, from equations 1a or 1b and the above T - P relation,

$$W_{\text{in}}, \text{ ft-lb. per lb.} = \frac{U_2^2 - U_1^2}{64.34} + Jc_p T_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right] + JQ_{\text{out}} \quad (20) \\ \text{or} = \frac{U_2^2 - U_1^2}{64.34} + \frac{k}{k-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right] + JQ_{\text{out}}. \quad (20a)$$

Example 11. In the test of a centrifugal compressor taking its suction from the atmosphere, at 70° F., an impact tube at a point of mean velocity in the 5-in.-diameter discharge line gave a reading of 6.33 in. Hg. and a thermometer gave 145° F. The static pressure in the discharge line was 6.30 in. Hg. above atmosphere. A differential manometer between impact and static connections read 0.45 in. water.

From these data compute the rate of delivery of the compressor in lb. per second and in cubic feet of free air per minute, derive an effective or equivalent value of n for the compression state change and estimate the work required per pound and the

horsepower required to drive the compressor. For the work computation presume a total heat emission from the compressor in the amount of 5 per cent of the work input. (0.474 lb.; 3.22; 12.8 hp.)

Upon the basis of the estimated work requirements compute the corresponding over-all efficiency and compression efficiency of the compressor. (0.37; 0.38.)

180. Summary. — A compressor is in general a device the purpose of which is to deliver a compressible fluid from a region of lower pressure to one of higher pressure. For this purpose various types of pumps, fans, blowers, et cetera are employed. Some operate by positive displacement of the fluid and some by an initial impartation of kinetic energy to the fluid by a high speed rotor, combined with subsequent partial reversion of the kinetic energy to flow-work and internal energy by the use of diffusers. The centrifugal types operate on the latter schedule.

The work requirements for compression and delivery may be expressed by the conventional steady-flow energy equation arranged in the form

$$W_{\text{in}}, \text{ ft-lb. per lb. of fluid} = \frac{U_2^2 - U_1^2}{64.34} + J(H_2 - H_1) + JQ_{\text{out}}, \quad (1)$$

$$\text{or, for a gas} = \frac{U_2^2 - U_1^2}{64.34} + Jc_p(T_2 - T_1) + JQ_{\text{out}}, \quad (1a)$$

$$\text{or} = \frac{U_2^2 - U_1^2}{64.34} + \frac{k}{k-1} (P_2 V_2 - P_1 V_1) + JQ_{\text{out}}. \quad (1b)$$

For frictionless, mechanically reversible conditions the work input required may be expressed alternatively as

$$W_{\text{in}}, \text{ ft-lb. per lb.} = \frac{U_2^2 - U_1^2}{64.34} + P_2 V_2 + \int_2^1 P dV - P_1 V_1 \quad (2)$$

$$= \frac{U_2^2 - U_1^2}{64.34} + \int_1^2 V dP. \quad (2a)$$

From the last relation it is evident that any condition, such as energy emission as heat (Q_{out}), which will act further to reduce the volume and will reduce the temperature and enthalpy increase of the fluid during compression, will act jointly to reduce the work required per pound of fluid delivered. In recognition of this condition isothermal compression is regarded as an ideal one for gas compression.

For frictionless isothermal compression of a gas the work required is

$$W_{\text{in}}, \text{ reversible isothermal compression, ft-lb. per lb.} = \frac{U_2^2 - U_1^2}{64.34} + RT_1 \log_e \frac{P_2}{P_1} \quad (4a)$$

$$\text{or} = \frac{U_2^2 - U_1^2}{64.34} + P_1 V_1 \log_e \frac{P_2}{P_1}. \quad (4)$$

In this and the above expressions the kinetic energy term may or may not be negligible under the conditions of actual compressor operation but commonly will be relatively quite minor if the pressure range exceeds about 10 per cent of the initial pressure. If it is not negligible the use of the term may still be avoided if the pressure and temperature term in the foregoing expressions are regarded as *impact* data, rather than *static* data. For very low pressure ranges, not exceeding about 1 per cent of P_1 , the isothermal work will be expressible by the relation

$$W_{\text{in}}, \text{ at } (P_2 - P_1) < 0.01 P_1, = \frac{U_2^2 - U_1^2}{64.34} + (P_2 - P_1)V \quad (4d)$$

In this relation the two terms are recognized as measuring directly the kinetic energy increase and the flow-work required for the delivery of the fluid through the pressure range $P_2 - P_1$ if the density change of the fluid is negligible. The power required for ideal isothermal compression and delivery is readily obtained by taking the product of the work per pound, as determined by the above equations, and the mass-rate of delivery. The power determined for a low pressure fan by the use of equation 4d is also known as its *air horsepower*.

The work required for a reversible adiabatic compression of a gas may be determined by the relations

$$W_{\text{in}}, \text{ reversible adiabatic compression, ft-lb. per lb.} = Jc_p T_1 [(P_2/P_1)^{(k-1)/k} - 1] \quad (6)$$

$$\text{or} = \frac{k}{k-1} P_1 V_1 [(P_2/P_1)^{(k-1)/k} - 1]. \quad (6a)$$

In this and the following relations the kinetic energy term is omitted. If it is not negligible it may readily be reintroduced or again the kinetic energy may be accounted for automatically in test analyses by use of *impact* pressure and temperature data instead of *static* data. For conditions in which $(P_2 - P_1)/P_1$ does not exceed about 0.25 the exponential term in equation 6 may be eliminated by a binomial expansion which gives the relation

$$W_{\text{in}}, \text{ isentropic comp. with } (P_2 - P_1)/P_1 < 0.25 = R T_1 \frac{P_2 - P_1}{P_1} \left(1 - \frac{1}{2k} \frac{P_2 - P_1}{P_1} \right). \quad (6b)$$

If $(P_2 - P_1)/P_1$ does not exceed about 0.01 the equation may be further simplified to a form which is the equivalent of equation 4d.

In multi-stage compression the air may conveniently be recooled between stages. For two-stage compression with an interstage pressure of P' , with reversible adiabatic compression in each stage and with complete recooling to T_1 between stages the work required becomes

$$W_{\text{in}} = Jc_p T_1 [(P'/P_1)^{(k-1)/k} + (P_2/P')^{(k-1)/k} - 2]. \quad (8)$$

There is an optimum value of the inter-stage pressure P' which gives both a minimum total work requirement and equal work distribution between stages. Its value is such that

$$P'/P_1 = P_2/P' = (P_2/P_1)^{1/2}. \quad (9)$$

When the optimum value of P' is employed

$$W_{\text{in}} = 2 J c_p T_1 [(P_2/P_1)^{(k-1)/2k} - 1], \quad (10)$$

$$\text{or} = \frac{2k}{k-1} P_1 V_1 [(P_2/P_1)^{(k-1)/2k} - 1]. \quad (10a)$$

For compression in N stages

$$W_{\text{in}} = N J c_p T_1 [(P_2/P_1)^{(k-1)/Nk} - 1], \quad (10b)$$

$$\text{or} = \frac{Nk}{k-1} P_1 V_1 [(P_2/P_1)^{(k-1)/Nk} - 1]. \quad (10c)$$

The benefits of cooling during compression and of multi-staging with inter-cooling may be shown graphically to advantage.

Actual compressor performance may as usual be judged by comparison with ideal performance. For such comparison it is the practice to determine the ratio of the ideal work requirement to the actual and to term this ratio an efficiency. Isothermal compression is the outstandingly logical ideal but three other bases of comparison are employed, and four corresponding efficiencies. These are described or defined as follows:

Over-all efficiency

$$= \frac{\text{Work for isothermal compression}}{\text{Work actually required}}; \quad (11)$$

Compression efficiency

$$= \frac{\text{Work for isentropic compression}}{\text{Work actually required}}; \quad (12)$$

Equivalent compression efficiency, multi-stage compression

$$= \frac{\text{Work for isentropic compression but complete inter-cooling}}{\text{Work actually required}}; \quad (13)$$

Fan efficiency

$$= \frac{\text{Air horsepower output}}{\text{Horsepower input}}. \quad (14)$$

For estimating the *indicated work* actually required for operating a given reciprocating compressor the following relation may be employed:

Indicated work per cycle

$$= \frac{n}{n-1} P_b (V_b - V_d) \left[1 + C - C \left(\frac{P_c}{P_b} \right)^{1/n} \right] \left[\left(\frac{P_c}{P_b} \right)^{(n-1)/n} - 1 \right], \quad (17)$$

where n = effective exponent of the compression or clearance reëxpansion curve ($PV^n = \text{constant}$);

P_b = interior pressure during intake;

P_c = interior pressure during delivery;

C = clearance fraction = $\frac{\text{clearance volume}}{\text{piston displacement per stroke}}$. (15)

The reciprocating compressor is unable to deliver a free-air capacity equal to the piston displacement corresponding to the entire suction stroke. This is due to (a) the necessity for reëxpansion of the clearance air before induction of the fresh charge may begin and (b) reduction of the density of the incoming air because of pressure depression and temperature rise during entry of the air. The ratio of the volume or mass of air actually delivered by the compressor to the volume or mass which it might ideally deliver if no reëxpansion should be necessary and no entering air density reduction should occur, and also if no leakages should exist, is known as the volumetric efficiency of the compressor. This efficiency may be estimated or predicted by the relation

$$\text{Vol. eff.} = \frac{P_b}{P_1} \frac{T_1}{T_b} \left[1 + C - C \frac{T_b}{T_a} \left(\frac{P_c}{P_b} \right)^{1/n'} \right]. \quad (19)$$

The centrifugal compressor will in general exhibit a rather lower efficiency than does the reciprocating compressor, due to the inherently greater fluid friction and turbulence which will exist. No ready basis is available for rational estimate of the actual work requirements of a centrifugal compressor except as the probable temperature rise through the compressor may be predicted or the corresponding effective value of n may be estimated from experience. In that case the conventional energy equation may be employed for the work estimate. The value of n will exceed considerably the value of the specific heat ratio k , frequently attaining a value of 3 or more.

181. Review Questions and Topics. —

1. (a) Compare the general objectives of the engine and the compressor.
- (b) Classify compressors as to their pressure range and as to their methods of operation.
2. (a) From the steady-flow energy equation develop the general energy relation expressing the work requirements for compression and delivery of a compressible fluid and the special forms of that relation as applied to a gas.
- (b) Show analytically and graphically the net effect on ideal work requirements which will result from heat emission during compression and discuss the suitability of reversible isothermal compression as an ideal standard manner of compression of a gas.
3. (a) Deduce expressions for the work required for reversible isothermal compression and delivery of a gas and indicate the permissible modification for a pressure rise not exceeding 1 per cent.

(b) Discuss the permissibility of omission of the kinetic energy term and the possibility of suppressing the term but making an automatic accounting of the energy.

(c) Develop from the relations for work per pound expressions for the horsepower required for reversible isothermal compression and delivery; define the air horsepower of a fan.

4. (a) Develop the energy relations for evaluating the work required for isentropic compression and delivery of a gas and quote the modifications of this relation which may be applied for moderate and low pressure ranges.

(b) Outline the basis of selection of the proper inter-stage pressure in two-stage compression with inter-cooling and quote the relation for computation of the work for ideal multi-stage compression.

5. Represent isothermal and isentropic compression and isentropic compression with inter-cooling on P - V , T - S and H - S coordinates and discuss the items indicated by the diagrams.

6. Define the several "efficiencies" by which actual compressor performances are judged.

7. (a) Define clearance fraction and discuss the general characteristics of the indicator diagram of the actual compressor.

(b) Quote and interpret the relation by which the actual indicated work required per cycle of a reciprocating compressor may be evaluated.

8. (a) Define volumetric efficiency from both volume and mass aspects and indicate the items which cause the actual volumetric efficiency of a reciprocating compressor to depart from unity.

(b) Quote and interpret or discuss the relation by which the volumetric efficiency of a reciprocating compressor may be estimated.

9. (a) What features occasion the lower "efficiencies" in the centrifugal compressor as compared with those of the usual reciprocating compressor?

(b) Quote and interpret the relation for estimating the work required for operating a centrifugal compressor.

Symbols and Abbreviations, Chapter XVII

C = clearance fraction, or clearance volume per piston displacement per stroke.

c_p = specific heat at constant pressure ($= 0.241$ for air).

E = internal energy, B.t.u. per pound.

H = enthalpy ($E + PV/J$), B.t.u. per pound.

Hp. = horsepower ($= 550$ ft.-lb. per second).

J = Joule's equivalent ($= 778$ ft.-lb. per B.t.u.).

k = specific heat ratio, c_p/c_v ($= 1.40$ for air).

L = length of piston stroke, feet.

$\log_e$ = logarithm to base e ($= 2.3026 \times$ common log).

M' = mass-rate of flow or delivery, pounds per second.

m.e.p. = mean effective pressure, pounds per square foot.

N = number of stages in a multi-stage compressor, and also number of cycles per minute in reciprocating compressor.

n = exponent in state change equations of the form $PV^n = \text{constant}$ or $T/P^{(n-1)/n} = \text{constant}$.

P = absolute pressure, pounds per square foot and to be interpreted as static pressure or as impact pressure as circumstances may dictate.

P_m = mean effective pressure, pounds per square foot.

P' = interstage pressure, multi-stage compressor.

Q = energy in transition as heat, B.t.u. per pound.

R = gas constant (= 53.3 for air, in ft-lb.-°F. system).

S = entropy, per pound, and as subscript to denote constancy of entropy.

T = absolute temperature, deg. Rankine (= 460 + degrees Fahrenheit).

U = velocity, feet per second.

V = volume, usually specific (cubic feet per pound) but also used to designate the volume of any given mass.

V' = volume rate of flow, cubic feet per second.

W = energy in transition as work, foot-pounds per pound.

CHAPTER XVIII

REFRIGERATION

The study of the principles of refrigeration deserves the attention of the student of the science of thermodynamics not only because of the increasing use of refrigeration in ice-making, in the preparation and preservation of food, the cooling and humidifying of air and in many manufacturing processes but also because refrigeration serves so well to illustrate certain fundamentals of the science.

182. General Considerations. — The reader is aware that the action of the reversible heat engine has been the foundation upon which the most essential and characteristic features of thermodynamics have been based. He will also recall that that device, acting as an engine, is the ideal prototype of all of our practicable methods of power generation from the energy in the high temperature products of the combustion of fuels. The refrigerating machine is similarly the practical example of the *reversed* heat engine, in the feature that it acts to abstract energy as heat from a region of lower temperature and to deliver heat to a region of higher temperature and does this by the supplying of energy to the machine either and usually as shaft-work but also possibly as heat from some still higher temperature level. The refrigerating machine is thus a **heat pump** and is in its fundamental features simply a reversed heat engine. That viewpoint is considered further in Art. 183.

Having thus recognized the essential parallelism of the two devices it is also well to observe certain aspects in which they diverge. For example, the objective of the heat engine is the transformation to work of a maximum portion of the energy which is supplied from the high temperature source, but an unfortunately necessary accompaniment of the engine is the rejection of an unavailable residue of energy to that universal energy receiver, the atmosphere. The engine thus operates through a temperature range which extends from atmospheric temperature *upward*. In distinction, the refrigerating machine has for its practical objective only the securing and maintenance of a temperature which is below that of the atmosphere, but again the energy which it discards must go to the same general energy receiver, the atmosphere. Consequently the refrigerating machine operates through a temperature range which extends *downward* from the temperature of the atmosphere.

183. The Ideal or Carnot Refrigeration Cycle.—In Art. 43 the reversed Carnot cycle was described as one abstracting energy from a lower temperature region and returning it to a region of higher temperature by means of the work supplied. Examining this reversed Carnot

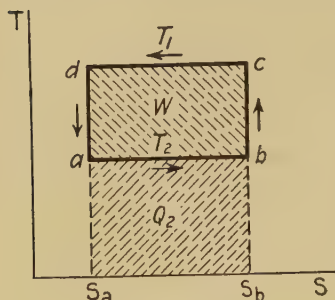


FIG. 123

cycle in more detail by the aid of the T - S diagram of Fig. 123, it is evident that, by operating the cycle in the direction indicated, heat Q_2 may be abstracted from a region at the lower temperature T_2 in the amount $T_2(S_b - S_a)$, and heat Q_1 may be delivered to a region at the higher temperature T_1 in the amount $T_1(S_c - S_d)$, or $T_1(S_b - S_a)$. It follows further from the general energy equation (see Art. 40, Eq. 1) that for the operation of this complete cycle work will be

required in the amount $Q_1 - Q_2$, or alternatively,

$$W/J = Q_1 - Q_2 = (T_1 - T_2)(S_b - S_a).$$

As a natural objective in refrigeration would be the removal of energy from the lower temperature region with a minimum expenditure of work input it follows that a suitable index of the performance of a refrigeration machine would be the ratio between the amount of heat removed, or the **refrigerating effect**, and the work required. This ratio is called the **Coefficient of Performance** of the machine, or

$$\text{Coefficient of performance (C.P.)} = \frac{\text{Heat removed } (Q_2)}{\text{Work required } (W/J)}. \quad (1)$$

For the reversed Carnot cycle the coefficient thus becomes

$$\begin{aligned} \text{C.P.} &= \frac{Q_2}{W/J} = \frac{T_2(S_b - S_a)}{(T_1 - T_2)(S_b - S_a)} \\ &= \frac{T_2}{T_1 - T_2}. \end{aligned}$$

It may be shown that this Carnot or reversible cycle coefficient is the maximum coefficient of performance conceivably obtainable with *any* refrigerating machine operating in any manner with any fluid between a given (constant) lower temperature T_2 and a given (constant) upper temperature T_1 . Thus, applying a line of reasoning similar to that used in Art. 46 to prove proposition I of the Carnot Principle and referring to Fig. 8 of that article, let Y be a reversible refrigerating machine having a coefficient of performance of $75/50 = 1.5$. Let X be another refrigera-

tion machine assumed to have a greater coefficient of performance, say $100/50 = 2$. Now reverse machine Y so that it acts as a heat engine, as shown, and couple it to machine X which acts as a refrigerating machine. The 50 units of work output of Y is just sufficient to drive the refrigerating machine X , and so the combined unit would be self-acting. However, the combined machine continuously delivers 25 units of energy from the low temperature to the high temperature region with no external supply of energy. This is contrary to the Second Law of Thermodynamics (Clausius' Statement, see Art. 46). Therefore the maximum coefficient of performance of a refrigerating machine is that of the reversible Carnot refrigeration machine Y , which has been shown to be $\frac{T_2}{T_1 - T_2}$.

This principle governing the performance of refrigeration machines has associated with it several corollaries complementary to those associated with the Carnot Principle:

I. No refrigeration machine delivering heat from a region at a lower temperature to one at a higher temperature can have a greater coefficient of performance than that of a reversible refrigerating machine operating between those temperatures.

II. The coefficient of performance of all reversible refrigerating machines working between the same temperatures is the same, irrespective of the cycles or of the working fluids used in the machines.

III. The coefficient of performance of a reversible refrigerating machine depends only on the temperatures of the lower and higher temperature regions.

The proofs of these complimentary propositions are wholly parallel to those made in Art. 46 for the like propositions of the Carnot Principle and need not be repeated.

The Carnot refrigeration cycle may ideally be realized with any vapor. It is instructive to see how a fluid such as water might be used. Referring to the T - S diagram of a vapor, as presented in Fig. 124, and starting with the substance at

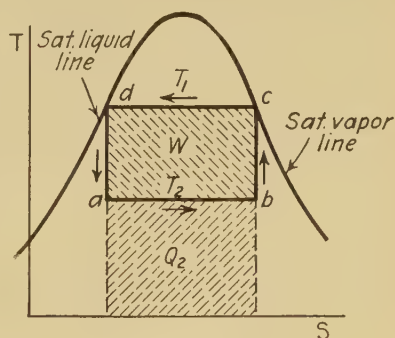


FIG. 124

state d in a saturated liquid state at the upper temperature (and pressure) of the cycle, let the liquid be expanded adiabatically and reversibly in a cylinder to state a . As it expands the temperature drops, part of the liquid vaporizes and some work output is obtained. Between states a and b additional liquid is caused to vaporize at the

constant lower temperature and pressure of the cycle by the absorption of energy as heat from the cold region. Here the refrigeration is accomplished. After the partial vaporization to some suitable state b , work energy is supplied and the vapor and liquid mixture is compressed isentropically to a state c which in the figure is the saturated vapor state at the upper pressure and temperature. The vapor is now condensed at the constant upper temperature and pressure to the original state at d , condensation being effected by heat energy rejection in an amount equal to that absorbed from the lower temperature region plus the *net* work of the cycle. The net work is that required for the compression less that regained by the expansion.

Example 1. What is the ideal (Carnot) coefficient of performance of a refrigerating machine for ice-making with the upper temperature region, or heat energy receiver, at a (atmospheric) temperature of 62° F.

Solution. —

$$\begin{aligned}\text{Coefficient of performance} &= \frac{T_2}{T_1 - T_2} \\ &= \frac{32 + 460}{(62 + 460) - (32 + 460)} \\ &= 16.4.\end{aligned}$$

The interpretation of the example is that for each unit of mechanical energy or work supplied 16.4 units of energy might ideally be removed as heat from the 32° F. cold region and 17.4 units would thus be delivered as heat to the 62° F. atmosphere.

Example 2. Compute the Carnot cycle coefficient of performance (α) for an upper temperature of 80° F. and lower temperatures from - 10° F. to + 40° F. and (b) for a lower temperature of 20° F. and upper temperatures of 50° F. to 100° F. Do these for even 10° temperature intervals; show by curves the influence of the temperatures on the coefficient and discuss the practical significance of the results.

Example 3. A Carnot refrigeration cycle is operated between the temperatures of 50° F. and 100° F. What vapor pressures would be required if the fluid were steam and what would be the coefficient of performance? What refrigeration would be accomplished per pound of steam?

Solution. — From the saturated steam tables the necessary pressure would be 0.178 and 0.95 lb. per sq. in. abs., respectively. The coefficient of performance would equal $(50 + 460)/(100 - 50)$ or 10.2. The heat rejected per pound of steam to the upper temperature region at 100° F. would be the enthalpy or "latent heat" of evaporation at that temperature or, from the steam tables, 1036.3 B.t.u. per lb. From the definition of coefficient of performance and the necessary energy relations of the cycle,

$$10.2 = \frac{\text{refrigeration}}{\text{work}} = \frac{1036.3 - \text{work per lb.}}{\text{work per lb.}}.$$

Solving this equation for the work term, the net work required per pound of steam circulating equals 92.2 B.t.u. Finally therefore the refrigeration accomplished per pound of steam = $Q_1 - W/J = 1036.3 - 92.2 = 944.1$ B.t.u.

Example 4. Solve example 3 by ascertaining the qualities of the steam at states *a* and *b* in Fig. 124, the refrigerating accomplished by the vaporization between those states, and the work respectively secured and required in the expansion from *d* to *a* and in the compression from *b* to *c*.

184. Refrigerating Capacity. — For evaluating the rate of available energy delivery from an engine or power plant we have employed the horsepower (or the kilowatt) as the conventional unit of capacity. The analogous unit which is used for expressing the rate of heat energy intake from the cold region of a refrigerating machine, or its **refrigerating capacity**, is the **tons of refrigeration per 24 hours**. This term is frequently abbreviated and the rate of refrigerating effect is stated simply in **tons**.

The standard commercial ton is arbitrarily defined as a removal of energy as heat from the cold region at the rate of 288,000 B.t.u. per 24 hours or $\left(\frac{288,000}{24 \times 60} =\right)$ 200 B.t.u. per minute. The unit originates from the fact that the "latent heat of fusion" of ice is approximately 144 B.t.u. per lb. or $(144 \times 2000 =)$ 288,000 B.t.u. per short ton, whence a refrigerating machine which is operating at a capacity of one ton is absorbing energy at a rate equal to that which would exist if one ton of ice were melting in the refrigerated region each 24 hours.¹

An index of refrigerating machine performance which is employed in practice rather more than is the Coefficient of Performance is a derived one which is associated with the ton unit of capacity, namely, **horsepower required per ton** of refrigeration per 24 hours. The relation between the performance so expressed and the Coefficient of Performance is readily obtained by recalling that

$$\begin{array}{l} \text{Refrigerating effect, B.t.u.} \\ \text{per minute} \end{array} = \text{tons (per 24 hours)} \times 200, \quad \text{and}$$

$$\begin{array}{l} \text{Work required, B.t.u.} \\ \text{per minute} \end{array} = \text{Hp.} \times \frac{2545}{60} = \text{Hp.} \times 42.4,$$

whence

$$\text{Coefficient of Performance} = \frac{\text{tons} \times 200}{\text{hp.} \times 42.4} = \frac{4.71}{\text{hp. per ton}}. \quad (2)$$

Example 5. Compute the horsepower ideally required per ton of refrigerating effect per 24 hours for the conditions of examples 1 and 3 and at the extremes of temperature for parts (*a*) and (*b*) of example 2. Discuss the evidence of the latter.

¹ To avoid confusion in this connection it is to be observed that in the actual manufacture of one ton of ice, due to heat leakage, et cetera, much more than one ton of refrigerating effect will be required.

185. Practical Refrigeration Cycles. — It will be recalled (Arts. 42 and 43) that the Carnot engine mechanism was a rather fantastic one in which the successive state changes of the cycle were presumed to be carried out with the fluid retained within an engine cylinder, but also that, with some modifications, the cycle might be adapted to practical conditions by causing the fluid to flow progressively through various separate devices in each of which one of the individual phases of the cycle might be performed advantageously. The Rankine cycle and the modified forms of that cycle, as employed and exemplified in the conventional steam power plant, constituted the practicable adaptations of the Carnot engine cycle.

In a similar manner it is practically advantageous to modify the reversed Carnot cycle, or Carnot refrigeration cycle, in some particulars and to carry out the resulting fluid cycles in a succession of apparatus through which the fluid is caused to flow progressively and in each of which one phase of the cycle is accomplished. These practical adaptations fall in general into three classifications:

- (a) **Vapor cycles** in which the transfer of the vapor from the lower temperature and pressure of the cycle to the upper is accomplished through the agency of work supply to a **compressor**;
- (b) **Gas (air) cycles** in which again the transfer of the fluid from the lower temperature and pressure to the upper is effected by a **compressor**;
- (c) **Vapor cycles** in which the transfer of the vapor from the lower pressure and temperature to the upper is accomplished in an **absorption system**, primarily through the agency of heat supplied at a still higher temperature.

These several practical cycles are considered in some detail in the following.

186. The Vapor Compression System. — The practical vapor compression system of refrigeration differs in principle from the Carnot cycle in two features. The one is that which has already been mentioned, the segregation of the several phases of the cycle into various devices to and through which the vapor is caused to flow progressively. The other is that, due to the failure to develop as yet a practicable engine or turbine in which to accomplish the expansion of the condensed liquid from the upper temperature and pressure of the cycle to the lower, this expansion is in practice done in an irreversible throttling process which takes place in an **expansion valve**.

The resulting cycle as it would appear with these modifications, but still idealized in some respects, is represented in the T - S and H - S dia-

grams of Figs. 125 and 126, respectively. A diagrammatic representation of the arrangement of the apparatus appears in Fig. 127.

Starting at point *a* in each figure the fluid, which is called the **refrigerant**, leaves a storage tank in which it exists as a liquid at about atmospheric temperature and at its corresponding saturation pressure P_1 . Between *a* and *b* the fluid is permitted to escape under control through the expansion valve to the low pressure region of the cycle. Due to the throttling character of the process the enthalpy of the fluid at state *b*

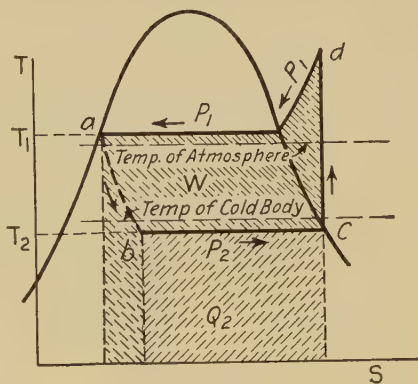


FIG. 125

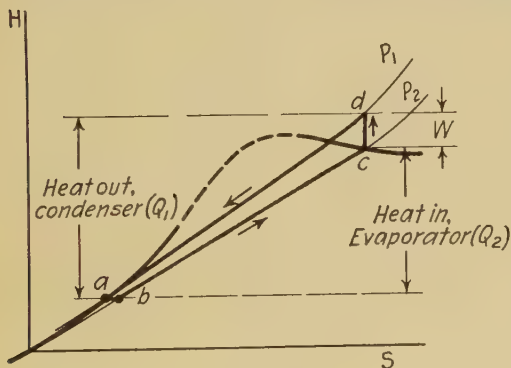


FIG. 126

must equal that at state *a*, with the result that a sensible portion of the liquid must vaporize, but due to the lower pressure of the vapor its temperature must have fallen. Specifically it must have dropped to the saturation temperature corresponding to the lower pressure. After leaving the expansion valve the low temperature and low quality vapor mixture enters the **evaporator**. Here

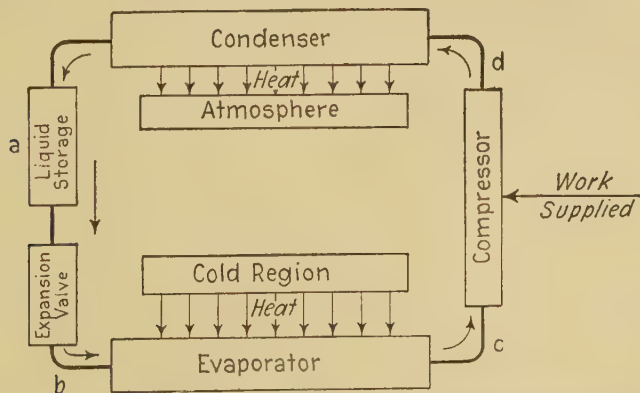


FIG. 127. Elements of vapor-compression type of refrigerating plant.

the provision is made for the absorption of energy as heat by the fluid from the region or substance which it is desired to refrigerate. The pipe coils which constitute the evaporator may be placed directly in the region that is to be cooled, in which event the system would be said to be one with **direct expansion**, or they may be placed in and act to cool some liquid with low freezing point, such as calcium chloride brine, which is circulated to the region that is to be cooled and which thus acts simply as a convective energy carrier. The latter system would be known as one with **indirect expansion**. In either event the reception of energy by the refrigerant will act to cause it to evaporate. The amount of energy received by each pound of the refrigerant and thus the degree of completeness of its evaporation will depend on the rapidity of its circulation, on the temperature difference between the fluid in the evaporator and its environs, and on other operating conditions. In the figures the fluid is shown as leaving the evaporator at state *c*, which is the saturated vapor state. It may in fact leave as a wet vapor mixture or even moderately superheated. In connection with the process of evaporation in the evaporator it is to be observed that the temperature therein must be *below* the temperature of the cold region in order that heat transition may occur in the desired direction. The pressure maintained therein must be the saturation pressure corresponding to the temperature, which pressure we shall designate as P_2 .

From the evaporator the refrigerant passes to the **compressor**. The function of the compressor is two-fold. The one function is that of withdrawing the fluid from the evaporator at a rate sufficient to maintain the necessary reduced pressure and temperature in the evaporator. The other is that of compressing and delivering the fluid at a temperature which is adequately *above* that of the atmosphere or of the region or substance to which the fluid must next discard its "load" of energy. The increase of temperature of the vapor must be accompanied by increase of pressure. In this compression process, lacking any natural substance of a sufficiently low temperature that may act to withdraw energy as heat *during* the compression and thus reduce the work requirement (see end of Art. 172) a reversible adiabatic or isentropic compression must be regarded as the best that might be obtained. The figures show such a compression. Evidently it is one that will tend to dry and to superheat the refrigerant. The state change is represented by line *c-d*.

After leaving the compressor at state *d* the vapor enters the **condenser** in which it must first be de-superheated and then condensed. The departing energy passes as heat to **circulating water** which may be expected

to be at or about atmospheric temperature. The temperature during the condensation must moderately exceed that of the water in order that the heat transition may proceed in the desired direction. The refrigerant pressure which must be delivered by the compressor is the saturation pressure P_1 corresponding to the condensing temperature. The state change of the refrigerant within the condenser is obviously represented in the figures by line $d-a$.

A variety of fluids have been used as the refrigerant in vapor compression systems. For larger commercial installations ammonia is used almost universally at the present time. For the small self-contained household installations sulphur dioxide vapor is very popular. In marine installations on combatant ships neither of the foregoing is considered desirable on account of their active toxicity, and carbon dioxide is preferred. Various hydrocarbons such as ethyl chloride, propane, et cetera, are also used. Ammonia offers the advantages of moderate condenser pressures and moderate specific volume at the evaporator pressures. Sulphur dioxide offers the further advantage of even more moderate pressures but requires considerably more compressor displacement on account of fairly high specific volumes at the evaporator pressure. Carbon dioxide has the disadvantage of requiring very high pressures in both the condenser and the evaporator.²

The vapor compression cycle as described in the foregoing is idealized in the features (a) that the liquid leaving the condenser or storage tank was presumed to be *saturated* liquid, at the saturation temperature corresponding to the condenser pressure, and (b) that the compression was taken as frictionless and adiabatic. The actions in the actual cycle will customarily differ in various respects, the major ones of which are (a) that the liquid going to the expansion valve may in practice be moderately sub-cooled to a state such as that indicated by point a' in Figs. 128 and 129, (b) that the compression will scarcely be an isentropic one, and (c) that there will be slight pressure drops accompanying the flow of the refrigerant through the condenser and through the evaporator. Also the vapor leaving the evaporator and entering the compressor may readily be moderately superheated, as at state c' in the figures, or might be moderately wet, as at state c'' .

A subcooling of the liquid is evidently advantageous as it increases the available refrigerating effect per pound of refrigerant by the amount $H_a - H_{a'}$. The subcooling may be promoted in practice by causing

² For a complete description of the various refrigerants, a discussion of their adaptability under various conditions and tables and charts of their thermodynamic properties see Macintire, Handbook of Mechanical Refrigeration, published by John Wiley & Sons.

the cooler incoming circulating water to flow first to that portion of the condenser from which the condenser refrigerant departs (i.e., by provision for "counter-current" flow).

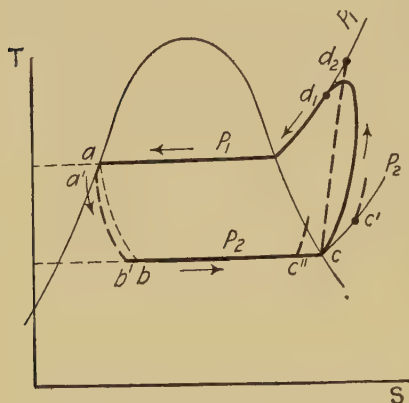


FIG. 128

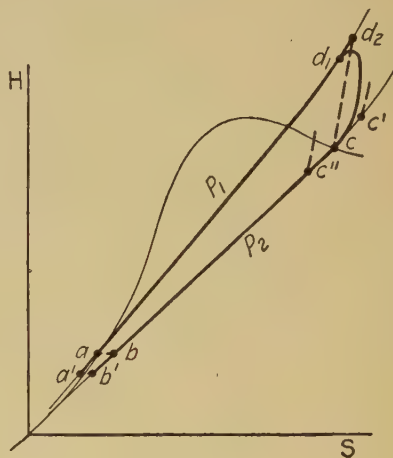


FIG. 129

The actual state change in the compressor will vary in character as that device may be a reciprocating compressor, of the centrifugal type, an ejector, et cetera. The positive-displacement, reciprocating-piston type of compressor is the more usual one. A reasonable representation of the state change occurring enroute through such a compressor is shown by line cd_1 in the figures. The state change through a centrifugal compressor would be better represented by a line such as cd_2 . In the reciprocating compressor the phenomena of initial evaporation, of subsequent recondensation, of pressure drop through the valves, et cetera, will affect the character of the state change and the work required, in a manner similar but opposite to the analogous phenomena in the reciprocating steam engine (see Art. 138). In a centrifugal compressor a progressive increase of entropy would as usual result from turbulence and friction. In either type of compressor heat energy emission to a water-jacket *from the fluid during the compression* would as usual act to reduce the work required and advantage could be taken of this fact if water of a sufficiently low natural temperature were available. The advantage of water-jacketing is problematical, however, unless the vapor should enter the compressor dry or superheated and during the compression there should thus tend to be considerable superheating of the vapor above the saturation temperature corresponding to the condenser pressure — and such a condition would in itself be disadvantageous as regards both plant capacity and plant performance.

Probably the most effective means of improving the performance of a refrigerating process, but at the same time one that is too frequently over-looked, is that of providing adequate, well designed and well maintained heat transfer surfaces in the evaporator and in the condenser and an ample amount of cool circulating water to the condenser. The effect is to reduce as far as practicable the total temperature range, $T_1 - T_2$, through which the refrigerant must pass by enabling its lower temperature T_2 to be a minimum amount below the requisite cold region temperature and its upper temperature T_1 to be a minimum amount above the circulating water supply temperature.

The energy relations for the several phases of the cycle, both for actual operation and for the ideal cycle, are (regarding all kinetic energy terms as negligible), per pound of vapor circulated,

for the expansion valve,

$$H_a = H_b; \quad (3)$$

for the evaporator,

$$\begin{aligned} H_b + Q_{\text{in}, b-c} &= H_c, \text{ or} \\ Q_{\text{in}, b-c} &= H_c - H_b = H_c - H_a; \end{aligned} \quad (4)$$

for the compressor,

$$\begin{aligned} JH_c + W_{\text{in}, c-d} &= JH_d + JQ_{\text{out}, c-d}, \text{ or} \\ W_{\text{in}, c-d} &= J(H_d - H_c) + JQ_{\text{out}, c-d}^3 \\ &= J(H_d - H_c)_S \text{ for isentropic compression;} \end{aligned} \quad \begin{aligned} (5) \\ (5a) \end{aligned}$$

for the condenser,

$$\begin{aligned} H_d &= H_a + Q_{\text{out}, d-a} \text{ or} \\ Q_{\text{out}, d-a} &= H_d - H_a; \end{aligned} \quad (6)$$

for the complete cycle,

$$W_{\text{in}} = J[(Q_{\text{out}, d-a} - Q_{\text{in}, b-c}) + Q_{\text{out}, c-d}] = J(H_d - H_c + Q_{\text{out}, c-d}).^3 \quad (7)$$

Coefficient of Performance

$$= \frac{H_c - H_a}{H_d - H_c + Q_{\text{out}, c-d}} \quad (8)$$

$$= \frac{H_c - H_a}{(H_d - H_c)_S}, \text{ for isentropic compression.} \quad (9)$$

³ As with any compressor, energy departure as heat which might act to reduce the enthalpy increase acts to reduce instead of increase the work requirement (see Art. 172). In distinction, any heat emission occasioned by mechanical friction in moving parts obviously acts to increase the work.

In these relations the several subscripts are to be interpreted as referring to locations with respect to the various apparatus of the plant rather than to particular states indicated by the letters on the foregoing H - S or T - S diagrams.

Example 6. For an indirect-expansion ice-making installation operating with ammonia assume that the brine leaves the cooler (evaporator) at 14°F. and that to obtain this temperature the necessary vapor temperature in the evaporator is 5°F. Also assume that the cooling water is supplied at 70°F. and leaves at 80°F. , enabling the ammonia to condense at 86°F.

Presuming dry saturated vapor at the compressor suction, isentropic compression and no subcooling of the liquid from the condenser ascertain the pressures in the evaporator and the condenser, the temperature after compression and the ideal coefficient of performance and horsepower per ton of refrigeration. Compare with the Carnot cycle performance between the vapor temperatures in the evaporator and condenser and with a Carnot cycle performance between the lowest brine temperature and the cooling water supply temperature. Indicate the reasons for the lower performance of the vapor cycle as compared with the Carnot between the same temperature limits.

Solution. — From tables of the properties of ammonia (Bureau of Standards);

Evaporator (suction) pressure = sat. pressure corresponding to 5°F. = 34.3 lb. per sq. in. abs. (= 19.6 lb. gage);

Condenser (discharge) pressure = sat. pressure at 86°F. = 169.2 lb. per sq. in. abs. (= 154.5 lb. gage).

For saturated vapor at 5°F. , $H = 613.3$ (= H_c) and $S = 1.3253$ (= S_c).

After compression at a constant entropy of 1.3253 to 169.2 lb. abs., t (from charts) = 210°F. and $H = 713$ (= H_d).

For saturated liquid at 86°F. , $H = 138.9$ (= H_a and H_b).

Coefficient of performance

$$= \frac{613.3 - 138.9}{713 - 613.3} = \frac{474.4}{99.7} = 4.76.$$

Horsepower per ton of refrigerating effect per 24 hours

$$= \frac{4.71}{4.76} = 0.99.$$

Carnot performance between $(460 + 5 =)$ 465°R. and $(460 + 86 =)$ 546°R.

$$= \frac{465}{546 - 465} = 5.74.$$

Carnot performance between $(460 + 14 =)$ 474°R. and $(460 - 70 =)$ 530°R.

$$= \frac{474}{530 - 474} = 8.47.$$

The ammonia cycle coefficient is lower than the Carnot (4.76 vs. 5.74) because of (a) the irreversible throttling in the expansion valve and (b) the higher temperature at which the heat is rejected during the desuperheating of the ammonia vapor leaving the compressor.

Example 7. If in example 6 the use of direct expansion would enable an evaporator (cooling coil) temperature of 14°F. and if by means of lower cooling water tem-

perature the condenser temperature could be reduced to 70° F., find the effect on the coefficient of performance and the hp. per ton.

Example 8. If in example 6 by counter-flow in the condenser the condensed ammonia might be subcooled to 75° F., find the effect on the performance and the horsepower per ton.

Example 9. For the conditions of example 6 determine the same items for vapor systems using (a) CO₂, (b) SO₂.

Example 10. The following data were secured from a carbon-dioxide refrigerating machine:

Condenser —

Vapor pressure, 990 lb. abs.; vapor temperature entering, 182° F. (H = 133 B.t.u. per lb.); liquid temperature leaving, 65° F. (H = 20); $t_{\text{sat.}} = 82^\circ \text{F.}$

Circulating water temperature entering, 58° F.; temperature leaving, 68° F.; rate of supply, 32 lb. per minute.

Evaporator —

Vapor pressure, 303 lb. abs.; vapor temperature leaving, 20° F. (H = 108 B.t.u. per lb.); vapor temperature entering ($= t_{\text{sat.}}$) = - 0.5° F.

Brine temperature entering, 24° F.; temperature leaving, 17.5° F.; rate of brine circulation, 58 lb. per minute; specific heat of brine, 0.66.

Compute from these data the capacity in tons at which the plant was operating; estimate from both the condenser and the evaporator data the mass-rate of circulation of the CO₂ (lb. per minute); estimate the power required to drive the compressor (assuming an energy emission from the compressor amounting to 30 per cent of the shaft-work input); estimate the coefficient of performance and horsepower per ton of refrigeration. Also estimate the volumetric efficiency of the compressor if it is single-acting, 2-in. bore by 3½-in. stroke, and is operating at 187 r.p.m. The specific volume of the gas at the state leaving the evaporator is 0.3 cu. ft. per lb.

(1.25 ton; C.P. = 2.5; Vol. eff. = 0.72.)

The superheating of the vapor during compression may be avoided by starting the compression with a wet mixture, such as at point *b*, Fig. 124, so that at the end of adiabatic compression the vapor will be just saturated, as at point *c* of the same figure. This procedure is known as **wet compression**. The wet condition may be obtained by allowing incomplete evaporation in the evaporator, or by expanding liquid directly from the storage tank into the compressor cylinder at the beginning of compression. It may be seen from the *T-S* or *H-S* diagrams, Fig. 125 or 126, that this reduces the amount of work required per pound of refrigerant; but it also reduces the refrigerating effect, so that the net gain is very small. Because of operating difficulties and the small possibilities of gain, the practice of wet compression has fallen into disfavor.

Example 11. For a wet compression cycle operation with the pressures of example 6, compute the necessary quality at the beginning of compression, and the coefficient of performance. (0.89; 5.07)

187. The Air Compression System. — In some circumstances air has been preferred as a refrigerant, due to the undesirably toxic character of some of the vapor refrigerants and the explosive character of others. The apparatus required for an air compression system is in principle the equivalent of that required for a vapor system, *except that* for obtaining the low air temperature it is necessary that the compressed air shall do work in its expansion, instead of being permitted simply to throttle through an expansion valve. An air engine is therefore employed for effecting the pressure and temperature drop which must occur as the air passes to the cooling coils.

The necessity for an energy emission as work in the air expander becomes obvious when it is recalled that with a *perfect* gas the simple throttling or “constant H ” process would cause no temperature drop in the gas, since for such a gas $dH = f(dT)$ (see Arts. 77 and 89). Although with an actual gas such as air there would be some temperature drop on throttling through a considerable pressure range it would be inadequate for purposes of continuous industrial refrigeration.

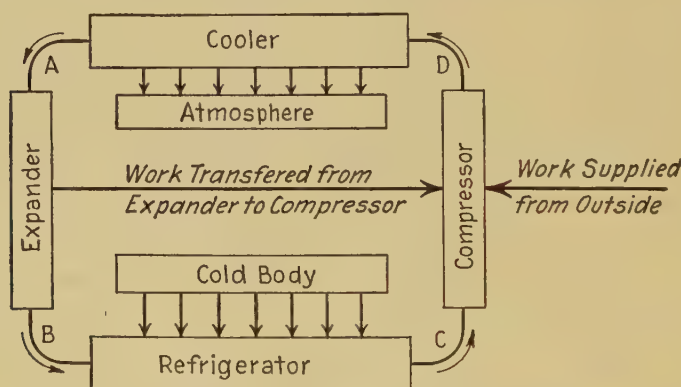


FIG. 130. Elements of air compression type of refrigerating plant.

The arrangement of the elemental components of an air compression system of refrigeration is shown diagrammatically in Fig. 130 and the ideal state changes of the cycle are represented in the T, H - S and P - V diagrams of Figs. 131 and 132, respectively.

Referring to the several locations and state-points indicated on the figures, state a represents air at slightly above atmospheric temperature but under a pressure considerably above atmospheric. The air at this state enters the **expander cylinder** or engine through which it would ideally expand isentropically to a state b , at which state the pressure commonly would still exceed considerably the atmospheric pressure but the

temperature would be 50° F. or more below the temperature of the region which it is desired to refrigerate. The work output from the expander engine is available for and would actually be used for assisting in driving the compressor of the system.

Between states *b* and *c* the air passes at ideally constant pressure through the refrigerating coils and there absorbs energy by heat transition from the cold region, departing from the refrigerating coils at a temperature still moderately below that of the cold region.

After leaving the coils at state *c* the air enters the compressor cylinder and is compressed to its original pressure at *a* but to a temperature considerably above that of the atmosphere. An isentropic state change, *c-d*, is represented in the figures but as usual heat emission from the air

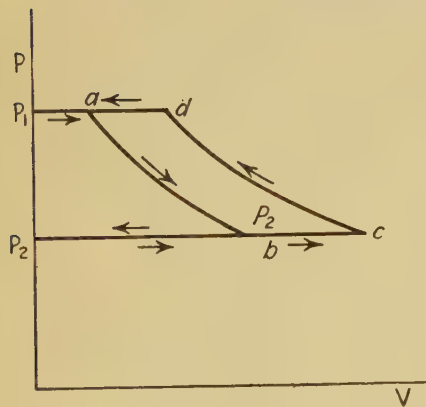


FIG. 132

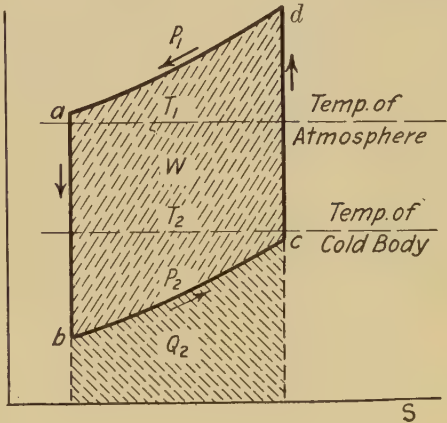


FIG. 131

during its compression would act to reduce the work required for the compression. The work necessary for driving the compressor comes partly from an external source and partly from the expander engine.

Upon leaving the compressor at state *d* the hot air passes at ideally constant pressure through a cooler where it emits energy as heat to the circulating water and is thus recooled to the original state *a*.

Presuming the ideal conditions of constant pressure cooling and warming and isentropic expansion and compression the energy relations for the various phases of the cycle are;

for the expander cylinder, per pound of air,

$$\begin{aligned} W_{out} &= Jc_p(T_a - T_b) \\ &= Jc_pT_b[(P_1/P_2)^{(k-1)/k} - 1]; \end{aligned}$$

for the refrigerating coils,

$$Q_{\text{in}} = c_p(T_c - T_b);$$

for the compressor,

$$\begin{aligned} W_{\text{in}} &= Jc_p(T_d - T_c) \\ &= Jc_pT_c[(P_1/P_2)^{(k-1)/k} - 1]; \end{aligned}$$

for the air cooler,

$$Q_{\text{out}} = c_p(T_d - T_a);$$

for the complete cycle,

$$W_{\text{in, net}} = Jc_p(T_c - T_b) [(P_1/P_2)^{(k-1)/k} - 1].$$

Coefficient of performance

$$\begin{aligned} &= \frac{T_c - T_b}{T_d - T_a - T_c + T_b}, \quad \text{or} \\ &= \frac{1}{(P_1/P_2)^{(k-1)/k} - 1}, \quad \text{or by Eq. 14 of Art. 87,} \\ &= \text{either } \frac{T_b}{T_a - T_b} \quad \text{or} \quad \frac{T_c}{T_d - T_c}. \end{aligned}$$

It may be seen from the T - S diagram of Fig. 131 or from the last expressions above that for a useful temperature range $T_a - T_c$ the air cycle coefficient of performance must be materially lower than that of the Carnot cycle, due to the necessary temperature depression between b and c and the temperature elevation between d and a . At a given capacity the performance may be improved by a more rapid mass-rate of circulation of air, due to the consequent possibility of reducing the temperature range in the refrigerator ($T_c - T_b$) and in the air cooler ($T_d - T_a$).

The air compression system once held sway in marine installations, as the **Dense Air** system, but now it is practically abandoned due partially to its relatively poorer coefficient of performance and partially to operating difficulties arising from the freezing of any moisture in the air and the consequent stoppage of the expander valves, et cetera.

188. The Absorption System; Refrigeration through the Supplying of Energy as Heat. — In the refrigerating systems hitherto considered the energy required for passing the refrigerant from the lower temperature and pressure of the cycle to the upper temperature and pressure was supplied as work at the compressor. This work energy may be regarded as the *available* portion of a quantity of energy which had been

supplied to an engine cycle from some source at a still higher temperature than the upper temperature of the refrigerating cycle.

It is of value to investigate the ideal *over-all performance* obtainable from a composite engine-refrigerator system. Presuming the engine to take energy in the amount Q' from a higher temperature source at temperature T' and to operate between that temperature and an atmospheric temperature of T_1 , then

$$W_{\text{engine}} = Q' \frac{T' - T_1}{T'}.$$

Similarly presuming the heat pump or refrigerator to absorb energy in the amount Q_2 from the cold region at a temperature T_2 and to operate between that temperature and the same atmospheric temperature T_1 , then (from Eq. 1 et sequi, Art. 183)

$$W_{\text{refrigerator}} = Q_2 \frac{T_1 - T_2}{T_2}.$$

If the engine work is directed wholly to driving the heat pump, then

$$Q' \frac{T' - T_1}{T'} = Q_2 \frac{T_1 - T_2}{T_2}.$$

An over-all coefficient for the ideal composite engine-refrigerator system would thus be

$$\frac{\text{Heat from cold region}}{\text{Heat from hot source}} = \frac{Q_2}{Q'} = \frac{T' - T_1}{T'} \frac{T_2}{T_1 - T_2}.$$

To illustrate the quantitative aspects of this relation, presume an engine source at 212°F. , an atmospheric temperature of 62°F. , and a cold region temperature of 32°F. For these conditions the ideal over-all performance would be

$$\frac{\text{Heat from cold region}}{\text{Heat from hot source}} = \frac{212 - 62}{212 + 460} \frac{460 + 32}{62 - 32} = 3.66.$$

It thus appears that each unit of energy passing reversibly from a source at higher temperature through a fairly large temperature *head* could effect the elevation of 3.66 units of energy from the lower temperature region through a moderate temperature *head*.

Returning our attention however to the means for the return of the refrigerant from the lower temperature of the cycle to the upper temperature in the system under consideration, instead of effecting this by work supply through a power-driven compressor, arrangements may also be made whereby it is accomplished through a direct supply of heat energy from a high temperature source. Several typical forms and

arrangements of apparatus have been devised and are in use for this purpose, all of which may be designated, however, as **absorption systems**. All of those in practical use employ ammonia as the refrigerant and act by the alternate absorption of the cold ammonia vapor by some *adsorbent* and subsequent elimination of the ammonia at a higher temperature by the application of heat energy. The system in most common use employs water as the ammonia adsorbent.

The elements of an ammonia absorption system are shown diagrammatically in Fig. 133. The usual condenser, expansion valve and evaporator are evident. Through these occurs the customary progressive flow of the refrigerant. The distinctive items of the system are the **absorber**, **liquid pump**, **heat exchanger**, and **generator**. These jointly

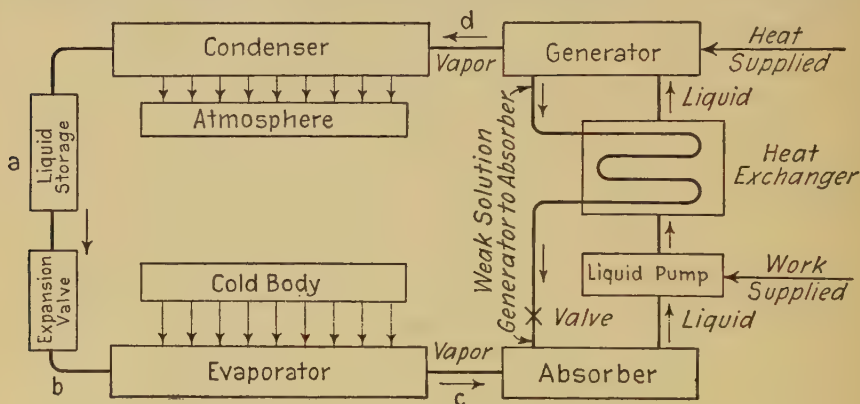


FIG. 133. Elements of vapor-absorption type of refrigerating plant.

replace the conventional compressor of the compression system and through them there is a closed circuit of an ammonia-water solution. The general actions in this circuit are as follows.

The cold vapor leaving the evaporator passes to the absorber and is there avidly absorbed by a weak solution of water and ammonia known as the **weak aqua**. This absorption process has the characteristic of emitting energy, whence it is necessary that cooling water be circulated through coils in the absorber in order to maintain the temperature of the liquid sufficiently low and thus permit the building up of an adequate ammonia concentration. The weak aqua in the absorber thus becomes eventually a **strong aqua** and is then delivered by the liquid pump through the heat interchanger to the generator. In the generator heat energy is supplied, usually by a low pressure steam coil, and the temperature of the strong aqua is thereby raised sufficiently that the ammonia which had been taken up in the absorber is eliminated from the strong aqua

and is thus delivered as ammonia vapor to the condenser. The resulting weak aqua then completes the aqua circuit by returning through the heat exchanger to the absorber. The action in the exchanger is the mutual one of warming the strong aqua which is enroute from the absorber to the generator by the cooling of the weak aqua which is returning from the generator to the absorber.

In certain smaller absorption systems the liquid pump, which is observed to be the only moving part required in the system, is dispensed with by employing the same element alternately as the absorber and as the generator, with a consequent intermittent action of the system. Also there is an ingenious system in use in household units in which the pump is dispensed with by introducing hydrogen gas in the evaporator and absorber, with the result that the system is under virtually the same *total* pressure throughout. Circulation is brought about by differences in the densities of the liquids in various parts of the system. The total pressure in the evaporator is the sum of the partial pressures of the hydrogen gas and the ammonia vapor, but the temperature is established only by the partial pressure of the saturated ammonia vapor.

Various other adaptations of the absorption system employ other absorbents, such as anhydrous ammonium nitrate, silica gel, et cetera.

The energy relations for the condenser, expansion valve and evaporator of an absorption system are in principle the same as those for a compression system but in practical application they may become more complicated because there may be a carry-over of some of the adsorbent (such as water vapor in the aqua system) from the generator to and through the condenser and evaporator. For the absorber, pump, exchanger and generator, together with the various accessory apparatus which are found in the actual plant, the energy relations become still more involved by reason of the semi-chemical character of the processes of vapor absorption by and elimination from the adsorbent. For these reasons no energy analysis of the processes of the absorption system will be presented. The reader may find these in the more recent texts and research reports on refrigeration.

It will have been observed that the elemental requisites for refrigeration are simply a supply of a liquid refrigerant under some pressure, an expansion valve and the coils in which the expanded refrigerant may evaporate at the lower pressure and temperature. If an unlimited supply of the liquid refrigerant were available and if its saturation pressure at the desired refrigeration temperature should perchance be above the pressure of the atmosphere, then continued refrigeration might be secured simply by letting the refrigerant escape. It would thus appear

that the greater portion of the equipment which is employed for effecting a practical, economical and continuously operating refrigeration system is required only in order that the refrigerant may be recovered and be reliquefied at atmospheric temperature.

Example 12. The following data apply to an ammonia absorption system:

Steam coil: pressure, 50 lb. gage; quality, 100 per cent; drain temperature, 84° F.; steam condensed per hour, 1200 lb.

Ammonia condenser: pressure, 150 lb. per sq. in. gage.

Ammonia evaporator: pressure, 20 lb. per sq. in. gage; refrigerating effect, 150,000 B.t.u. per hour.

Compute the refrigerating effect obtained per unit (B.t.u.) of heat supplied by the steam and compare with that ideally obtainable with the existing steam, condenser and evaporator temperatures.

189. Summary. — The refrigerating process is that of the reversed heat engine. Heat is removed from a region at a temperature below atmospheric by the application of available energy in the form of work or of heat supplied at a temperature above atmospheric. The coefficient of performance is the ratio of the heat removed from the cold region to the available energy supplied. For the Ideal Carnot refrigerating machine the coefficient of performance is $\frac{T_2}{T_1 - T_2}$, where T_2 is the refriger-ator temperature and T_1 is the condenser or atmospheric temperature. This is the highest coefficient of performance attainable for refrigerating machines operating between temperatures T_1 and T_2 .

The vapor compression system differs in principle from the Carnot cycle in the substitution of a throttling expansion of the vapor for the reversible adiabatic expansion process of the Carnot cycle. The vapor absorption system replaces the compression process by the absorption of the vapor by some adsorbent and its elimination under a higher temperature and pressure by the application of heat.

The air compression system retains both the compression and the expansion processes of the Carnot cycle, but it is unable to realize or even approach the constant temperature absorption or constant temperature rejection of heat by and from the refrigerant which is typical of the Carnot cycle.

190. Review Questions and Topics. —

1. Describe the Carnot refrigeration cycle and derive expressions for its coefficient of performance.
2. Prove that the coefficient of performance of the Carnot refrigeration cycle is the maximum attainable for a given temperature range.
3. How may the coefficient of performance of a refrigerator be improved?
4. Describe the vapor compression refrigeration system and derive expressions for its coefficient of performance.

5. How does the actual vapor compression system differ from the ideal? What are the possibilities for improving economy of this system?

6. Describe the air compression refrigeration system and derive its coefficient of performance in terms of temperatures. Why is this system less economical than the vapor compression system?

7. Describe the vapor absorption system. How does it differ from the vapor compression system? What is its field of use?

Symbols and Abbreviations, Chapter XVIII

C.P. = coefficient of performance.

c_p = specific heat at constant pressure.

H = enthalpy, B.t.u. per pound of fluid.

J = Joule's equivalent (778 ft-lb. per B.t.u.).

k = specific heat ratio, c_p/c_v .

P = pressure, pounds per square foot, absolute.

P_1 = pressure of refrigerant in condenser or cooler.

P_2 = pressure of refrigerant in evaporator or refrigerator.

Q = energy transferred as heat, B.t.u. per pound of fluid.

S = entropy per pound mass of fluid.

T = temperature, degrees Rankine (degrees Fahrenheit absolute).

V = specific volume, cubic feet per pound.

PART V

CHAPTER XIX

GENERAL THERMODYNAMIC EQUATIONS

191. Foreword. — The equations which have been developed and used in the preceding parts of this work have been of three types: (a) equations which express the character of the energy relations during a process, such as the steady-flow energy equation; (b) equations representing relations existing between the various properties of *particular substances*, such as the characteristic equation, or *equation of state*, of a perfect gas; and (c) equations which are combinations of these two, such as the equation for the flow of air through nozzles. Besides these types there is a group of equations expressing various general relations which must and invariably will be found to exist between and among the various thermodynamic properties of any substance. These equations are called the **General Thermodynamic Equations**, because their application is perfectly general and they will express the inter-relations between the thermodynamic properties of *any homogenous substance whatsoever*.

These equations would rarely need to be used today in the ordinary applications of *engineering* thermodynamics, for the reason that there exist fairly accurate and complete tables of the numerical values of all needed properties of the various engineering fluids. However, the very evolution of these tables by the physicists became possible only through the application of the general equations to certain limited, experimentally determined data concerning the properties of those fluids. The general equations also play an important part in developing the general theory of thermodynamics, and further they open up and lead to a wide field for the application of the science of thermodynamics in the sciences of physics and chemistry.

It is the purpose of this **Part V** to develop and illustrate the use of the general thermodynamic equation in a brief but fairly comprehensive manner. Before undertaking the study of this material, however, the student should review that part of the differential calculus which pertains to partial and total differentiation and to exact and inexact differentials. Articles 192 and 193 are presented to assist in this review. Part V also presupposes a knowledge and understanding of Parts I and

II of the present work. Such an understanding is necessary for the reason that a grasp of the mathematical method of thermodynamics depends fully as much upon a clear appreciation of the physical meaning of the quantities and equations considered as upon a facility in making the mathematical transformations which are involved.

192. Exact Differentials; Point Functions. — It is shown in the differential calculus that if, in mathematical parlance, a variable z is a continuous function of two other variables x and y ; that is, if, in mathematical symbols,

$$z = f(x, y), \quad (1)$$

then

$$dz = \left(\frac{\partial z}{\partial x} \right)_y dx + \left(\frac{\partial z}{\partial y} \right)_x dy. \quad (2)$$

A physical interpretation of these mathematical concepts may readily and advantageously be made if one recalls (Art. 10) that two properly selected physical properties of a fluid serve to determine its state and thus to determine any other properties at that state. To illustrate, for a perfect gas given values of P and V denote one particular state and thus one unique value of T ; whence, according to the equation of state,

$$\begin{aligned} T &= PV/R, \quad \text{or} \\ T &= f(P, V). \end{aligned}$$

Recall further in this connection that if the three properties are employed as linear magnitudes on three coördinates, as represented graphically in Fig. 134, then the locus of all points representing possible states of the fluid would be a surface in space, a portion of which surface for a gas is represented in the figure by the surface $ABCD$. If two given states of the fluid are denoted by points 1 and 2 on the surface, then it becomes evident that the total change of temperature associated with a state change from 1 to 2 would be the same irrespective of the path which might be followed in passing from 1 to 2. Let us presume therefore for simplicity a composite path made up as shown of, first, a constant pressure state change and then a constant volume state change. Unquestionably the total temperature change dT would equal the algebraic sum of the temperature changes during each of the individual components of the total state change. But the portion of the total temperature change which would occur during the constant pressure component of the state change must equal the *rate* of change of temperature with respect to change of volume times the change of volume, or $(\partial T)_P = \left(\frac{\partial T}{\partial V} \right)_P dV$. Similarly the portion of the total temperature

change which would occur during the constant volume component of the state change would equal the *rate* of temperature change with respect to change of pressure times the change of pressure, or $(\partial T)_V = \left(\frac{\partial T}{\partial P}\right)_V dP$. Therefore

$$dT = (\partial T)_V + (\partial T)_P = \left(\frac{\partial T}{\partial P}\right)_V dP + \left(\frac{\partial T}{\partial V}\right)_P dV.$$

The identity of this relation and equation 2 is apparent.

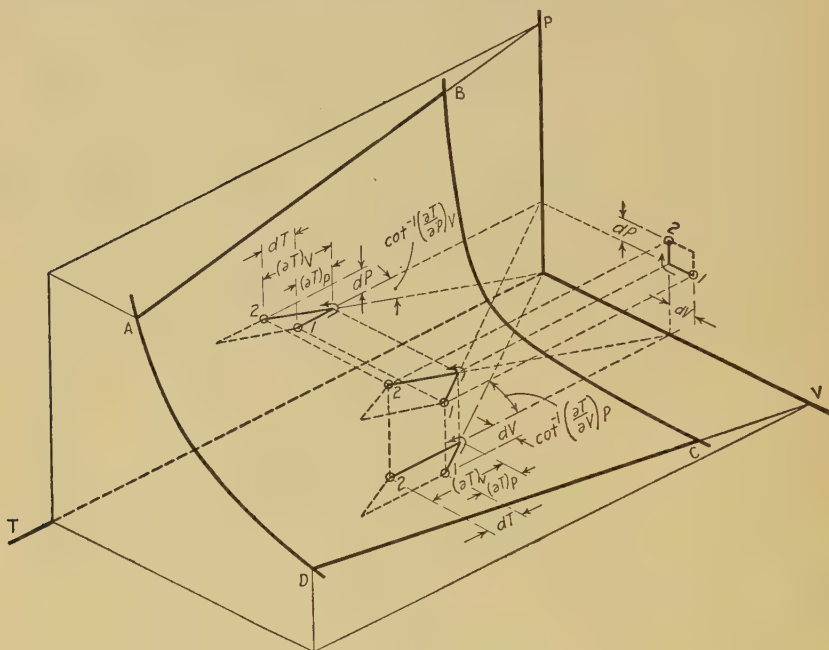


FIG. 134. P - V - T surface for a gas.

It is convenient to rewrite equation 2 in the form

$$dz = M dx + N dy. \quad (2a)$$

In the above, and again employing mathematical parlance,

$\left(\frac{\partial z}{\partial x}\right)_y = M$ = the **partial derivative** (or *differential coefficient*) of z with respect to x , that is, with x variable and y constant;

$\left(\frac{\partial z}{\partial y}\right)_x = N$ = the partial derivative of z with respect to y , that is, with y variable and x constant;

$M dx$ = the **partial differential** of z with respect to x ;

$N dy$ = the partial differential of z with respect to y ;

dz = the **total differential** of z .

In connection with the above physical interpretation of these several mathematical relations it is again to be emphasized that each of the variables which were involved in the relations (as T , P and V) were *properties* and as such were items the magnitude of which would have an invariable value at any specific state of a fluid irrespective of the manner in which the fluid might have arrived at that state. Furthermore, as properties, they were items the total change in the magnitude of which would be the same for any state change from one particular state of the fluid to a second particular state irrespective of the manner or *path* by which the state change were effected.

In mathematical parlance such properties, regarded now simply as variables between which there will exist for any particular fluid an inflexible **functional relation** (such as an equation of state of the fluid), would be known as **point functions**. Also their change, in any state change of infinitesimal magnitude, would be regarded and designated as an **exact differential**. With respect to such functions and differentials, the differential calculus further shows that the following important relation will hold between the partial differential coefficients M and N of equation 2a:

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y. \quad (3)$$

To prove this relation note that, as $M = \left(\frac{\partial z}{\partial x}\right)_y$, then

$$\left(\frac{\partial M}{\partial y}\right)_x = \frac{\partial^2 z}{\partial x \partial y}.$$

Similarly, as $N = \left(\frac{\partial z}{\partial y}\right)_x$, then

$$\left(\frac{\partial N}{\partial x}\right)_y = \frac{\partial^2 z}{\partial y \partial x}.$$

Since the order of differentiation is immaterial the truth of equation 3 is thus verified.

The general relationship of equation 3 is of great utility in evolving various of the general thermodynamic relations between the properties of fluids and will be applied quite extensively in the following developments. The relation has as well a utility as a test for ascertaining if any proposed thermodynamic equation has the nature of a functional relation between properties. Expressed alternatively, the equation serves

to distinguish between *exact* differentials and the so-called *inexact* differentials. This feature is considered in the next article.

Several related characteristic features which pertain to the point function and its exact differential are:

- (a) An exact differential dz may be integrated directly into $z_2 - z_1$, without knowledge of the manner of variation of z between the points or states 1 and 2. Thus $\int_1^2 dV = V_2 - V_1$.
- (b) The process of integrating dz around a closed cycle is represented by the symbol $(\oint) dz$. If dz is an exact differential $(\oint) dz = \text{zero}$.

It is well to recall the various items which have been shown, in Parts I and II of this work, to be fluid properties and thus to be variables between which definite functional relations will exist for any fluid and further to be items an infinitesimal change of which may be taken as an exact differential. Additional useful properties will be considered in subsequent articles. Those to which attention has been given to this point are, in the order of their earlier presentation:

Internal energy, E , which was described (Art. 5) as the measure of the relative amount of energy stored at a particular state of a fluid in the system of molecules, atoms, electrons et cetera which comprises the fluid. From its very significance the item is a property and point function. *The notion of its being a property is frequently regarded as the fundamental embodiment of the First Law of Thermodynamics.*

Pressure, P , which was regarded (Art. 8) as the result of a molecular bombardment phenomenon and thus as an external evidence of internal state. It is one of the few properties for the direct measurement of which we have convenient physical instruments.

Specific volume, V , (or density, D), which (Art. 9) upon purely physical considerations is obviously a function of the state. This property is indirectly measureable but rather less easily than is pressure.

Temperature, T , which initially was regarded as a further external evidence of internal state (Art. 10), but to which additional extremely important and fundamental attributes were subsequently ascribed (Chapter VI). Temperature is the other of the properties for the direct measurement of which we have convenient physical instruments.

Enthalpy, H , which was seen (Art. 32) to be a composite property, $E + PV$, of marked convenience in connection with the analysis of all steady-flow processes.

Entropy, S , which is an indirect measure of the relative unavailable energy attributable to a fluid in any state (Art. 53) and the change of which during any process was shown (Art. 54) to depend only on the initial and the final state of the fluid. Entropy thus possesses the fundamental attribute of a property or point function and in the following needs scarcely to be regarded from any viewpoint other than that of a property. *This property characteristic of entropy is sometimes regarded as the fundamental embodiment of the Second Law of Thermodynamics.*

Example 1. An equation of state which was proposed by *van der Waals* for representing the P - V - T relation for actual (imperfect) gases and vapors is as follows:

$$P = \frac{RT}{V-b} - \frac{a}{V^2},$$

where a , b and R are constants.

Determine the partial derivatives of P with respect to T and to V and, by comparing the further derivatives of these with respect to the other variable $\left(\frac{\partial^2 P}{\partial T \partial V}\right)$, ascertain if mathematically the proposed equation of state is a suitable functional relation for correlating the properties.

Solution.

$$\begin{aligned} dP &= \left[\frac{\partial \left(\frac{RT}{V-b} - \frac{a}{V^2} \right)}{\partial T} \right]_V dT + \left[\frac{\partial \left(\frac{RT}{V-b} - \frac{a}{V^2} \right)}{\partial V} \right]_T dV \\ &= \left[\frac{R}{V-b} - 0 \right] dT + \left[\frac{-RT}{(V-b)^2} + \frac{2a}{V^3} \right] dV \\ M &= \frac{R}{V-b}; \quad N = \frac{-RT}{(V-b)^2} + \frac{2a}{V^3} \\ \left[\frac{\partial M}{\partial V} \right]_T &= \frac{\partial^2 P}{\partial T \partial V} = \frac{-R}{(V-b)^2} \\ \left[\frac{\partial N}{\partial T} \right]_V &= \frac{\partial^2 P}{\partial V \partial T} = \frac{-R}{(V-b)^2}. \end{aligned}$$

The proposed equation therefore satisfies the mathematical requirements

Example 2. A general equation of state for vapors and imperfect gases which was proposed by *Callendar* is:

$$V = \frac{RT}{P} - \frac{c}{T^n} + b,$$

where b , c , n and R are constants.

Determine the partial derivatives of V with respect to T and to P and thus express the total differential of V and proceed as in example 1 to check the general mathematical propriety of the equation as a functional relation between properties.

$$\left(\frac{\partial^2 V}{\partial T \partial P} = -\frac{R}{P^2} \right).$$

193. Inexact Differentials. — There are many circumstances in which differential equations which are of the same form as equation 2a may correctly express a relation between several physical quantities but at

the same time may not be based upon any general functional relation between the several variables. As an illustration from the engineering thermodynamics, recall that for any device through which steady-flow occurs (but without friction and with no appreciable kinetic energy change — see Art. 36, Eq. 9) it develops that

$$\begin{aligned} W &= - \int_1^2 P dV + P_2 V_2 - P_1 V_1 \\ &= - \int_1^2 P dV + \int_1^2 d(PV). \end{aligned}$$

Writing the last equation in differential form

$$\begin{aligned} dW &= -P dV + d(PV) \\ &= -P dV + P dV + V dP \\ &= 0 dV + V dP. \end{aligned} \tag{4}$$

Referring now to equation 2a ($dz = M dx + N dy$), equation 4 is seen to be of quite the same form, with the several coefficients M and N having the particular values

$$M = 0; N = V.$$

However, if we take the second derivatives of the coefficient, that is, $\left(\frac{\partial M}{\partial P}\right)_V$ and $\left(\frac{\partial N}{\partial V}\right)_P$, it develops that

$$\left[\frac{\partial M}{\partial P}\right]_V = 0; \left[\frac{\partial N}{\partial V}\right]_P = \left[\frac{\partial V}{\partial V}\right]_P = 1.$$

The two are not the same and the required test of equality of the two second derivatives, as established by equation 3, is not met and it must be concluded that W may not be taken as a function of P and V .

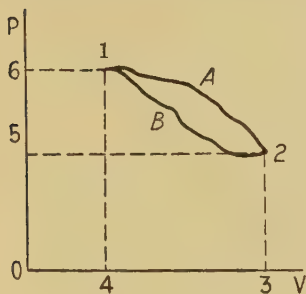


FIG. 135

A physical interpretation of this mathematical evidence may readily be obtained by recalling that, referring for illustration to an expansion process in an engine, a fluid may exist at a given initial state 1 and may pass by any of several processes or *paths* to a second given state 2 but may effect distinctly different amounts of work delivery in the two processes if the particular paths between the given initial and final states should differ. Thus, in Fig. 135, if path 1A2 should be followed in instance A the ideal work output would equal area 41A23, whereas if

in instance B the path $1B2$ should be followed its work output would equal area $41B23$. The conclusion is that the work obtained in passing between the two specified states 1 and 2 is not fixed by the values of the properties at those states and that therefore W may not be taken as a function of P and V but depends instead on their manner of variation in passing between the two states. This is in distinction to the fact that the ultimate change in any property in passing from state 1 to state 2 would depend solely on the initial and final state and would be wholly independent of the path followed.

In the above analysis the symbol dW was employed to represent the small quantity of work energy transferred during a state change between two states which differed infinitesimally. Because of an apparent similarity between the term dW in equation 4 and the term dz in equation 2a, which latter term is properly designated as a *differential*, dW might perhaps be likewise termed a differential, although probably incorrectly so. However if that phraseology is employed the fact that W is not a function of the variables P and V and that dW does not depend solely on the magnitude of their change may be designated by describing the quantity as an *inexact differential*. This is in distinction to the term *exact differential*, which was employed in the preceding article to denote the change in any one variable during a change of any other variables of which the first is a definite function. As a symbolical distinction between the exact and inexact differential we shall hereafter employ d' instead of d , as for example $d'W$ instead of dW .

It will be observed that $d'W$ is a *small quantity of work energy* which is transferred during an infinitesimal state change but which varies in amount with the character of the state change even though the change may be between two specific states. In a like manner it might have been developed that the *amount of heat energy* $d'Q$ which is transferred during an infinitesimal state change is not an exact differential but depends on the character of the state change. Alternatively, Q is not a property and no functional relation may exist between Q and the properties of a fluid.

194. The General Energy Equation.—In evolving any general thermodynamic relations which shall apply to the properties of all fluids the sole wholly general starting points are the First and Second Laws and the Carnot Principle. The First Law is expressed most conveniently in the energy equations. In Chapter III it was found advantageous for the subsequent developments of the engineering aspects of the thermodynamics to present two particular equations namely,

For non-flow processes (Art. 34)

$$W + Q = E_2 - E_1 \quad \text{and}$$

For steady-flow processes (Art. 31)

$$\begin{aligned} \frac{gZ_1}{32.17} + \frac{U_1^2}{64.34} + P_1V_1 + E_1 + W + Q \\ = \frac{gZ_2}{32.17} + \frac{U_2^2}{64.34} + P_2V_2 + E_2, \quad \text{or} \\ \left[W + \frac{g(Z_1 - Z_2)}{32.17} + \frac{U_1^2 - U_2^2}{64.34} + (P_1V_1 - P_2V_2) \right] + Q = E_2 - E_1. \end{aligned}$$

In these equations the symbol W was assigned solely to that particular mechanical energy which might be in transition as *shaft-work*. For convenience let the significance of the symbol be now extended to include all energies of a mechanical nature, that is, all *Mechanical Effects* (Art. 36). This is reasonable because all of the mechanical forms of energy will have their origin in the action of some form of force through a distance. It also enables the writing of a *general* energy equation in the simple form,

$$W + Q = E_2 - E_1. \quad (5)$$

Scrutinizing this equation it will be observed that although all terms are energy quantities only the internal energies E_1 and E_2 are properties. In order, therefore, that general property relations may be evolved from this energy equation it will be necessary to re-express the items W and Q in terms of properties. It is significant to note that in effecting this necessary transition it will be permissible to evaluate W and Q for processes which are idealized in the feature of being *mechanically reversible*, even though experience indicates that actually such processes are never attained. This is permissible for the reason that any functional

¹ In these and all subsequent relations of Part V it is implied that a consistent system of (energy) units is used throughout, for which reason no symbol for the conversion of units (such as the Joule equivalent, J) is employed. This is in conformity with the common practice in scientific work, in which an energy unit such as the erg is employed uniformly for all mechanical, thermal or other forms of energy. In such work specific heats are likewise evaluated in the same basic energy unit. Also when such a system of units is employed there is no need for the use of the proportionality constant 32.17 (see footnote 1, Art. 3) which has had to be introduced in the foregoing by reason of the engineering usage of the pound *force* and the pound *mass*. Alternatively, the factor is *unity* in the scientific system of units.

As a consequence of these considerations the latter of the above relations would frequently appear in the form

$$W + g(Z_1 - Z_2) + \frac{U_1^2 - U_2^2}{2} + (P_1V_1 - P_2V_2) + Q = E_2 - E_1.$$

relation between properties which would hold for the ideal process must likewise hold for any manner by which a given state change might be effected. This follows from the inherent *point-function* characteristic of a property.

Relative to the term W , it was shown in Art. 36 that for a mechanically reversible process the composite mechanical effects may be uniquely evaluated by the quantity $-\int P dV$. Recognition of this enables the elimination of the non-point function W and the introduction of the point functions P and V in the energy equation.

Relative to the term Q , Part II of this work was devoted almost exclusively to a development of the entropy property from the Carnot Principle and the Second Law, together with the Kelvin temperature scale. This development yielded the important relation that, again for mechanically reversible processes, $Q = \int T dS$. Application of this relation will serve to introduce the Carnot Principle and the Second Law into the general energy equation, all of which were noted above as fundamental starting points. It will also eliminate the non-point function Q and will introduce instead the point functions S and T .

By thus replacing W and Q in the general energy equation there is obtained the very fundamental and wholly general property relation

$$-\int P dV + \int T dS = E_2 - E_1, \quad (6)$$

or expressed in differential form,

$$T dS - P dV = dE. \quad (6a)$$

Equation 6a contains only exact differentials and point functions and becomes the thermodynamic basis of the entire mathematical development which follows.

195. Additional Thermodynamic Properties. — Of the properties which have already been considered and which were listed in Art. 192 those appearing in equation 6a are P , V , T , S and E . These may be regarded as the fundamental state functions and the absolute minimum number which are required in the science.

The composite **enthalpy** property, $E + PV$, has also been noted as an additional one of special convenience in connection with any steady-flow process. This property has the further convenient attribute that for any constant pressure process its change invariably evaluates the

energy transition to or from the fluid as heat. Thus, employing equation 6,

$$\begin{aligned}(Q)_P &= E_2 - E_1 + \int_1^2 P dV = E_2 - E_1 + P(V_2 - V_1) \\ &= E_2 - E_1 + P_2 V_2 - P_1 V_1 \\ &= (E_2 + P_2 V_2) - (E_1 + P_1 V_1) \\ &= H_2 - H_1,\end{aligned}$$

where, by definition,

$$H = E + PV. \quad (7)$$

Other names that have been assigned to this property and that have been discussed in Art. 32, are; *total heat*, *heat content*, *thermodynamic potential*, and *X (chi)-function* (Gibbs). Other symbols are *h*, *i* and *X*.

Further composite properties which are found to be of outstanding utility in the applications of thermodynamics to chemical processes (including combustion) are: $E - TS$ and $E - TS + PV$, or $H - TS$.

The function $E - TS$ has the special convenient attribute that for a reversible isothermal process its change evaluates the mechanical effects. Thus, again employing equation 6,

$$\begin{aligned}(W)_T &= E_2 - E_1 - \int_1^2 T dS = E_2 - E_1 - T(S_2 - S_1) \\ &= E_2 - E_1 - T_2 S_2 + T_1 S_1 \\ &= (E_2 - T_2 S_2) - (E_1 - T_1 S_1) \\ &= \Psi_2 - \Psi_1,\end{aligned}$$

where, by definition,

$$\Psi = E - TS. \quad (8)$$

The symbol Ψ (psi) for the composite property or state function $E - TS$ is that employed by the eminent physicist, Willard Gibbs, who first recognized the utility of the function. It has also been designated by the symbols F , F_v and A . Similarly it has been assigned a considerable and confusing variety of names; *free energy*, *thermal potential*, *work content*, et cetera. We shall employ the name used by Gibbs, the **Ψ -function**.

If the pressure and temperature are both constant during a reversible process (such as the formation of vapor from a liquid at constant pressure) we have that

$$\begin{aligned}(W)_P &= - \int_1^2 P dV = -P(V_2 - V_1) \\ &= -P_2 V_2 + P_1 V_1,\end{aligned}$$

and also, from equation 8, that

$$(W)_T = (E_2 - T_2 S_2) - (E_1 - T_1 S_1).$$

Therefore, if both P and T are constant and thus $(W)_P = (W)_T$,

$$(E_2 - T_2 S_2) - (E_1 - T_1 S_1) = -P_2 V_2 + P_1 V_1$$

or
$$E_2 + P_2 V_2 - T_2 S_2 = E_1 + P_1 V_1 - T_1 S_1$$

and
$$(Z_2)_{P,T} = (Z_1)_{P,T} \quad \text{or} \quad (\Delta Z)_{P,T} = 0,$$

where, by definition, $Z = E + PV - TS = H - TS. \quad (9)$

The property $(E + PV - TS)$ is thus an *energy* function which is constant during a mechanically reversible process conducted at constant temperature and pressure. For this function we shall employ the symbol Z (zeta), following Gibbs. It has also been represented variously by $-G, F, F_p, \Psi$ and Θ . It has been designated as the *Gibbs function* and also by the same names as were assigned to the Ψ -function that is, *thermal potential* and *free energy*. It has seemed preferable in this work to avoid the use of semi-descriptive names (the use of which has too frequently led to confusion of thought) and to designate the function simply as the **Z-function**.

The eight foregoing functions are the working material of the mathematical analysis. Summarizing, they are P, V, T and S , which four may be regarded as more directly *property* functions, and E, H ($= E + PV$), Ψ ($= E - TS$) and Z ($= E + PV - TS$). These last four will be observed either to measure energy or to have the dimensions of energy and so may be regarded as *energy* functions (sometimes *potentials*).

Example 3. Verify the conclusions given in the last three columns for mechanically reversible processes in which certain properties are held constant as indicated in the first column.

Property or properties held constant	Mechanical Effects	Heat	Equalities of properties
P	$P_1 V_1 - P_2 V_2$	$H_2 - H_1$	
V	zero	$E_2 - E_1$	
T	$\Psi_2 - \Psi_1$	$T_2 S_2 - T_1 S_1$	
S	$E_2 - E_1$	zero	
$P \& T$	$P_1 V_1 - P_2 V_2$	$T_2 S_2 - T_1 S_1$	$Z_1 = Z_2$
$V \& T$	zero	$E_2 - E_1$	$\Psi_1 = \Psi_2$

196. The Maxwell Relations and Other Relations between State Functions. — A very great number of relations may be written connecting the eight state functions.² In this work only a few are evolved that

² Bridgman in his *Condensed Collection of Thermodynamic Formulas* (Harvard University Press) presents an ingenious collection of tables by which any of the general thermodynamic relations may be readily obtained. He notes that including relations involving W and Q 11×10^6 relations between first derivatives are possible and 9.5×10^{21} between second derivatives.

have been found to be particularly useful in connection with the engineering aspects of thermodynamics. A first group of these consists of relations derived from the energy equation (Eq. 6 or 6a) and those defining the several composite functions H , Ψ and Z (Eq. 7, 8 and 9). Their derivation is effected by equating partial derivatives (differential coefficients) in accordance with the method of equation 3. The application of this device is permissible because of the point function character of those composite functions.

In the following the relations are developed quite directly but in a manner which it is believed will be self-explanatory. In the tabular arrangement the sources appear in the left columns, together with certain modified forms of the original defining relations. The result of the equating of differential coefficients appears in the right columns.

Source Relation, etc.	Resulting (Maxwell) Relations
$dE = T dS - P dV$ (6a)	$\left(\frac{\partial T}{\partial V}\right)_S = - \left(\frac{\partial P}{\partial S}\right)_V$ (13)
$dH = dE + d(PV)$ (7) $= (T dS - P dV) + (P dV + V dP)$ $= T dS + V dP$ (10)	$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$ (14)
$d\Psi = dE - d(TS)$ (8) $= (T dS - P dV) - (T dS + S dT)$ $= -S dT - P dV$ (11)	$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$ (15)
$dZ = dH - d(TS)$ (9) $= (T dS + V dP) - (T dS + S dT)$ $= -S dT + V dP$ (12)	$\left(\frac{\partial S}{\partial P}\right)_T = - \left(\frac{\partial V}{\partial T}\right)_P$ (16)

Equation 6a is the customary energy equation but now may be regarded simply as a definitive relation correlating the change of internal energy with the change or the magnitude of the four property functions P , V , T and S . Equations 10, 11 and 12 are obtained by combining equations 7, 8 and 9 with the energy equation (6a) and provide alternative definitive relations which correlate the three additional energy functions with the change or magnitude of the four property functions.

Equations 13, 14, 15 and 16 are known as the **Maxwell Relations**. Each is evidently developed by equating the partial differential coefficients of equations 6a, 10, 11 and 12 respectively. They provide significant relations between the property functions of a fluid and are relations

that must maintain for any fluid whatsoever. Their physical significance is illustrated in a subsequent example.

By writing equations 6a, 10, 11 and 12 for the special cases of constancy of one of the variables another set of unique property function and energy function relations may be evolved. These again are relations that must be found to hold for any fluid. The following tabulation showing sources, the constant property and the resulting relations should be self-explanatory.

Sources	Resulting Relations
From eq. 6a, if S is constant, $\left(\frac{\partial E}{\partial V}\right)_S = -P$; and from eq. 11, if T is constant, $\left(\frac{\partial \Psi}{\partial V}\right)_T = -P$.	$\left(\frac{\partial E}{\partial V}\right)_S = \left(\frac{\partial \Psi}{\partial V}\right)_T = -P \quad (17)$
From eq. 6a, if V is constant, $\left(\frac{\partial E}{\partial S}\right)_V = T$; and from eq. 10, if P is constant, $\left(\frac{\partial H}{\partial S}\right)_P = T$.	$\left(\frac{\partial E}{\partial S}\right)_V = \left(\frac{\partial H}{\partial S}\right)_P = T \quad (18)$
From eq. 10, if S is constant, $\left(\frac{\partial H}{\partial P}\right)_S = V$; and from eq. 12, if T is constant, $\left(\frac{\partial Z}{\partial P}\right)_T = V$.	$\left(\frac{\partial H}{\partial P}\right)_S = \left(\frac{\partial Z}{\partial P}\right)_T = V \quad (19)$
From eq. 11, if V is constant, $\left(\frac{\partial \Psi}{\partial T}\right)_V = -S$; and from eq. 12, if P is constant, $\left(\frac{\partial Z}{\partial T}\right)_P = -S$.	$\left(\frac{\partial \Psi}{\partial T}\right)_V = \left(\frac{\partial Z}{\partial T}\right)_P = -S \quad (20)$

The physical significance of the foregoing general equations is illustrated by the following examples, in which approximate checks of several of the equations are made by analyses of actual property data from tables of the properties of steam.

Example 4. Illustrate and make an approximate check of equation 10 by use of steam table data, doing so by evaluating ΔH and $(T\Delta S + V\Delta P)$ for some small state change and comparing the two. Any character of state change will serve. (The results for a change of superheated steam from 88 lb. per sq. in. abs. and 330° F. to 89 lb. and 320° F., using arithmetical averages for T and V during the state

change, are $\Delta H = -4,980$ ft.-lb. and $(T\Delta S + V\Delta P) = -4,800$ ft.-lb. The approximateness of the check results from the finite magnitude of the state change.)

Example 5. Selecting any desired reference state of some fluid for which tables of properties are available, determine from the tabular data and by any necessary computations several values of T and V for an isentropic state change through the reference state and then plot by use of these data a constant entropy curve as it would appear on T - V coordinates. Secure data for at least one point near to but on each side of the reference state point. Proceed similarly to develop a constant volume curve on P - S coordinates through the same reference point. By finding the scalar slope of the two curves at the reference point evaluate the several partial derivatives of equation 13 and thereby check their equality.

Solution.—For steam at a reference condition of dry saturated steam at 200 lb. per sq. in. abs.

At reference point, $S = 1.5450$, $V = 2.285$, $T = 841.8^\circ \text{R.}$

At $S = 1.5450$ and $T = 850^\circ \text{R.}$, $V = 2.209$.

At $S = 1.5450$ and $T = 839.7^\circ \text{R.}$, $V = 2.337$.

At $V = 2.285$ and $P = 206$ lb. per sq. in., $S = 1.5553$.

At $V = 2.285$ and $P = 195$ lb. per sq. in., $S = 1.5224$.

By plotting the constant entropy curve and ascertaining its scalar slope at the reference point it appears that at that point

$$\left(\frac{\partial T}{\partial V}\right)_S = (\text{closely}) - 43.$$

By plotting the constant volume curve and ascertaining its scalar slope at the reference point it appears that at that point

$$\left(\frac{\partial P}{\partial S}\right)_V = (\text{closely}) + 44.$$

Example 6. Selecting any desired reference state for some fluid such as steam, proceed along lines parallel to those of example 5 and make approximate checks of equations 14, 15, and 16.

Example 7. Selecting any desired reference state for some fluid such as steam proceed along lines parallel to those of example 5 and make approximate checks of both portions of equation 18.

197. Thermal Capacities and Thermal Capacity Relations.—In endeavoring to determine the characteristics and properties of any particular fluid the types of data that are most readily obtainable in the physical laboratory are of three general classes. The first class includes the determinations of simultaneous values of the property functions, pressure, temperature and specific volume (P , T and V), at a considerable variety of states. Certain of the inter-relations between those property functions and the other energy functions have been presented in the preceding article.

The data of the second class are secured by ascertaining the character of the property changes which result if the fluid is caused to pass through an adiabatic but wholly irreversible throttling process. The significance and utility of such data are considered in a following article.

The data of the third class involve the calorimetric determination of the energy required as heat to effect a *unit* change in the magnitude of the pressure, the temperature, or the specific volume of a *unit* mass of the fluid if or when one of those properties is maintained constant. Such a characteristic of the fluid is known as a **thermal capacity**. The familiar *specific heat at constant pressure*, c_P , is thus one of the several thermal capacities, being the amount of heat energy required to change the temperature of one pound of the substance one degree while the pressure is maintained constant. It is the purpose of the present article to present several significant relations between the thermal capacities and the property functions.

In conformity with the above general definitions of thermal capacity there might be a total of six thermal capacity ratios. Only four however are of particular practical utility. These, together with their names and conventional symbols, are:

$$\left(\frac{\partial'Q}{\partial T}\right)_P = c_P; \text{ Specific heat at constant pressure.} \quad (21)$$

$$\left(\frac{\partial'Q}{\partial T}\right)_V = c_V; \text{ Specific heat at constant volume.} \quad (22)$$

$$\left(\frac{\partial'Q}{\partial P}\right)_T = l_P; \text{ Latent heat of pressure change.}^3 \quad (23)$$

$$\left(\frac{\partial'Q}{\partial V}\right)_T = l_V; \text{ Latent heat of expansion.}^3 \quad (24)$$

The notation $\partial'Q$ which appears in these relations is a reminder of the important consideration that Q is not a point function. It follows that the thermal capacities are not in themselves exact partial derivatives. This consideration is given further attention in the next paragraph. In the interim recall, however, that, in accordance with the Second Law as embodied in the expression for the entropy function, $\partial'Q = T \partial S$ for a mechanically reversible process (Arts. 54 and 55).

In order to evolve the various desired relations between the several thermal capacities and the property functions it needs further to be recalled that, as entropy is a point function and the change of entropy dS between any two states of a fluid is therefore independent of the path followed, it is permissible to express the entropy change for any total state change as the sum of the several changes in any desired sequence and types of paths. Thus, in a form parallel to equation 2,

³ Due to the familiar usage of the term *latent heat* in connection with the common process of vaporization of a saturated liquid at constant temperature the term persists in connection with any isothermal process with any fluid.

$$dS = \left(\frac{\partial S}{\partial T}\right)_P dT + \left(\frac{\partial S}{\partial P}\right)_T dP$$

or

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV.$$

But

$$\left(\frac{\partial S}{\partial T}\right)_P = \left(\frac{1}{T} \frac{\partial' Q}{\partial T}\right)_P = \frac{c_P}{T} \text{ (from eq. 21);}$$

$$\left(\frac{\partial S}{\partial P}\right)_T = \left(\frac{1}{T} \frac{\partial' Q}{\partial P}\right)_T = \frac{l_P}{T} \text{ (from eq. 23);}$$

$$\left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{1}{T} \frac{\partial' Q}{\partial T}\right)_V = \frac{c_V}{T} \text{ (from eq. 22);}$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{1}{T} \frac{\partial' Q}{\partial V}\right)_T = \frac{l_V}{T} \text{ (from eq. 24).}$$

These relations denote that, just as $\partial' Q$ is not an exact differential but $\partial' Q/T$ is, so the thermal capacities are not exact partial derivatives but when divided by T they become so.

Introducing these coefficients in the above differential expressions for dS ,

$$dS = \frac{c_P}{T} dT + \frac{l_P}{T} dP; \quad (25)$$

or

$$dS = \frac{c_V}{T} dT + \frac{l_V}{T} dV. \quad (26)$$

By interpreting these relations for the particular condition of an isothermal process it develops that

$$(\partial S)_T = \frac{l_P}{T} (\partial P)_T; \quad l_P = T \left(\frac{\partial S}{\partial P}\right)_T = -T \left(\frac{\partial V}{\partial T}\right)_P \text{ (from eq. 16);} \quad (27)$$

and

$$(\partial S)_T = \frac{l_V}{T} (\partial V)_T; \quad l_V = T \left(\frac{\partial S}{\partial V}\right)_T = T \left(\frac{\partial P}{\partial T}\right)_V \text{ (from eq. 15).} \quad (28)$$

By similar interpretations for the particular condition of an isentropic process and by utilizing the conclusions of equations 27 and 28 it develops that

$$\frac{c_P}{T} (\partial T)_S = -\frac{l_P}{T} (\partial P)_S; \quad c_P = -l_P \left(\frac{\partial P}{\partial T}\right)_S = T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial P}{\partial T}\right)_S; \quad (29)$$

$$\frac{c_V}{T} (\partial T)_S = -\frac{l_V}{T} (\partial V)_S; \quad c_V = -l_V \left(\frac{\partial V}{\partial T}\right)_S = -T \left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_S. \quad (30)$$

By equating the two expressions for dS of equations 25 and 26 and interpreting for the particular condition of a constant pressure process (or a constant volume process) and again utilizing the conclusions of equation 28 (or 27) it develops that

$$\begin{aligned} c_P - c_V &= l_V \frac{dV}{dT} - l_P \frac{dP}{dT} \\ &= l_V \left(\frac{\partial V}{\partial T} \right)_P \left[\text{or} = - l_P \left(\frac{\partial P}{\partial T} \right)_V \right] \\ &= T \left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial V}{\partial T} \right)_P, \end{aligned} \quad (31a)$$

$$\text{or} = - T \left(\frac{\partial P}{\partial T} \right)_V^2 \left(\frac{\partial V}{\partial P} \right)_T, \quad (31b)$$

$$\text{or} = - T \left(\frac{\partial V}{\partial T} \right)_P^2 \left(\frac{\partial P}{\partial V} \right)_T. \quad (31c)$$

By writing the ratio between equations 29 and 30,

$$\begin{aligned} c_P/c_V = k &= - \frac{\left(\frac{\partial V}{\partial T} \right)_P \left(\frac{\partial P}{\partial T} \right)_S}{\left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial V}{\partial T} \right)_S} = - \left(\frac{\partial V}{\partial T} \right)_P \left(\frac{\partial T}{\partial P} \right)_V \left(\frac{\partial P}{\partial V} \right)_S \\ &= \left(\frac{\partial V}{\partial P} \right)_T \left(\frac{\partial P}{\partial V} \right)_S. \end{aligned} \quad (32)$$

By observing again that equations 25 and 26 are exact differential equations and that c_P/T , c_V/T , l_P/T and l_V/T are exact differential coefficients it becomes evident that the methods of equation 3 may be employed and equations written between the partial derivatives of the coefficients. Thus,

$$\begin{aligned} \left[\frac{\partial}{\partial P} \left(\frac{c_P}{T} \right) \right]_T &= \left[\frac{\partial}{\partial T} \left(\frac{l_P}{T} \right) \right]_P \quad (\text{from equation 25}) \\ &= - \left(\frac{\partial^2 V}{\partial T^2} \right)_P \quad (\text{from equation 27}); \end{aligned}$$

whence

$$\left(\frac{\partial c_P}{\partial P} \right)_T = - T \left(\frac{\partial^2 V}{\partial T^2} \right)_P \quad (33)$$

⁴ These follow from the mathematical relation that, for a three-variable functional relation, $\left(\frac{\partial X}{\partial Y} \right)_Z \left(\frac{\partial Y}{\partial Z} \right)_X = - \left(\frac{\partial X}{\partial Z} \right)_Y$, that is, $\left(\frac{\partial V}{\partial T} \right)_P \left(\frac{\partial T}{\partial P} \right)_V = - \left(\frac{\partial V}{\partial P} \right)_T$.

Also

$$\begin{aligned}\left[\frac{\partial}{\partial V}\left(\frac{c_V}{T}\right)\right]_T &= \left[\frac{\partial}{\partial T}\left(\frac{l_V}{T}\right)\right]_V \text{ (from equation 26)} \\ &= \left(\frac{\partial^2 P}{\partial T^2}\right)_V \text{ (from equation 28);}\end{aligned}$$

whence

$$\left(\frac{\partial c_V}{\partial V}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V. \quad (34)$$

The utility of equations 27 to 34, correlating as they do the thermal capacities and the property functions P , V , T and S , is quite varied. One lies in the opportunity which they afford for cross-checking the consistency of the more difficultly and generally less accurately measurable calorimetric data on the thermal capacities by use of the more accurately determinable relations among the property functions of a fluid.

Equation 28, which is in effect the historical **Clapeyron's equation**, has the particular utility that it enables the computation of the latent heat of vaporization of a fluid solely from data on the change of volume during vaporization and the relation between the pressure and temperature of saturated vapor. Thus, interpreting the latent heat of expansion l_V in terms of the heat supplied during vaporization at constant temperature,

$$l_V = \frac{Q_{\text{vaporization}}}{\Delta V_{\text{vaporization}}} = \frac{H_{fg}}{V_{fg}}, \quad (\text{See Art. 61})$$

whence, from equation 28,

$$H_{fg} = V_{fg} T \left(\frac{dP}{dT} \right).$$

In this expression, since the ratio $\partial P / \partial T$ for a *saturated* vapor is the same for any process whether at constant volume or otherwise, it is written as the general derivative instead of the partial derivative. Physically it is simply the rate of change of the saturation pressure with temperature (the slope of the P - T curve) at the temperature T .

Example 8. At some point in the steam tables check the tabular value of the latent heat of vaporization (H_{fg}) from V_{fg} and the rate of change of saturation pressure with temperature at that point, by use of the Clapeyron equation. (*Note* — Consistent units must be used throughout. If P is taken in pounds per square foot and V in cubic feet per pound mass, in what units will H be obtained?)

Example 9. The *compressibility*, $\frac{(\partial V / \partial P)_T}{V}$, of mercury at atmospheric pressure and 22° C. is -3.91×10^{-6} units per 10^6 dynes (megadynes) per sq. cm., its *coefficient of cubical expansion*, $\frac{(\partial V / \partial T)_P}{V}$, is 182×10^{-6} units per deg. C., its specific volume

is 0.07385 cu. cm. per gr. and its specific heat at constant pressure (c_P , is 0.03322 cal. per deg. C. or $(41.8 \times 10^6 \times 0.03322 =) 1.389 \times 10^6$ ergs per deg. C., all at the above pressure and temperature (Smithsonian Tables). From these data and equation 31c compute the specific heat of mercury at constant volume (c_V).

$$(c_V = 1.389 \times 10^6 - 0.184 \times 10^6 = 1.205 \times 10^6 \text{ ergs per deg. C.} = 0.0288 \text{ cal. per deg.})$$

198. Expressions for dE and dH .—Several useful expressions which correlate the internal energy and enthalpy with the thermal capacities and the property functions follow from equations 6a and 10 (Art. 196) and certain of the above.

Thus, from equations 6a, 26 and 28, in sequence,

$$\begin{aligned} dE &= T dS - P dV \\ &= c_V dT + l_V dV - P dV \\ &= c_V dT + \left[T \left(\frac{\partial P}{\partial T} \right)_V - P \right] dV. \end{aligned} \quad (35)$$

From this last relation, interpreted respectively for constant temperature and constant volume processes,

$$\left(\frac{\partial E}{\partial V} \right)_T = T \left(\frac{\partial P}{\partial T} \right)_V - P, \quad (36)$$

and

$$\left(\frac{\partial E}{\partial T} \right)_V = c_V. \quad (37)$$

From equations 10, 25 and 27, in sequence,

$$\begin{aligned} dH &= T dS + V dP \\ &= c_P dT + l_P dP + V dP \\ &= c_P dT + \left[- T \left(\frac{\partial V}{\partial T} \right)_P + V \right] dP. \end{aligned} \quad (38)$$

From this last relation, interpreted respectively for constant temperature and constant pressure processes,

$$\left(\frac{\partial H}{\partial P} \right)_T = V - T \left(\frac{\partial V}{\partial T} \right)_P, \quad (39)$$

$$\left(\frac{\partial H}{\partial T} \right)_P = c_P. \quad (40)$$

199. The Joule-Thomson Experiment.—A type of physical experiment which is of much value in assisting to ascertain and check the characteristics of a fluid is the *Joule-Thomson* experiment. In this experiment the fluid is caused to flow to, throttle through, and pass from a porous plug, without energy reception or departure as heat and with-

out appreciable velocity or velocity change in any phase of the process. The pressures and temperatures are measured on each side of the plug.

The energy equation for this pure throttling and wholly irreversible process is (Art. 33) $H_1 = H_2$. Alternatively, its characteristic is that $dH = 0$. Introducing this consideration in equation 38, the relation for the process becomes

$$c_P(\partial T)_H = \left[T \left(\frac{\partial V}{\partial T} \right)_P - V \right] (\partial P)_H,$$

or

$$\left(\frac{\partial T}{\partial P} \right)_H = \frac{1}{c_P} \left[T \left(\frac{\partial V}{\partial T} \right)_P - V \right]. \quad (41)$$

The coefficient $(\partial T / \partial P)_H$ in this expression is called the **Joule-Thomson coefficient** (μ). It is evidently to be interpreted as the change of temperature of the fluid per unit fall of pressure under the above irreversible adiabatic throttling conditions. It will be seen in the following article that for a perfect gas, which has the state equation $PV = RT$, there would be no temperature change. With most gases, excepting hydrogen, there is a temperature drop or *cooling effect* at normal temperature levels, but a temperature *rise* at temperature levels above the so-called *inversion temperature*. For most gases the inversion temperature is high but for hydrogen it is at about -80°C .

This experiment is further useful for reducing the readings of a gas thermometer to the energy or thermodynamic scale of temperature and for determining the location of the absolute thermodynamic zero of temperature. Results obtained in this manner closely corroborate those secured by the method considered in Art. 48.

The cooling effect which is characteristic of the actual gases is used regeneratively for the liquefaction of gases by the *Linde* process.

200. Perfect Gas Properties. — It is not the intent in this material to give attention to the detailed procedures by which the physical properties of the various engineering fluids are ascertained and cross-checked and tabular presentations are evolved. It is desired, however, to verify from the foregoing *general* thermodynamic equations certain characteristics which were ascribed to the perfect gas in various earlier chapters. For those refer to Chapter VI, Art. 48, and Chapter IX, Arts. 75 to 78.

The sole fundamental characteristic which would need to be assigned to the perfect gas is that its equation of state shall be

$$\frac{PV}{T} = \text{a constant, } R.$$

For a gas which conforms to that equation of state the following characteristics must follow. These characteristics, which have been ascribed

to the hypothetical perfect gas in earlier articles and which it is now proposed to verify, are:

(a) $E = f(T)$ and is independent of any other state function (the gas conforms to *Joule's "Law,"* Arts. 48 and 75).

(b) $dE = c_V dT$. (Art. 76)

(c) $c_V = f(T)$ and is independent of any other state function. (Art. 76)

(d) $dH = c_P dT$. (Art. 77)

(e) $c_P = f(T)$ and is independent of any other state function. (Art. 77)

(f) $c_P - c_V =$ the constant R . (Art. 78)

In connection with the basic equation of state of the perfect gas observe that, if

$$PV = RT, \quad (A)$$

or $P dV + V dP = R dT,$

then $P(\partial V)_T = -V(\partial P)_T; \left(\frac{\partial V}{\partial P}\right)_T = -\frac{V}{P}; \quad (B)$

$$P(\partial V)_P = R(\partial T)_P; \left(\frac{\partial V}{\partial T}\right)_P = \frac{R}{P} = \frac{V}{T}; \quad (C)$$

$$V(\partial P)_V = R(\partial T)_V; \left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V} = \frac{P}{T}; \quad (D)$$

$$\left(\frac{\partial^2 P}{\partial T^2}\right)_V = \left[\frac{\partial}{\partial T}\left(\frac{R}{V}\right)\right]_V = 0; \quad (E)$$

$$\left(\frac{\partial^2 V}{\partial T^2}\right)_P = \left[\frac{\partial}{\partial T}\left(\frac{R}{P}\right)\right]_P = 0. \quad (F)$$

Verification of the above characteristics of the perfect gas, but wholly by the general thermodynamic equations, is secured as follows:

(a) From equations 36 and (D),

$$\left(\frac{\partial E}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P = P - P = 0.$$

Furthermore,

$$\left(\frac{\partial E}{\partial P}\right)_T = \left(\frac{\partial E}{\partial V}\right)_T \left(\frac{\partial V}{\partial P}\right)_T = 0.$$

It follows that if, as shown, the change of molecular energy with change of volume or with change of pressure is invariably zero if the temperature is unchanged, then E may be a function only of the temperature T .

(b) From equations 35 and (D),

$$\begin{aligned} dE &= c_V dT + \left[T \left(\frac{\partial P}{\partial T} \right)_V - P \right] dV \\ &= c_V dT + (P - P) dV = c_V dT. \end{aligned}$$

Alternatively, from equation 37, if $(\partial E / \partial T)_V = c_V$ and if for the perfect gas constancy or inconstancy of V has no influence on E (by conclusion a), then $\left(\frac{\partial E}{\partial T} \right)_V = \frac{dE}{dT} = c_V$ and $dE = c_V dT$.

(c) From equations 34 and (E),

$$\left(\frac{\partial c_V}{\partial V} \right)_T = T \left(\frac{\partial^2 P}{\partial T^2} \right)_V = 0.$$

As c_V is thus shown to be unchanged by change of volume (and consequently of pressure) so long as the temperature is constant it follows that c_V may be a function of T only.

It is to be observed in this connection that the general equations offer no evidence that c_V should be or need be constant. It is recalled (Art. 82) that physical evidence shows instead that c_V does vary (increase) with temperature.

(d) From equations 38 and (C),

$$\begin{aligned} dH &= c_P dT + \left[-T \left(\frac{\partial V}{\partial T} \right)_P + V \right] dP \\ &= c_P dT + (-V + V) dP = c_P dT. \end{aligned}$$

(e) From equations 33 and (F),

$$\left(\frac{\partial c_P}{\partial P} \right)_T = -T \left(\frac{\partial^2 V}{\partial T^2} \right)_P = 0.$$

As c_P is thus independent of P (or V) it may be a function only of T . It may however vary with T .

(f) From equations 31a, (D) and (C),

$$\begin{aligned} c_P - c_V &= T \left(\frac{\partial P}{\partial T} \right)_V \left(\frac{\partial V}{\partial T} \right)_P \\ &= T \times \frac{P}{T} \times \frac{R}{P} = R. \end{aligned}$$

The several special characteristics of the perfect gas are thus verified. It is interesting to observe further that for an isentropic process with the

perfect gas, from equations 32 and (B),

$$\left(\frac{\partial P}{\partial V}\right)_S = k\left(\frac{\partial P}{\partial V}\right)_T = -k\frac{P}{V}.$$

Therefore $\left(\frac{\partial P}{P}\right)_S = -k\left(\frac{\partial V}{V}\right)_S$ and $\log_e\left(\frac{P_2}{P_1}\right) = -k\log_e\left(\frac{V_2}{V_1}\right)$,

or $\left(\frac{P_2}{P_1}\right)_S = \left(\frac{V_1}{V_2}\right)^k$; $(P_1 V_1^k)_S = (P_2 V_2^k)_S$.

This is equation 13 of Art. 87.

Finally, for the Joule-Thomson experiment and with a perfect gas, from equations 41 and (C),

$$\begin{aligned}\left(\frac{\partial T}{\partial P}\right)_H &= \frac{1}{c_P}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right] \\ &= \frac{1}{c_P}(V - V) = \text{zero}.\end{aligned}$$

For a perfect gas there would thus be neither cooling effect nor warming effect in pure adiabatic throttling.

201. Summary. — For practical utilization of the principles of thermodynamics in the analyses of the various engineering machines it is necessary that physical data be available concerning the magnitude and the manner of variation of a number of properties or *functions of state* of those fluids which may be employed in the machines. Those properties for which data must be available are at least six in number.

Of these properties the pressure, P , is the sole one that without general exception may be measured directly and with reasonable accuracy by convenient engineering instruments. Convenient instruments are also available for ready and reasonably accurate measurement of the temperature, T , when or if the fluid velocity is low, the fluid temperature is not changing rapidly and the temperature of the environs does not differ greatly from that of the fluid. (It is an interesting observation that the science of engineering thermodynamics is in one sense an effort to develop thermodynamic analyses which may be used to advantage under these practical conditions of a general ability to measure only the pressure property and a limited ability to measure the temperature.)

The third fluid property, specific volume, V , is directly measurable in the physical laboratory but may scarcely be regarded as readily measurable in engineering practice and is therefore a property for which the engineer must depend on laboratory determinations for data. This condition is likewise and even more outstandingly true for the remaining

necessary properties or functions entropy, S , internal energy, E and enthalpy, H . Furthermore, even the laboratory determinations of the relative magnitude of these can not be made directly but instead must be accomplished by quite indirect means.

In order to supply this absolutely essential information concerning the properties, and to do so in spite of the very limited character of the direct physical measurements which can be made even in the laboratory, there have been devised various general but highly ingenious relationships through the use of which physical determinations of the magnitudes and the manner of change of the few measurable properties may be employed to determine indirectly the relative magnitudes and manners of change of the remaining unmeasurable ones. It is to those relationships that attention has been given in the foregoing articles.

It will not be attempted to summarize here the wide range of general relations which may be developed. However a few general considerations may perhaps be emphasized with advantage.

The general thermodynamic relations originate wholly from the three basic natural laws which are known as the First and Second Laws of Thermodynamics and the Carnot Principle. To assist in the routine development of the relations an outstandingly useful tool is the mathematical observation that, in any exact functional relation between three mutually dependent variables, the result of successive partial differentiation of one variable with respect to the other two is the same irrespective to the order of differentiation.

Those physical data for which direct laboratory measurements may be made for a given fluid are:

- (a) the simultaneous values and manner of mutual dependence of properties P , T and V at a wide variety of states of the fluid;
- (b) certain of the thermal capacities of the fluid at a variety of states, obtaining the capacity data by calorimetric tests; and
- (c) the Joule-Thomson coefficient, $(\partial T/\partial P)_H$, of the fluid as obtained by throttling experiments.

Data of the first group enable the development of an empirical equation of state which inter-relates the properties P , V and T . By analyses of this equation or by graphic representations of the property relations, used jointly with the Maxwell equations of Art. 196, information is made available concerning the manner of variation of the entropy of the fluid. Also by the use of relations such as those of equations 17, 18, 19 and 20 of Art. 196 further conclusions may be drawn as to the manner of change of the properties E and H (and, if desired, ψ and Z).

Data of the second group may be checked by or may serve to check

data of the first group by the use of relations like those of equations 25 to 34 of Art. 197. They further enable the check or verification of otherwise evolved information concerning the manner of change of the properties E and H , by the use of relations such as those of equations 35 to 40 of Art. 198.

Data of the third group of Art. 199 offer further means for check and verification of information which had otherwise been evolved in aforementioned manners, and it likewise furnishes certain unique information of its own.

The mathematical development of this chapter also forms the point of departure for the applications of general thermodynamics in the fields of chemistry and physics.

Symbols and Abbreviations, Chapter XIX

c_P = specific heat at constant pressure.

c_V = specific heat at constant volume.

d = " differential of."

∂ = " partial differential of."

E = internal energy per unit mass.

f = " function of."

$H = E + PV$, or enthalpy per unit mass.

l_P = latent heat of pressure change.

l_V = latent heat of expansion.

M = a partial derivative.

N = a partial derivative

P = pressure, force per unit area.

$\Psi = E - TS$, or " psi " function, per unit mass.

Q = energy in transition by conduction or radiation, per unit mass.

R = the gas constant (1545/mol. wt., in ft.-lb.-°F. system, and 82.9×10^6 /mol. wt., in cm-gram-°C. system).

S = entropy per unit mass.

T = absolute temperature (Kelvin energy scale).

U = velocity.

V = volume per unit mass.

W = mechanical effects, per unit mass of fluid (also shaft-work prior to eq. 5).

x, y, z = any variables of a three-variable functional relation.

$Z = E + PV - TS$, or " zeta " function, per unit mass; also elevation, ft.

INDEX

A

Absolute pressure, 14, 130, 155, 178, 329
 Absolute temperature, 90, 97, 109, 113
 computation of, 98
 zero of, 90, 98
 Absorption system, refrigeration, 502-506
 Acoustic velocity, 236
 Adiabatic processes, 59
 irreversible, 114, 147, 173, 176
 isentropic, 113, 121
 pseudo-, 215
 reversible, 77, 145, 171, 211
 with air, 171, 173, 176
 with mixtures, 211
 with vapors, 145, 147
 Admission to turbine, 379
 Adsorbent, absorption system, 504, 505
 After-burning, 415
 Air (atmospheric),
 composition of, 392, 393
 humidity of, 201
 isentropic state change of, 211
 molecular weight of, 194, 393
 normal pressure of, 14
 required for combustion, 395
 supplied for combustion, 399-401
 Air compression, 459
 system of refrigeration, 500-502
 Air compressors (see Compressors)
 Air conditioning, 207, 208, 487
 Air ejector, 271
 Air, flow of, 226, 257
 Air horsepower, 467, 473
 Air injection, 441
 Air standard cycles, 432
 Diesel, 438
 Otto, 433
 Airless injection, 441
 American Society of Mechanical Engineers,
 Fluid Meter Research Report, 247
 Power Test Codes, 306, 307, 472
 Steam Properties Research, 129

Ammonia, as refrigerant, 495, 504
 Approach velocity, correction for,
 by correction factor, 225, 240
 by impact tube data, 240, 252, 465
 Aqua, weak and strong, 504
 Areas, significance of,
 on indicator diagram, 329, 475
 on P - V diagram, 60, 78, 137, 175, 181, 284
 on T - S diagram, 113, 115, 139, 285, 340, 450
 Ash, 392
 Atmosphere (see also Air, atmospheric)
 as energy receiver, 74, 99, 104, 290, 487
 Atomic weights, 394
 Atomization, need for, 441
 Atoms, 7
 Auxiliary machinery, 281, 298, 314
 Availability of energy, 72
 Available energy, 72, 80, 100, 103, 423
 meter for, 103
 Avogadro's law, 165, 393
 Axial forces, turbine blading, 372

B

Baker, J. B., 434
 Balance, energy, 312-315, 418
 Bernoulli theorem, 57
 Binary-fluid cycles, 305
 Blading, steam turbine, 350
 diagram efficiency of, 362, 371
 energy analysis of, 356-358, 376
 kinematic analysis of, 358-365
 speed ratio of, 359, 363
 velocity coefficient of, 358
 Bleeding, steam turbine, 294
 Blower (see Fan)
 Boiler, 282
 energy equation for, 54
 Bomb calorimeter, 402
 Brake horsepower, 329, 444
 Brake mean effective pressure, 331, 448
 Bridgman, P. W., 519

British thermal unit (B.t.u.), 34

Brownian movement, 8

C

Callendar, 513

Calorific value of fuels, 401

constant pressure, 403

constant volume, 402

higher, 406-408

influence on combustion temperature, 411

lower, 406-408

of fuel-air mixture, 408, 447

volumetric, 409

Calorimeter,

fuel, 401

bomb (constant volume) type, 402

constant pressure type, 402

energy equation for, 402

throttling (steam), 148

Carbon dioxide,

as refrigerant, 495

in combustion products, 397, 399

meter, 399

Carburetor, 209, 426

Carnot cycle, 76, 278, 293

efficiency of, 79, 288, 305

reversal of, 79, 488

Carnot Principle, 83, 94, 220, 299, 489, 515

Carnot, Sadi, 83

Carrier, W. H., 203

Centrifugal compressor, 460

characteristic of, 173, 468, 479

energy equation for, 480

Characteristic equation, 128

for imperfect gases, 513

for perfect gases, 156

Chemical energy, 11, 390, 402, 411

losses in combustion, 415

Chemical equations, combustion, 393, 413

Chemical equilibrium, 413

Circulating water, 494

Clapeyron's equation, 526

Clausius, 85

Clearance,

linear, 377, 381

ratio, 325

volume, 325, 427, 474

Coefficient,

blade velocity, 358

of discharge, 245, 268

of performance, 488, 491, 497

of velocity, 244, 262

pressure, 94-96

volume, 94-96

Combustion,

chemical equations for, 393, 395, 413

constant pressure, 403, 426

constant volume, 403, 426

dual, 442

energy equations for, 402, 411

for energy release, 11, 72, 390

losses in, 415, 416

products of, 394, 395, 398

analysis of, 399

dewpoint of, 404, 407

moisture in, 403

water recovery from, 207

temperature developed by, 410-415

Compounding, 343

Compression, air,

irreversible adiabatic, 175, 480

isentropic, 468, 469

with inter-cooling, 470, 471

isothermal, 170, 464-467

polytropic, 180, 476

Compression ratio, 433, 437, 440

effect on engine efficiency, 434, 439

effect on temperature, 436, 441

Compressors, 459

centrifugal, 173, 468, 479, 480

energy equation for, 461-464

performance standards for, 463, 472

positive displacement, 460

reciprocating, 460

mechanical performance of, 473

volumetric efficiency of, 477

refrigerating cycles, 494, 496

Condenser, 208, 285, 494

Condition curve, turbine, 381

Conduction, of heat, 23

Conservation of Energy, 32, 44

Constant, of gases, 156, 164

equivalent, for mixtures, 193

universal, 165

Constant entropy process (see Isentropic or Reversible adiabatic)

- Constant pressure processes,
 with gases, 169
 with gas-vapor mixtures, 207
 with vapors, 142
 Constant temperature processes (see Isothermal)
 Constant volume processes,
 with gases, 169
 with gas-vapor mixtures, 206
 with vapors, 142
 Critical pressure,
 for flow, 231, 234, 235
 of vapors, 136
 Critical ratio,
 for gas, 236
 for steam, 260, 265
 Critical temperature of steam, 137
 Cumulative enthalpy (heat) drop, 383
 Curtis stage, or turbine, 371
 Cut-off, point of, 325
 ratio, 325, 331
 variation, effect of, 333, 342
 Cycles, 73
 air standard, 432, 433, 438
 Carnot, 75-80, 278, 488
 Diesel, 426, 437-442
 dual-combustion, 442
 efficiency of reversible, 90, 98
 Ericsson, 76
 four-stroke, 424, 427
 fluid, 72, 278
 general energy equation of, 74
 heat engine, 79, 278
 heat pump, 80, 488
 machine, 72, 278
 multiple-fluid, 292, 302
 non-flow, 73
 Otto, 426, 432-436
 Rankine, 279-290
 refrigeration, 492
 regenerative, 76, 292
 reheating, 292, 300
 reversible, 75, 98
 steady-flow, 72
 Stirling, 76
 temperature engine, 98, 99
 two-stroke, 428, 436
 Cylinder,
 initial condensation in, 338
 steam engine, actions in, 337
 steam turbine, 350
- D
- Dalton's law, 192
 David, W. T., 414
 Degrees of superheat, 127
 deLaval turbine, 367
 Density, 14
 Derivatives, 510
 Detonation, 426, 435, 440
 Dewpoint, 202
 determination of, 203-205
 of combustion products, 404, 407
 Diagram efficiency, blading, 362, 371
 Diagrams, 135
 areas on (see Areas)
 enthalpy-entropy ($H-S$), 112
 of gases, 182, 183
 of steam, 139
 indicator, 177, 328, 474
 Mollier (see enthalpy-entropy)
 pressure-volume, 61
 of gases, 181
 of steam, 136
 temperature-entropy ($T-S$), 112, 113
 of gases, 182, 183
 of steam, 138
 velocity, turbine, 352, 358, 370, 375
 Diaphragm, impulse turbine, 368
 Diatomic gases, specific heat of, 182
 Diesel cycle engine, 426, 437
 air excess in, 439, 440
 air standard analysis of, 438
 compression ratios in, 437, 439, 441
 expansion ratios in, 437, 439, 441
 injection in, 441
 mean effective pressures in, 448
 $T-S$ analyses of, 451
 Differential coefficient, 510
 Differentials, exact, 511
 inexact, 512, 513
 partial, 511
 total, 511
 Diffuser, 248, 269
 Diphenyl oxide, 304
 Discharge, coefficient of, 245, 268
 mass-rate of, gas, 255, 256
 vapor, 258, 265, 266
 Displacement, piston, 325, 448, 474
 Dissociation, 413
 Double-acting engine, 323
 Driving force, blading, 364, 372, 379

Dryness fraction, 128
 Dual-combustion cycle, 442, 452
 Dynamometer, 307, 329

E

Effects, mechanical, 63, 284, 462, 516
 Efficiency, 472
 compression, 472, 473
 diagram, blading, 362
 ejector, 272
 engine, 306
 fan, 473
 heat engine, 79, 90, 98
 internal combustion engine, 431, 452
 Diesel cycle, 438-440
 dual combustion cycle, 442
 Otto cycle, 434-436
 mechanical, 330, 444, 475
 nozzle, 241, 244, 261
 over-all, 472
 Rankine cycle, 287, 288
 ratio, 306
 regenerative cycle, 297
 reheating cycle, 301
 reversible cycles, 83-87, 90, 98
 steam power plant, 311
 thermal, 79, 287, 309
 type, 305
 volumetric, 446, 477-479
 Ejector, 269
 Electrical energy, 23, 44
 Electromagnetic waves, 23, 25, 44
 Electrons, 7
 Elements, chemical, 7
 of reaction turbine, 374, 377
 Energy, 1
 availability of, 72
 available, 71, 72, 80, 100, 103, 423
 balance, 312-315, 418
 chemical, 11, 390, 402, 411, 415
 conservation of, 32
 classifications of, 2, 11, 25, 43
 entropy of, 116
 free, 518, 519
 functions, 519
 internal, 8, 10, 16, 48, 512
 of gas, 157, 159
 of liquid, 133
 of vapors, 131

Energy,
 internal, of vaporization, 131
 intrinsic, 10
 kinetic, 4, 16, 47, 226, 350
 mechanical, 6
 molecular, 9, 10
 potential, 2, 16, 47, 51
 stored, 2, 278, 390
 sub-atomic, 11
 temperature scale, 89, 90
 transitional, 2, 25
 unavailable, 71, 72, 80, 100, 103, 108
 units of, 33
 Energy equations, 42
 for blading, 354, 376
 for boiler, 54
 for combustion, 411
 for compressors, 56, 461-471
 for fuel calorimeters, 402
 for internal combustion engine, 428, 430
 for nozzles, 55, 224, 353, 376
 for Rankine cycle, 281-288
 for refrigeration cycles, 497, 502
 for steam engine, 54
 for throttle, 55
 general, 44, 58, 515
 non-flow, 57, 515
 steady-flow, 46-51, 53, 515
 Engine, 79, 283
 Carnot, 76, 279
 efficiency, 306
 internal combustion (see Internal combustion engine)
 temperature, 99
 thermal efficiency of, 79
 steam (see Steam engine)
 Engineering thermodynamics, 1, 11, 122, 508
 Enthalpy, 52
 as a property, 52, 512, 518
 of gases, 160, 162, 529
 of liquids, 134
 of vapors, 130, 132, 135
 Enthalpy-entropy diagram, 112
 of gases, 182, 183
 of liquids and vapors, 139
 Entropy, 108
 as a coördinate, 112, 139
 as a property, 111, 220, 513

Entropy,

- change of, 108, 109
- computation of its change, 109
- inadequate definition of, 111
- of energy, 116, 220, 316
- of gases, 163
- of liquids, 133
- of vapors, 130, 132
- unit of, 108

Equations,

- characteristic (see Equation of state)
- energy (see Energy equations)
- of state, 128, 156, 509, 511, 513

Equilibrium,

- chemical, 412
- in expansion of vapor, 360
- thermal, 126

Ericsson cycle, 76**Evaporator, 314, 493****Excess air, 398, 440****Expander cylinder, 500****Expansion ratio,**

- internal combustion engine, 433, 437, 439
- nozzles, 262
- steam engine, 325

Expansion valve, 492**Exponential relations, 178****F****Fan, 459, 460**

- air horsepower of, 467
- efficiency of, 466
- energy equation of, 473

Feed-water heating,

- Rankine cycle, 293, 298
- regenerative, 292-300
- stage, 294

Felbeck, G. T., 166, 212**First law, of Thermodynamics, 33, 220**

- energy balance, 312

Fliegner's equation, 234**Flow of gases,**

- through nozzle, 223-236, 241-245
- through orifice, 237-239, 245-247
- through Venturi meter, 248
- summarized formulas for, 253-256

Flow of vapors, 257-269

- supersaturation in, 260, 263-267

Flow-work, 48**Flow nozzle, 222, 245****Four-stroke cycles, 424, 427****Free air, 446, 467, 469, 478****Free energy, 518, 519****Friction,**

- fluid, 30, 119, 242, 260, 306, 357
- horsepower, 330, 444, 475
- mechanical, 21, 28, 30, 119, 306

Fuels, 72, 390

- forms and characteristics of, 391
- heating values of, 401-410

Functions,

- energy, 519
- point, 511
- property, 519
- state, 13

Fundamental interval, 94**G****Gage pressure, 14****Gas, perfect, 94, 155**

- enthalpy of, 162, 170, 185, 529
- entropy of, 163, 185
- internal energy of, 157, 162, 185
- mixtures, (see Mixtures)
- pressure coefficient of, 94
- P - V - T relation for, 155, 185
- specific heats of, 160, 161, 162, 166, 182
- state changes of, 169-180, 186
- volume coefficient of, 94

Gas constant, 156, 164

- of gas mixtures, 193
- universal, 165

Gases, imperfect, 124, 513**Gibbs, Willard, 53, 141, 518, 519****Goodenough, G. A., 166, 182, 212, 414, 434, 436, 441****Governing, steam engine, 341****H****Head, Bernoulli equation, 57**

- temperature, 118
- thermal, 53
- static, 466
- velocity, 466

Heat, conducted, 23
 definition of, 24
 latent, 130, 131, 523, 526
 radiation, 24
 specific, 35, 513
 Heat balance, (see Energy balance)
 Heat content, 52, 130, 518
 Heat drop, 383
 Heat engine, 79
 as temperature engine, 99
 ideal efficiency of, 90, 98
 Carnot Principle for, 83-87
 practical aspects of, 99
 reversible, 75, 278
 Carnot cycle for, 76-80, 278, 279
 Heat pump, 80, 487
 Heat rate, 311
 Heaters, feed-water,
 Rankine cycle, 293, 298
 regenerative cycle, 292-300
 Heating value of fuels (see Calorific)
 Helium, 95, 97, 165
 Horsepower, 35
 air, 467, 473
 friction, 330, 444, 475
 indicated, 307, 330, 444, 475
 per ton of refrigeration, 491
 shaft, (or brake), 307, 331, 444, 475
 Horsepower-hour (hp-hr.), 35, 307
 Hotness, 74, 97
 Humidity, 199, 201
 absolute, 201
 determination of, 203-205
 relative, 201, 202
 specific, 201
 Hydrocarbons, 392, 397, 399
 Hydrogen, 95, 97, 165, 167
 in fuels, 403, 407
 Hygrometer (or psychrometer),
 dewpoint type, 203
 wet and dry bulb type, 203
 charts and computations for, 205
 energy equation for, 204

I

Ice point, 96
 Ignition, 426, 437
 Impact, tube, 240, 249
 pressure, 250, 465, 467, 468

Impact,
 temperature, 250, 465, 467, 468
 Imperfect gas, 124, 513
 Incomplete combustion, 398, 411, 415
 Incomplete expansion, 332
 Indicated horsepower, 307, 330, 444,
 475
 Indicated m.e.p., 330, 448, 475
 Indicator diagram, 177, 328, 475
 Initial condensation, 338
 Initial velocity (see Approach velocity)
 Injection, 426, 441, 437
 air, 441
 solid, 441
 Inter-cooling, air compression, 468, 470
 Internal combustion engine,
 advantages of, 423, 452
 capacity factors for, 443-449
 characteristics of cycles, 424-428
 compression ratios for, 436, 440
 Diesel cycle analyses, 437-442
 dual-combustion cycle, 442, 443
 efficiency of,
 expression for ideal, 431
 mechanical, 445
 values of ideal, 435, 436, 439-441
 volumetric, 446
 energy equations of, 428, 430
 four-stroke cycle, 424, 427
 Otto cycle analysis, 432-436
 T-S analyses of, 449-453
 two-stroke cycle, 428, 436
 Internal energy, 8, 10, 16, 512
 of gases, 157, 159, 162, 529
 of liquids, 133
 of vapors, 131, 135
 Intrinsic energy, 10
 Irreversibility,
 criteria of, 26
 mechanical, external, 30, 119
 internal, 30, 119
 thermal, external, 32, 117, 299
 internal, 32, 117, 299
 Irreversible adiabatic process,
 with gases, 173
 with vapors, 147
 Isentropic process,
 with gases, 171
 with gas-vapor mixtures, 211
 with vapors, 144

Isothermal, compression, 464
 processes, 60
 with gases, 170, 176
 with gas-vapor mixtures, 209
 with vapors, 137

J

Jackets, water, 55, 428, 460, 464, 475
 Joule, Dr., 34
 Joule, definition of, 35
 Joule's equivalent, 34, 516
 Joule's experiment, 158
 Joule's law, 157
 Joule-Thomson coefficient, 528
 Joule-Thomson experiment, 159, 176, 527

K

Kelvin, Lord, 90
 Kelvin temperature scale, 90, 98
 realization of in thermometry, 91-97
 Kinematics of turbine blading, 358-363
 Kinetic energy, 2, 4, 6, 47, 241, 269, 353, 467
 Kilowatt-hour (kw-hr.) definition of, 35, 307

L

Labyrinth packing, 369
 Latent heat, 130, 523, 526
 Leaving losses, turbine, 354
 Linde process, gas liquefaction, 528
 Liquids,
 in equilibrium with vapor, 126
 properties of,
 saturated, 127
 subcooled, 127, 133
 super-pressure, 133
 Lower calorific value, 406-408

M

Marine power plant, 314, 423
 Mass-rate of flow, nozzle or orifice, 232-239, 241-247, 255-269
 Matter, composition of, 7
 Maximum discharge, orifice, 234, 237

Maxwell's relations, 519, 520
 Mean effective pressure, 330, 448
 brake, 331, 448
 indicated, 330, 448, 475
 shaft (see brake)
 Mechanical Effects, 63, 284, 462, 516
 Mechanical efficiency, 330, 444, 475
 Mechanical energy, 6
 Mercury-water cycle, 305
 Mixtures, 124, 125, 190
 gas, 190-195
 analysis mutations of, 191, 192
 gas constant, of, 193
 partial pressures in, 192, 193
 specific heats of, 194, 195
 state changes of, 195
 gas-vapor mixtures, 196-215
 dewpoint of, 202, 203-205, 404
 examples analyzing, 196-198
 humidity of, 199, 201-205
 processes with,
 constant pressure, 207
 constant volume, 206
 isentropic, 211
 isothermal, 209
 saturated, 198-200
 Mol mass, 182, 394
 Mol volume, 165, 394
 Molecular energy, 9, 10
 losses of, 416
 Molecular weights, 165, 394
 of air, 194, 393
 of gas mixtures, 194
 Molecules, 7
 Mollier diagram (see Enthalpy-entropy diagram)
 Moss, S. A., 234, 251
 Multiple-expansion engines, 343
 Multiple-fluid cycles, 302
 Multi-staging,
 compressors, 460, 461, 468, 470
 impulse turbine, 367

N

Napier's equation, 265
 Nature, energy stores in, 11, 72, 390
 Nitrogen, 96, 165, 167
 in atmosphere, 392-394
 in combustion products, 395-401

Non-expansion engine, 336
 Non-flow processes, 45, 57
 applications, 59
 energy equation for, 58
 Nozzle, 220
 convergent, 223, 232
 divergent, 223, 232
 efficiency of, 241, 243, 261
 energy equation for, 224
 expansion ratio in, 262
 form characteristics of, 220
 for gas flow, 230-232
 for vapor flow, 259
 ideal flow in, 226
 mass-rate of flow in, 232-243, 255,
 259, 261, 264
 steam turbine, 350, 353
 supersaturation in, 260, 263-267
 throat, 223, 236, 259
 velocities produced in, 225, 227, 228,
 236, 243, 258, 262

O

Orifice, 220
 actual flow through, 245-247, 268
 characteristic flow conditions, 238, 268
 discharge coefficients of, 245, 268
 ideal flow in, 238, 239, 258, 268
 rounded-entrance, 223, 245, 268
 sharp-edged, 223, 246, 268
 Orsat apparatus, 396
 Otto cycle, 426
 air standard analysis for, 433
 compression ratios in, 435
 efficiency of, 434-436
 mean effective pressure of, 448
 T-S analysis of, 450, 451
 Over-expansion, 262
 Oxygen, 165, 167
 in atmosphere, 390, 392-395
 in combustion products, 397, 399

P

Parsons turbine, 380
 Partial pressure, 192
 in gas mixtures, 192, 193
 in gas-vapor mixtures, 198, 202
 of vapor in mixtures, 137, 140, 155,
 165, 197

Partington and Shilling, 166, 167, 182
 Perfect gas, 94, 97, 155, 528
 constant of, 156, 165
 constant pressure process, 169
 constant volume process, 169
 enthalpy of, 160, 529
 entropy of, 163
 internal energy of, 157, 169, 529
 irreversible adiabatic process, 173
 isentropic process, 171
 isothermal process, 179
 polytropic process, 177
 pressure coefficient of, 94
 P-V-T relation of, 155, 528
 specific heats of, 160-162, 166, 179,
 529
 summary of process relations, 168
 summary of property relations, 185
 temperature scale, 94, 96
 throttling process with, 176, 531
 volume coefficient of, 94
 Performance, coefficients of,
 ir compression system, 502
 Carnot heat pump, 488
 over-all, absorption system, 503
 vapor compression system, 497
 versus hp. per ton, 491
 Perpetual motion machines, 29
 Phase, 12, 136
 Phase rule, 141
 Piston displacement, 330, 447, 448, 474,
 478
 Piston speed, 331, 445
 Pitot tube, 253
 Point function, 511
 Poisson's equation, 171
 Polytropic process, 115, 177
 Potential energy, 2, 16, 47, 51
 Potential, thermal, 53, 519
 Pound, dual use of name, 5
 Power, units of, 35
 Power plant cycles, 278-320
 multiple-fluid, 302
 Rankine, 279
 regenerative, 292
 reheating, 300
 Pressure, 13, 512
 absolute, 14
 critical,
 for flow, 231, 235

Pressure, critical,
 for vapors, 136
 gage, 14
 impact, 250, 465, 467, 468
 mean effective, 330, 448, 475
 partial, 192, 193, 197, 202
 saturation, 127
 unit of, 14
 Pressure coefficient, 94–96
 Pressure staging, turbine, 367
 Principle, Carnot, 83
 of Energy Conservation, 32, 44
 Processes, 19
 adiabatic, 59, 77, 79, 145, 147, 171,
 173, 211
 constant pressure, 60, 144, 169,
 207
 constant volume, 59, 142, 169, 206
 isentropic, 108, 144, 171, 211
 isothermal, 60, 170, 209
 polytropic, 115, 177
 specifications of, 141, 168
 throttling, 55, 147, 176, 527
 with gas mixtures, 195
 Products of combustion, 394, 395,
 398
 dewpoint of, 405, 407
 losses in, 415
 moisture in, 403
 Properties, 12, 124, 512, 517
 as point functions, 511
 entropy as one of the, 111, 513
 of gases, summarized, 185
 of saturated liquids, 128–130
 of saturated vapor, 128–130
 of subcooled liquids, 133
 of superheated vapors, 132
 of wet vapors, 134
 thermodynamic, 512, 517
 Property functions, 519
 Proton, 7
 Psi-functions, 518
 Psychrometers (see Hygrometers)
 Pumps, air, 209, 459
 feed-water, 281, 287
 regenerative cycle, 295, 298, 300
 heat, 80, 487
 $P dV$, 63, 284, 462, 516
 PV product, 48, 52
 P - V diagrams, 136, 181

Q

Quality, liquid-vapor mixture, 128, 134

R

Radiation, heat, 23
 by electromagnetic waves, 23
 Rankine temperature scale, 90, 98
 Rankine vapor cycle, 279–291
 description of, 279–285
 efficiency of, 287–291
 handicaps of, 291, 292
 Rateau type turbine, 368
 Ratio of compression (see Compression)
 Ratio of expansion, (see Expansion)
 Reaction, per cent of, 374
 Reaction turbine, 373–381
 Receiver, 74
 atmosphere as, 74, 99, 104, 290, 487
 Refrigerants, 495, 500, 504
 Refrigerating capacity, 491
 Refrigerating effect, 80, 488
 Refrigeration,
 absorption system, 502–506
 air compression system, 500–502
 as reversed heat engine operation, 80,
 487
 Carnot cycle of, 488
 vapor compression system, 492–499
 Regenerative cycle, 76, 292–300
 Regions (vapor property diagrams)
 gas, 137
 saturation, 136, 140
 subcooled liquid, 136, 138, 140
 superheat, 136, 140
 Reheat factor, 383
 Reheating cycle, 300, 301
 Reheating-regenerative cycle, 302
 Reversibility,
 criteria of, 26
 mechanical, 27
 thermal, 30, 117, 298
 Reversible cycles, 75, 98, 488
 as availability meter, 103
 efficiency of, 99
 Rho (ρ), 359
 Ricardo, 434
 Rotor, steam turbine, 350, 377

S

Saturated liquid, 127, 151
 liquid line, 136, 138, 139
 mixture, 198
 vapor, 127, 151
 vapor line, 136, 138, 140
 Saturation pressure, 127
 region, 136, 138
 temperature, 127, 151
 Scavenger blower, 428
 Second Law, of Thermodynamics, 85,
 220, 299, 513
 energy balance, 315
 Shaft horsepower, 307, 329, 444, 475
 Shaft-work, 20, 50
 Shilling, 166, 167, 182
 Simple engine, 343
 Solid injection, 441
 Source (of energy), 74
 Specific heat, 35, 523
 at constant pressure, 36, 161, 523
 at constant volume, 36, 160, 523
 for a polytropic, 179
 of gas mixture, 194, 195
 of gases, 166, 182
 ratio, 162
 representation on T - S coordinates, 115
 Specific volume, 14, 512
 State, designation of, 12, 141
 equation of, 128
 function, 13
 State changes (see Processes)
 Steady-flow process, 45
 applications, 53-56
 energy equation for, 46-52, 53
 Steam,
 rate, 308
 saturated, 129
 subcooled, 267
 superheated, 127, 132
 supersaturated, 260, 263, 267, 384
 tables, 129
 wet, 128, 134
 Steam engine, 283
 compound, 343
 governing, 341
 multiple-expansion, 343
 reciprocating, 323
 simple, 343

Steam engine,
 uniflow, 344
 Steam point, 96
 Steam power plant cycles, 278-321
 compared with Int. Comb. engine, 453
 energy balance in, 311-316
 irreversibility in, 299
 multiple-fluid, 302
 Rankine, 279-290
 regenerative, 76, 292
 reheating, 300, 301
 reheating-regenerative, 302
 Steam turbine, 283, 295, 323, 350
 condition curve, 381
 Curtis, 371
 deLaval, 367
 diaphragm, 368
 energy relations in, 353-355
 impulse, 351, 356-372
 pressure staging in, 367
 reaction, 351, 373-381
 re-entry type, 372
 reheat in, 383
 supersaturation in, 384
 velocity staging, 369
 Stirling cycle, 76
 Subcooled liquid, 127, 133, 151
 region, 136, 138, 140
 steam, 267
 Supercharger, 446
 Superheat, degrees of, 127
 region, 136, 140
 Superheated vapor, 127, 132, 151
 Super-pressure liquid, 133
 Supersaturation, 260, 263, 267, 384
 System, 42

T

Tables of,
 energy forms, in transition, 25
 stored, 11
 summarized, 43
 fuels and fuel properties, 391, 404,
 409, 410
 gas properties, 165, 167, 185, 186
 vapor properties, 129, 131, 132
 Temperature, 14
 absolute, 90, 97
 absolute zero of, 90, 98
 as an efficiency influence, 87, 90, 99

Temperature,
 as a property, 15, 512
 Centigrade scale of, 98
 critical, 137
 energy scale of, 89
 Fahrenheit scale of, 98
 impact, 250
 Kelvin scale of, 90, 98
 obtainable by combustion, 413
 perfect gas scale of, 94, 96
 Rankine scale of, 90, 98
 realization of Kelvin scale, 91-97
 saturation, 127, 151
 thermodynamic scale of, 88
 Temperature engine, 99
 Temperature-entropy diagram, 113
 of gases, 182, 183
 of vapors, 138
 Thermal capacities, 36, 522, 523
 efficiency, 79, 287, 309
 head, 53
 irreversibility, 32, 117, 299
 potential, 53, 518, 519
 Thermodynamics,
 engineering, 1, 11, 122, 508
 First Law of, 33, 512, 515
 mathematical treatment of, 509
 Second Law of, 85, 220, 299, 513, 515
 Thermodynamic equations, 508
 Thermometers, 92
 calibration of, 92
 hydrogen, 91
 Throat, of nozzle, 223
 mass-rate of flow in, 236
 velocity in, 236
 Throttling calorimeter, 148
 Throttling process, 55, 147, 176, 527
 Total heat, 53, 130, 518
 Traverse, impact tube, 252
 Turbine, internal combustion, 424
 steam (see Steam Turbine)
 Type efficiency, 305

U

Under-expansion, 262
 Uniflow steam engine, 344
 Universal gas constant, 165

V

van der Waals, 513

Vapor, 124
 in equilibrium with liquid, 126
 low pressure, 137, 140, 155, 156, 164,
 197, 203, 204
 saturated, 127, 151
 superheated, 127, 132, 151
 wet, 128, 134, 151
 Vapor compression system, refrigeration,
 coefficient of performance, 497
 features of, 492-496
 wet compression in, 499
 Velocity, 4, 46
 absolute, 4, 352
 acoustic, 236
 approach, 225, 240, 253
 coefficient, blading, 358
 nozzle, 244, 262
 nozzle, 225, 236, 243
 relative, 4, 352
 throat, 236
 Vena contracta, 246
 Venturi meter, 248
 Volume, coefficient, of gas, 94, 95
 specific, 14
 Volumetric efficiency,
 internal combustion engine, 446
 reciprocating compressor, 477-479

$\bar{W}$

Water, properties of, 133
 Wet bulb, 203
 Wet vapor, 128, 134, 151
 Work, 20
 flow, 48
 with adiabatic processes,
 irreversible, 147, 175
 reversible, 145, 173
 with constant pressure process, 144,
 170, 207
 with isothermal process, 170, 211
 with polytropic process, 179

Z

Zero, of temperature, 90, 98
 of enthalpy, 130, 184
 Zeta-function, 519

